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Mathematical modelling of metrological traceability chains

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Abstract

Metrological traceability is a property of the measurement result and requires an established multi-level calibration hierarchy. The metrological traceability chain is established through the calibration hierarchy and is used to establish the metrological traceability of the measurement result. Approaches to the establishment of metrological traceability chains are based mainly on graphic images.

Special international guidelines and European recommendations on the measurement uncertainty are used to evaluate the measurement uncertainty during the calibration of measuring instruments. The measurement uncertainty necessarily increases along the calibration sequence and is different for different levels of the metrological traceability chain. The measurement uncertainty depends on measurement standards or measuring instruments used during calibration.

A mathematical model of the metrological traceability chain for different levels of the calibration hierarchy is proposed. This model includes, as components, such basic metrological characteristics for a certain level of the chain as the measurement range, measurement uncertainty, measuring instrument or measurement standard. As additional parameters for the metrological traceability chain, it is proposed to use data from the calibration certificate of the corresponding measuring instrument or measurement standard used at a certain level of the chain.

Recommendations regarding practical application of the developed mathematical model of the metrological traceability chain, which can be used for chains of any measurements, are proposed. They can be used to establish the required metrological characteristics for a certain level of the metrological traceability chain. Such a model and recommendations for its practical application can be used in national metrology institutes and calibration laboratories.

Keywords: modelling; metrological traceability; measurement uncertainty; measurement standard; measuring instrument.

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1. Introduction

According to the International Vocabulary of Metrology, the metrological traceability (MT) is a property of a measurement result whereby the result can be related to a reference through a documented unbroken chain of calibrations, each contributing to the measurement uncertainty (MU). MT requires an established calibration hierarchy, which is a sequence of calibrations from a reference to the final measuring instruments (MIs) or measurement standards, where the result of each calibration depends on the result of the previous calibration [1].

The MU necessarily increases along the sequence of calibrations. The international Guide to the Expression of Uncertainty in Measurement [2] and special European recommendation for Evaluation of the Uncertainty of Measurement in Calibration [3] are used to evaluate the MU during the calibration of MIs.

Depending on measuring standards or MIs used during calibration, the level of MU is determined, which is different for different MT levels [4]. The MT chain is established through the calibration hierarchy and is used to establish the MT of a measurement result. Approaches to the establishment of MP chains are based mainly on graphical representations [5, 6].

International Laboratory Accreditation Cooperation (ILAC) attaches great importance to MT for accredited laboratories in the framework of Mutual Recognition Arrangement [7]. ILAC has a special Policy on Traceability of Measurement Results and Policy for Measurement Uncertainty in Calibration [8, 9]. For accreditation of calibration laboratories, the MT is the basis for establishing global MT.

In [10], the application of systems thinking for the establishment of MT chains is proposed, which is based on graphical representations of MT chains. In [11], the mathematical model of the system-oriented MIs is proposed. In [12], the mathematical model of the system of assessing the quality of the indicators of MIs is proposed. These two approaches can serve as the basis for mathematical modelling of MT chains.

2. The statement of the problem, purpose and objectives of the study

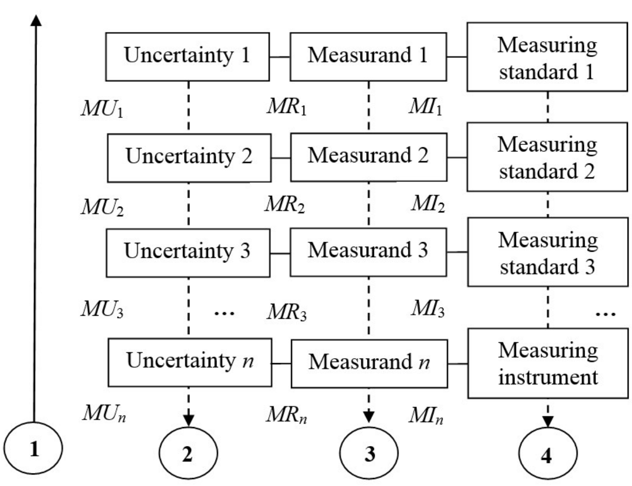
The purpose of the paper is to highlight the results of practical application of mathematical modelling of MT chains, in particular:

- to develop a mathematical model of MT chains based on the selected multiple mathematical approach;
- to define the principles for determining the required components of MT chains of different levels;
- to propose approaches for practical application of the model to ensure reliability of calibration results, and to reduce resource costs and time for calibrations.

3. Defining the principles for construction of the mathematical model of metrological traceability chains

The MT chains are characterized by a number of properties that are based on certain metrological characteristics (MCs) of measurement standards or measuring instruments. These MCs generally include the measurement range (MR) of a specific quantity and the MU of this quantity. The properties of the MT chain at different levels of the calibration hierarchy are different. They depend on MCs of the calibrated measurement standards or MIs.

