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Optimal data processing in the ballistic laser gravimeter under the effect of correlated interference

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Abstract

In order to develop optimal data processing algorithms in ballistic laser gravimeters under the effect of correlated interference, the method of generalized least squares is applied. In this case, to describe the interference, a mathematical model of the autoregression process is used, for which the inverse correlation matrix has a band type and is expressed through the values of the autoregression coefficients. To convert the "path-time" data from the output of the coincidence circuit of ballistic laser gravimeters to a process uniform in time, their local quadratic interpolation is used.

Algorithms for data processing in a ballistic gravimeter, developed on the basis of a method of weighted least squares using orthogonal Hahn polynomials, are considered. To implement a symmetric measurement method, the symmetric Hahn polynomials, characterized by one parameter, are used.

The method of mathematical modelling is used to study the gain in the accuracy of measuring the gravitational acceleration by the synthesized algorithms in comparison with the algorithm based on the method of least squares. It is shown that auto seismic interference in ballistic laser gravimeters with a symmetric measurement method can be significantly reduced by using a mathematical model of the second-order autoregressive process in the method of generalized least squares. A comparative analysis of the characteristics of the algorithms developed using the method of generalized least squares, the method of weighted least squares and the method of ordinary least squares is carried out.

Keywords: auto seismic interference; weight functions; correlated interference; polynomials; autoregressive process; gravitational acceleration.

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Introduction

One of the important factors that determine the accuracy of measuring the gravitational acceleration (GA) by ballistic gravimeters is the effect of seismic interference. In the classical works [1–3], devoted to this problem, the main attention is focused on measuring the GA using the method of ordinary least squares (OLS) when processing the data on the motion of the test body (TB). At the same time, as shown in [4–9], the use of the method of generalized LS (GLS) and the method of weighted LS (WLS) is often more promising for the GA measurements.

Practical application of these LS extensions requires knowledge of the statistical properties of the interference (a priori difficulty). In addition, the implementation of GLS in data processing is hampered by significant computational difficulties associated with the need to invert correlation matrices of large dimension. These difficulties can be overcome by using parametric time series models, in particular, autoregressive processes, in which it is assumed that the readings are equidistant in time. In order to take advantage of this opportunity, it is required to ensure the equidistance of the "path-time" readings at the input of the processing device.

The purpose of this work is to develop interference-immune methods for measuring the GA by ballistic laser gravimeters (BLGs) using modern data processing methods.

Additional opportunities for developing interference-immune methods for measuring the BLG GA arise in the case of uniform spaced "time-path" readings, when well-developed time series models can be used to describe the interference. To convert non-uniform "path-time" data from the output of existing BLG matching schemes to a series uniform in time, an approach based on local quadratic data interpolation is used [10].

As known [1, 3], the ballistic method of absolute measurements of the GA is based on the description of the motion of the TB in the vertical direction by the polynomial

$$x(t) = a_0 + a_1 t + a_2 t^2, \qquad (1)$$

where parameters a_0 and a_1 have a physical meaning in respect to the path and speed of the TB at the initial moment of time, and parameter a_2 is half of the value of the GA being measured.

Taking into account (1), the problem of measuring the GA is solved by the methods of polynomial regression analysis (RA), in which the GA value is estimated as twice the value of the second-order regression coefficient $\hat{g} = 2\hat{a}_2$.

Problem statement and methods of regression analysis

It is assumed that at the moments of time t_k, $k=0,\ldots,K$-1 data are recorded, which is the sum of the signal $x(t)$ in the form of a polynomial (1) and interference $\xi(t)$

$$z(t) = x(t) + \xi(t). \qquad (2)$$

It is required to find unbiased linear estimates of the coefficients a_0, a_1, a_2 with minimum standard deviations relative to the true values. It should be noted that in problems of gravimetry, only the coefficient a_2 is of practical importance.