The generalized metrological traceability chain is shown in Fig.1. A solid line with an arrow (1) indicates the direction of the MT chain. Dashed lines show the direction of the increasing MU (2), MR (3) and calibration hierarchy (4).



1 – MT; 2 – MU; 3 – MR; 4 – calibration hierarchy

Fig. 1. Generalized MT chain

For mathematical modelling, the method used in [12] was chosen, since it is more visual than the method used in [11], and does not require additional graphical interpretation.

The mathematical model of the MT chain can be summarized as follows:

$$Q_{MT} = f\begin{pmatrix} f\left(\left\{\left(Par, Met\right), B\right\}\right), f\left(\left\{Q_{MR}, B\right\}\right), \\ f\left(\left\{Q_{MU}, B\right\}\right), f\left\{Q_{MI}, B\right\} \end{pmatrix}, \quad (1)$$

where Q_{MT} are properties of an element of the MT chain;

Q_{MR} is MR for an element of the MT chain;

Q_{MU} is MU of an element of the MT chain;

Q_{MI} are properties of the MI (measurement standard) for an element of the MT chain;

(Par, Met) are parameters and methods of measurement for an element of the MT chain;

$B = \left\{b_1,...,b_i,...,b_{|Q_k|}\right\}$ is the characteristic vector of the corresponding sets;

Q_k are parameters of the level of the MT chain, k (k=(1|k|) is the level index, where |k| is the lower level.

4. Main components of the mathematical model of the metrological traceability chain

The mathematical model of evaluating the i-th level of the MT chain can be given in a general form:

$$Q_{MTi} = f\left(\left\{LC_{MTi}\right\}\right), \quad (2)$$

where LC_{MTi} is the generalized parameters of the i-th level of the MT chain, which, in turn, is equal to:

$$LC_{MTi} = f\begin{pmatrix} VerQ_{MTi}, ValQ_{MTi}, \\ LC_{ParQ_{MTi}}, LC_{MetQ_{MTi}} \end{pmatrix}, \quad (3)$$

where $VerQ_{MTi}$ is verification of MCs of the i-th level of the MT chain;

$ValQ_{MTi}$ is validation of MCs of the i-th level of the MT chain;

LC_{ParMTi} are parameters at the i-th level of the MT chain;

LC_{MetMTi} are methods at the i-th level of the MT chain.

Verification and validation of MCs of the i-th level of the MT chain:

$$Ver_{MTi} = f\begin{pmatrix} f\left(\left\{\left(Par, Met\right), B\right\}\right)_{Ver}, f\left(\left\{Q_{MR}, B\right\}\right)_{Ver}, \\ f\left(\left\{Q_{MU}, B\right\}\right)_{Ver}, f\left(\left\{Q_{MI}, B\right\}\right)_{Ver} \end{pmatrix}, \quad (4)$$

$$Val_{MTi} = f\begin{pmatrix} f\left(\left\{\left(Par, Met\right), B\right\}\right)_{Val}, f\left(\left\{Q_{MR}, B\right\}\right)_{Val}, \\ f\left(\left\{Q_{MU}, B\right\}\right)_{Val}, f\left(\left\{Q_{MI}, B\right\}\right)_{Val} \end{pmatrix}. \quad (5)$$

Methods at the i-th level of the MT chain are determined as:

$$LC_{MetQ_{MTi}} = \bigcup_b \{MetLCQ_b\}, \quad (6)$$

where b is an index of a certain level of the MT chain.

After certain generalizations, one can get:

$$LC_{MetQ_{MTi}} = f\left(\left\{MetLC_{Q_{MTi}}\right\}\right),$$
$$MetLC_{Q_{MTi}} = f\left(\left(VerQ_{MTi}\right),\left(ValQ_{MTi}\right)\right), \quad (7)$$

herewith $LC_{MetMTi} = \varnothing$, $MetLC_{MetMTi} = \varnothing$, $VerQ_{MTi} = \varnothing$, $ValQ_{MTi} = \varnothing$.

5. Practical aspects of mathematical modelling of metrological traceability chains

Mathematical modelling of MT chains for a certain level provides a required list of data for checking and approving the parameters of the MT level only in general. Specific analysis for the purpose of verification should be carried out at least of three main MCs of each level of the MT chain: MR, MU and MIs (measurement standard). It is also necessary to determine the calibration parameters and the calibration method for each level of the MT chain.

To ensure such an analysis, it is advisable to use a special table (Table 1) with specific MCs for a certain level of the MT chain. Two sub-parameters are used for MIs: MR and MUs of MIs (measurement standard).