Let us write the observation equation (2) in matrix form [9, 11]

$$\vec{\boldsymbol{Z}} = \boldsymbol{X}\,\vec{\boldsymbol{a}} + \vec{\xi},$$

where vectors

$$\vec{Z} = (z(t_0),)\ldots,z(t_{K\text{-}1}))^{tr}, \vec{\xi} = (\xi(t_0),)\ldots,\xi(t_{K\text{-}1}))^{tr},\ \vec{a} = (a_0, a_1, a_2)^{tr},$$

and the regression matrix

$$X = \begin{bmatrix} 1 & t_0 & t_0^{\,2} \\ 1 & t_1 & t_1^{\,2} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots \\ 1 & t_{K-1} & t_{K-1}^{\,2} \end{bmatrix}.$$

For signal processing in BLG, three main methods of regression analysis of data can be used [9, 11]: 1) OLS; 2) GLS; 3) WLS. When using them, the estimate of the vector of regression parameters is formed in accordance with the following expressions [11]:

1) OLS estimate

$$\hat{\vec{\boldsymbol{a}}} = (\boldsymbol{X}^{tr}\boldsymbol{X})^{-1}\boldsymbol{X}^{tr}\vec{\boldsymbol{Z}};$$

2) GLS estimate

$$\hat{\vec{\boldsymbol{a}}} = (\boldsymbol{X}^{tr}\boldsymbol{R}^{-1}\boldsymbol{X})^{-1}\boldsymbol{X}^{tr}\boldsymbol{R}^{-1}\vec{\boldsymbol{Z}}, \qquad (3)$$

where R is the interference correlation matrix;

3) WLS estimate

$$\hat{\vec{\boldsymbol{a}}} = (\boldsymbol{X}^{tr}W\boldsymbol{X})^{-1}\boldsymbol{X}^{tr}W\vec{\boldsymbol{Z}},$$

where W is the diagonal weight matrix.

Model of autoregression processes

Let us consider the advantages of describing the additive interference $\xi(t)$ by an autoregressive model (AR model) [12].

The autoregressive process is one of the most widely used models and has a form of a time series, which in many applied problems is generated from equidistant readings of continuous time processes: $\xi_k = \xi(k\Delta_t),\ k=0,1\ldots$

AR process is a time series model in which the values of the time series at a given moment are linearly dependent on the previous values of the same series. The values of AR process of order p adhere to the equation [12]

$$\xi_k = \varphi_1\xi_{k\text{-}1} + \varphi_2\xi_{k-2} + \ldots + \varphi_p\xi_{k\text{-}p} + \eta_k,$$

where $\varphi_1,\ldots,\varphi_p$ are the autoregressive coefficients; $\eta_k, k \in Z$ is the stationary uncorrelated random sequence with variance σ^2.

Let us consider in more detail the second-order AR model, which describes the processes in oscillatory systems exposed to random influences.

The power spectral density of the AR process of the second process is described by the expression [12]

$$p(f) = 2\sigma^2 |H(f)|^2, \quad 0 \le f \le 1/2,$$

where function

$$|H(f)| = \frac{1}{\left|1 - \varphi_1 e^{-j2\pi f} - \varphi_2 e^{-j4\pi f}\right|}$$

can be interpreted as the frequency response of the filter that forms the AR process from white noise.

If the condition $\varphi_1^2 + 4\varphi_2 < 0$ is satisfied for the AR coefficients, then the AR process is a pseudo-periodic one. In this case, the frequency response of the shaping filter and, accordingly, the spectral power density of the second-order AR process has a single maximum at the frequency $\nu_0 = (2\pi)^{-1}\arccos\,(0.5|\varphi_1| / \sqrt{-\varphi_2})$.

AR processes have a remarkable property: the inverse correlation matrix of their readings (sometimes of a very large dimension) has a band structure and is expressed in a known way through the values of the autoregressive coefficients [13]. The band structure of the inverse interference correlation matrix greatly simplifies the implementation of the GLS algorithm for estimating the regressive coefficients (3). To do this, one should estimate the coefficients of the AR model and avoid inverting high dimensional correlation matrices, which are often ill-conditioned.

Auto seismic interference and BLG mechanical system

The result of the GA measurement by ballistic gravimeters is influenced by the following additive interference [7, 8]: 1) external seismic interference; 2) auto seismic interference; 3) sampling interference.

A special danger for BLGs that implement a symmetric measurement method is represented by auto seismic interference that arise when pushing the TB. In this case, one speaks of the auto seismic effect, which can lead to systematic errors in the GA measurement. In gravimeters with an asymmetric method for measuring the GA, auto seismic interference also appear when the TB is released, however, its effect on the measurement error is much less [8].

Using the method of simulation modeling, we will show the possibility of reducing the auto seismic effect on the result of GA measurement by applying the GLS method in signal processing with the description of the interference by the AR model.

When modeling auto seismic effects, we will assume a rigid attachment of the reference reflector (RR) of the BLG interferometer relative to the foundation and use a model of the BLG mechanical system in the form of a mass m_0 mounted on a spring with a stiffness coefficient c_0 and a damper with a viscous friction coefficient b_0 [5–8]. Here m_0 is the total mass of the foundation, ballistic unit and devices installed on it, c_0 is the stiffness coefficient of the soil foundation, b_0 is the viscous friction of the foundation. Following [5–8], the parameters of the mechanical model are set as follows: $m_0 = 3000$ kg; $c_0 = 125.88$ MN/m; $b_0 = 73743.2$ N·s/n mass of the TB $m = 0.08$ kg initial speed of the TB $\nu = 1.4$ m/s. The acceleration of the TB is provided by the force action of a rectangular shape with an acceleration time of 50 ms.

As a result of modeling according to the procedure described in [5–8], we obtain damped oscillatory movements of the foundation of the ballistic gravimeter, which occur after each push. Obviously, such oscillations can be well described by the second-order AR model.

Representation of auto seismic interference by the AR process

In the case of the accepted model of the BLG mechanical system, the RR oscillatory movements together with the foundation cause the corresponding additive interference at the output of the "path-time" measurement scheme (PTM), which is described by a second-order differential equation. After local quadratic interpolation of data from the output of the PTM scheme with a certain step $\Delta = 0.5$ ms, we obtain a time sequence of equidistant time readings of the form

$$z_k = x_k + \xi_k,\ k = 0, \dots, K\text{-}1,$$

where $x_k = a_0 + a_0 k\Delta + a_2 (k\Delta)^2$ is the desired signal; $\xi_k = \varphi_1 \xi_{k\text{-}1} + \varphi_2 \xi_{k\text{-}2} + \eta_k$ is the interference that is described by the AR process.

The AR process can be represented as a process at the output of a shaping filter, which receives "white noise" at its input. The power spectral density of the AR process is determined in a known manner by the frequency response of the shaping filter. For the case of the accepted simulation conditions, it has a form typical for a second-order oscillatory system with a carrier frequency $f_0 = (2\pi)^{-1} \sqrt{c_0 / m_0}$.

Implementation of the generalized least squares method for measuring the gravitational acceleration based on the model of the second order AR process

To implement the GLS, knowledge of the interference correlation function (matrix) is required. It can be estimated by observing the interference readings $\xi(t)$ with the special sensors installed on the basement of the BLG or the reference arm of the interferometer, or by the residuals of the AR after excluding the second-degree polynomial from the "path-time" signal formed inside the BLG. To obtain sufficiently accurate estimates of the correlation functions, a large volume of samples is required, and even in this case, problems may arise when inverting the correlation matrices in algorithm (3). However, the situation is simplified when the interference is described by the AR model. In this case, to construct estimates of the interference correlation functions, several parameters should be estimated. In the case of the second-order AR model, there are only two parameters: φ_1 and φ_2. To estimate them, we use the covariance method [14], which minimizes the mean square of the linear prediction error of the AR process and, in the case under consideration, is reduced to solving the system of equations

$$\begin{cases} \sum_k \xi_k \xi_{k\text{-}1} = \varphi_1 \sum_k \xi_{k\text{-}1}^2 + \varphi_2 \sum_k \xi_{k-2} \xi_{k\text{-}1}; \\ \sum_k \xi_k \xi_{k\text{-}2} = \varphi_1 \sum_k \xi_{k\text{-}1} \xi_{k\text{-}2} + \varphi_2 \sum_k \xi_{k\text{-}2}^2, \end{cases}$$

where summation is performed over some set of readings $k \in S \subseteq \{1, \dots, K\text{-}1\}$.

The most important characteristics of the processing method used in BLG are the weight function (WF) of the gravimeter and its frequency response (FR) [7, 8]. The weight function characterizes the influence of the second derivative of the TB movement relative to the RR on the measured value of GA, while the FR of the gravimeter describes the effect of the frequency interference components on the GA measurement result.

The FRs of gravimeters with OLS and GLS processing with a processing time of 0.20 s are shown in Fig. 1 by dashed and solid curves, respectively. From the shape of the last curve, it can be seen that in the FR of the gravimeter, which implements the

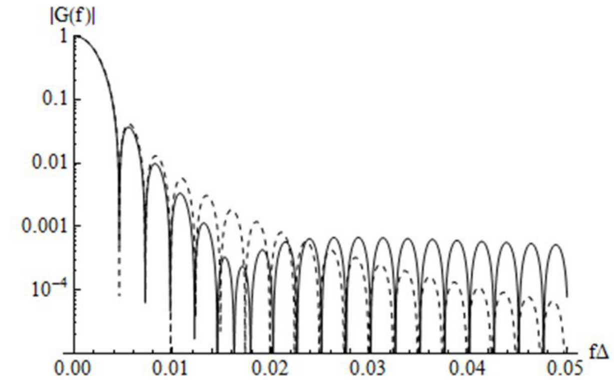

Fig. 1. Frequency response of the gravimeter that implements processing by the method of ordinary least squares and the method of generalized least squares

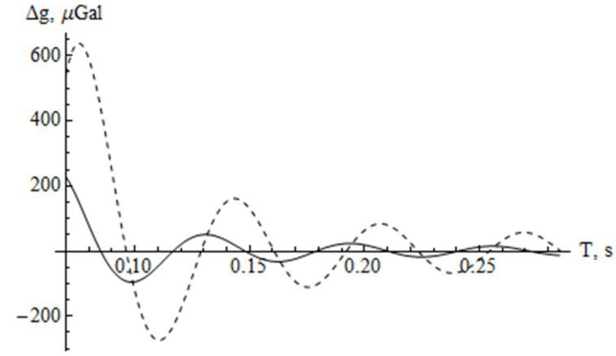

Fig. 2. Dependences of the auto seismic component of the error in measuring the gravitational acceleration on the duration of the processing interval for processing by the method of ordinary least squares and the method of generalized least squares

GLS processing, there is a dip in the frequency domain close to the resonance frequency of the auto seismic interference $f_0 \approx 32.5$ Hz, which corresponds to the normalized frequency $\nu_0 = f_0\Delta \approx 0.0163$ in the graph. This feature of the FR reflects the rejection of the most intense frequency components of the interference.

Fig. 2 shows the dependence of the auto seismic component of the error (ASE) of the GA measurement on the duration of the processing interval for the case of OLS processing (dashed curve) and GLS processing (solid curve).

From the course of the dependencies shown in Fig. 2, it can be concluded that the uncertainty of the GA measurement is approximately three times lower when the GLS processing is used in the BLG as compared to the OLS processing.

Rejection of auto seismic interference using the method of weighted least squares in signal processing

To reject the effect of auto seismic interference on the accuracy of the GA measurement, along with the GLS processing of signals in the BLG, the WLS processing can be used. Its implementation does not require exact knowledge of the correlation properties of the interference. The method of WLS assumes the use of weight windows. Hahn functions of the form [7] can be used as such windows, where the window shape is specified by a single parameter $\alpha > -1$.

In order for the WLS processing to be effective, it is necessary to choose a weight window taking into account the spectral composition of the interference. In the case of using Hahn polynomials, as shown in [7], the parameter value should be chosen positive if high-frequency spectral components prevail in the additive interference and negative if low-frequency components prevail.

Fig. 3 shows the FRs of gravimeters that implement the OLS (dashed curve) and WLS (solid curve) processing with a processing time of 0.20 s and a value of parameter $\alpha = 0.8$. It should be noted that this value of the parameter α was selected taking into account the specific properties of auto seismic interference in the model experiment.

From the course of the FRs shown in Fig. 3, it can be seen that the WLS processing provides a better rejection of high-frequency spectral components of the interference than the OLS processing.

Fig. 4 shows the ASE dependences of the GA measurement on the duration of the processing interval for the case of the OLS processing (dashed curve) and the WLS processing with the Hahn window at $\alpha = 0.8$ (solid curve), which were constructed by the simulation method for the conditions described above. From the course of these dependencies, it can be concluded that the uncertainty of the GA measurement is reduced by approximately one and a half times when using the

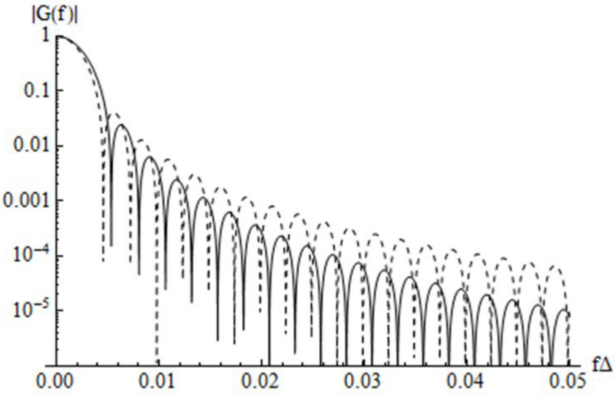

Fig. 3. Frequency response of the gravimeter that implements processing by the method of ordinary least squares and the method of weighted least squares

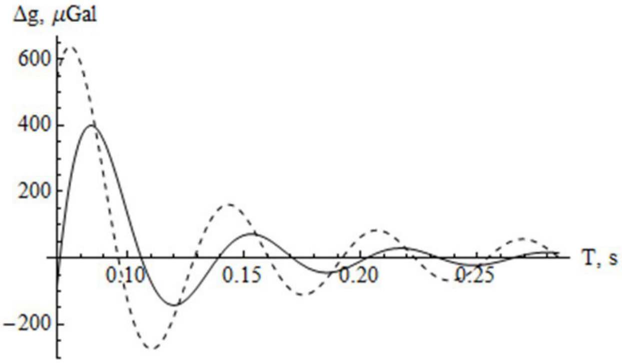

Fig. 4. Dependences of the auto seismic component of the error in the GA measurement on the duration of the processing interval for processing by the method of ordinary least squares and the method of weighted least squares

WLS processing in BLG as compared to the OLS processing.

Conclusions

1. The method of GLS can be simply implemented using an interference model in the form of an AR process. With known correlation characteristics of the interference, it provides unbiased estimates of the required parameters of polynomial regression with the minimum possible standard deviation.

2. The method of GLS makes it possible to effectively reject the effect of additive narrow-band interference in the middle frequency range in comparison with OLS due to the worse rejection of high-frequency interference. To implement the method, detailed knowledge of the characteristics of the interference is required.

3. The method of WLS is a kind of compromise between OLS and GLS. In it, a greater rejection of the effect of additive narrow-band interference in the middle frequency range in comparison with OLS is achieved due to the worse rejection of low-frequency interference. To implement the method, detailed knowledge of the characteristics of the interference is not required.

Оптимальна обробка даних у балістичному лазерному гравіметрі при дії корельованих завад

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Анотація

Роботу присвячено створенню алгоритмів оптимальної обробки даних у балістичних лазерних гравіметрах з урахуванням того, що сейсмічні завади є корельованими процесами.

Для побудови оптимальних алгоритмів обробки даних у балістичних лазерних гравіметрах на фоні корельованих завад застосовано узагальнений метод найменших квадратів. При цьому для опису завади використано математичну модель процесу авторегресії, для якої обернена кореляційна матриця має стрічковий характер і виражається через значення коефіцієнтів авторегресії. Для перетворення даних “шлях-час” із виходу схеми співпадіння балістичного лазерного гравіметра до рівномірного в часі процесу використовується їх локальна квадратична інтерполяція.

Розглянуто алгоритми обробки даних у балістичному гравіметрі, побудовані на основі зваженого методу найменших квадратів із застосуванням ортогональних поліномів Хана. Для реалізації симетричного способу вимірювання використано поліноми Хана симетричного виду, що характеризуються одним параметром.

Методом математичного моделювання досліджено виграш у точності вимірювання прискорення вільного падіння синтезованими алгоритмами у порівнянні з алгоритмом, заснованим на методі найменших квадратів. Показано, що автосейсмічні завади в балістичних лазерних гравіметрах із симетричним способом вимірювання можуть бути значно ослаблені при використанні в узагальненому методі найменших квадратів математичних моделей процесу авторегресії другого порядку. Виконано порівняльний аналіз характеристик алгоритмів, побудованих із використанням узагальненого методу найменших квадратів, зваженого методу найменших квадратів та звичайного методу найменших квадратів.

Ключові слова: автосейсмічна завада; вагові функції; корельовані завади; поліноми; процес авторегресії; прискорення вільного падіння.

Оптимальная обработка данных в баллистическом лазерном гравиметре при воздействии коррелированных помех

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Аннотация

Работа посвящена созданию алгоритмов оптимальной обработки данных в баллистических лазерных гравиметрах с учетом того, что сейсмические помехи являются коррелированными процессами.

Для построения оптимальных алгоритмов обработки данных в баллистических лазерных гравиметрах на фоне коррелированных помех применен обобщенный метод наименьших квадратов.

Рассмотрены алгоритмы обработки данных в баллистическом гравиметре, построенные на основе взвешенного метода наименьших квадратов с применением ортогональных полиномов Хана.

Методом математического моделирования исследован выигрыш в точности измерения ускорения свободного падения синтезированными алгоритмами по сравнению с алгоритмом, основанным на методе наименьших квадратов. Показано, что автосейсмические помехи в баллистических лазерных гравиметрах с симметричным способом измерения могут быть значительно ослаблены при использовании в обобщенном методе наименьших квадратов математической модели процесса авторегрессии второго порядка. Выполнен сравнительный анализ характеристик алгоритмов, построенных с использованием обобщенного метода наименьших квадратов, взвешенного метода наименьших квадратов и обычного метода наименьших квадратов.

Ключевые слова: автосейсмическая помеха; весовые функции; коррелированные помехи; полиномы; процесс авторегрессии; ускорение свободного падения.

References

1. Niebauer T.M., Sasagawa G.S., Faller J.E., Hilt R. and Klopping F. A new generation of absolute gravimeters. *Metrologia*, 1995, vol. 32, no. 3, pp. 159–180. doi: https://doi.org/10.1088/0026-1394/32/3/004
2. Rothleitner Ch., Svitlov S., Merimeche H., Hu H., Wang L.J. Development of new free-fall absolute gravimeters. *Metrologia*, 2009, no. 46, pp. 283–297. doi: https://doi.org/10.1088/0026-1394/46/3/017
3. Boliukh V.F., Vinnichenko O.I. Suchasni absoliutni lazerni hravimetry i yikhni metrolohichni kharak-terystyky [Modern absolute laser gravimeters and their metrological characteristics]. *Metrology and Instruments*, 2013, no. 5(43), pp. 47–56 (in Ukrainian).
4. Omelchenko A.V., Zanimonskiy Y.M., Vinnichenko A.I., Kupko V.S. Development of Methods for Data Processing in a Rise-and-Fall Gravimeter on the Basis of Polynomial Models. *Static and mobile measurements* (*TG – SMM 2013*): proceedings of IAG Symposium on Terrestrial Gravimetry (17–20 September 2013, Saint Petersburg, Russia). Saint Petersburg, 2013, pp. 143–147.
5. Omelchenko A.V., Boliukh V.F., Vinnichenko O.I. Avtoseismichna skladova pokhybky symetrychnoho balistychnoho hravimetra, zumovlena kolyvanniamy fundamentu [Autoseismic component of the error of symmetric ballistic gravimeter caused by foundation oscillations]. *Metrology and Instruments*, 2014, no. 3(47), pp. 51–56 (in Ukrainian).
6. Bolyukh V.F., Omelchenko A.V., Vinnichenko A.I. Vliyaniye avtoseysmicheskikh kolebaniy fundamenta na pokazaniya ballisticheskogo gravimetra s induktsionno-dinamicheskoy katapultoy [Influence of autoseismic oscillations of the foundation on the readings of the ballistic gravimeter with an induction-dynamic catapult]. *Measurement Techniques*, 2015, no. 2, pp. 19–22 (in Russian).

7. Omelchenko A.V., Bolyukh V.F., Vinnichenko A.I., Kupko V.S. Pomekhoustoychivaya obrabotka signalov v ballisticheskom lazernom gravimetre s simmetrichnym sposobom izmereniya uskoreniya svobodnogo padeniya [Anti-interference signal processing in a ballistic laser gravimeter with a symmetric measurement method of gravitational acceleration]. *Ukrainian Metrological Journal*, 2019, no. 1, pp. 54–62 (in Russian). doi: https://doi.org/10.24027/2306-7039.1.2019.164717
8. Vinnichenko O., Neyezhmakov P., Omelchenko A., Fedorov O., Bolyukh V. Analysis of the effects of seismic interference on the results of measuring the acceleration of free fall by ballistic laser gravimeters [Analysis of the effects of seismic interference on the measurement results of gravitational acceleration by ballistic laser gravimeters]. *Ukrainian Metrological Journal*, 2020, no. 1, pp. 51–61 (in Ukrainian). doi: https://doi.org/10.24027/2306-7039.1.2020.204228
9. Omelchenko A. Otsinyuvannya parametriv polinomialnoyi rehresiyi u vypadku korelovanykh pomylok sposterezhennya [Polynomial regression parameters estimation in a case of correlated errors of observation]. Visnyk TDTU, 2009, vol. 14, no. 3, pp. 124–133.
10. Omelchenko A.V., Vinnichenko O.I., Kupko V.S., Zanimonskyi Y.M. Zastosuvannia kvadratychnoi interpoliatsii u protsesi opratsiuvannia danykh u balistychnomu hravimetri [Application of quadratic interpolation during the data processing in ballistic gravimeters]. *Metrology and Instruments*, 2013, no. 4(42), pp. 50–55 (in Ukrainian).
11. Seber G. Lineynyy regressionnyy analiz [Linear regression analysis]. Transl. from Eng. V.P. Nosko, ed. M.B. Malyutov. Moscow, Mir, 1980. 456 p. (in Russian).
12. Box G., Jenkins G. Analiz vremennykh ryadov, prognoz i upravleniye [Time series analysis, forecasting and control]. Book 1. Transl. from Eng., ed. V.F. Pisarenko. Moscow, Mir, 1974. 406 p. (in Russian).
13. Wise J. The autocorrelation function and the spectral density function. *Biometrika*, June 1955, vol. 42, issue 1–2, pp. 151–159.
14. Markel J.D., Gray A.H. Lineynoye predskazaniye rechi [Linear prediction of speech]. Ed. Yu.N. Prokhorov and V.S. Zvezdin. Moscow, Svyaz Publ., 1980. 308 p. (in Russian).