Sub-parameters of MR and MUs of MIs (measurement standard) in Table 1 must fully correspond to the parameters of MR and MU for a certain level of the MT chain. Only in this case, it is possible to validate the considered parameters for a certain level of the MT chain. Non-compliance with the necessary metrological requirements of at least one of these components leads to the impossibility of approving such a combination of parameters for a certain defined level of the MT chain. MR and MU must be fully ensured by the used MIs (measurement standard). Verification and validation of MCs must be carried out for all used levels of the MT chain.

In addition, verification and validation of the parameters of each level of the MT chain as well as verification and validation of the methods that ensure the transition from a higher level to a lower level of the MT chain is necessary. To do this, it is necessary to analyse the calibration methods of the used MIs in order to validate them for a certain level of the MT chain.

Verification and validation of the parameters of each level of the MT chain should be carried out according to the ILAC Guidelines G8:09 [13] and standard ISO/IEC 17025 [14].

The ILAC Guidelines G8:09 provide an overview for laboratories regarding the rules for decision-making and compliance with the requirements. Since it does not provide detailed information on the application of basic statistics, staff in some laboratories may need to improve their knowledge of the rules for making risk-related decisions and associated statistics. The document provides a specific overview of the measurement uncertainty and risks associated with the decision-making.

The standard ISO/IEC 17025 recognizes that no single decision-making rule can address all statements of conformity across the diverse scope of testing and calibration. At the same time, the standard requires that the laboratory has authorized personnel to perform analyses of the results, including statements of conformity. In addition, it specifies that the measurement uncertainty must be expressed in the same units as the quantity being measured.

For verification and validation of the parameters of each level of the MT chain, the provisions of the standard ISO 10012 [15] can also be used. According to the standard, metrological confirmation must be established and implemented to ensure that metrological characteristics of the measuring equipment satisfy metrological requirements for the measurement process. It includes calibration of measuring instruments and their verification. The analysis of measurement uncertainties shall be completed before the metrological confirmation of the measuring equipment and validation of the measurement process.

The number of levels of the MT chain is determined by the MU required for a particular laboratory. This MU should be evaluated for different levels of the MT chain using international guidelines and regional recommendations [2, 3].

Table 1

The data for verification and approval of parameters for each level of the MT chain

Level of MT (Q_{MT})	MR (Q_{MR})	MU (Q_{MU})	MI (measurement standard)		Calibration certificate (LC_{ParMTi})	Calibration method (LC_{MetMTi})
			MR (SQ_{MRMI})	MU (SQ_{MUMI})		
First						
Second						
…						
n-th						

Conclusions

A mathematical model of the metrological traceability chain for different levels of the calibration hierarchy is proposed. This model includes, as components, such basic metrological characteristics for a certain level of the chain as the measurement range, measurement uncertainty, measuring instrument or measurement standard. As additional parameters for the metrological traceability chain, it is proposed to use data from the calibration certificate of the corresponding measuring instrument or measurement standard used at a certain level of the chain.

Recommendations regarding practical application of the developed mathematical model of the metrological traceability chain, which can be used for chains of any measurements, are proposed. They can be used to establish the required metrological characteristics for a certain level of the metrological traceability chain. Such a model and recommendations for its practical application can be used in national metrology institutes and calibration laboratories.

Практичні аспекти математичного моделювання ланцюгів метрологічної простежуваності

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Анотація

Метрологічна простежуваність є властивістю результату вимірювання і вимагає встановленої багаторівневої ієрархії калібрування. Ланцюг метрологічної простежуваності визначається через ієрархію калібрування та використовується для встановлення метрологічної простежуваності результату вимірювання. Підходи до побудови ланцюгів метрологічної простежуваності базуються переважно на графічних зображеннях.

Для оцінки невизначеності вимірювань під час калібрування засобів вимірювань використовуються спеціальні міжнародні настанови та європейські рекомендації щодо оцінки невизначеності вимірювань. Невизначеність вимірювання обов'язково зростає впродовж послідовності калібрування та є різною для різних рівнів ланцюга метрологічної простежуваності. Невизначеність вимірювання залежить від використовуваних еталонів або інструментів вимірювання, які використовуються під час калібрування.

Запропоновано математичну модель ланцюжка метрологічної простежуваності для різних рівнів ієрархії калібрування. Як компоненти, ця модель включає такі основні метрологічні характеристики для певного рівня ланцюга, як діапазон вимірювання, похибка вимірювання, засіб вимірювання або еталон вимірювання. Як додаткові параметри для ланцюга метрологічної простежуваності пропонується використовувати дані із сертифіката про калібрування відповідного засобу вимірювання або еталона, що використовується на певному рівні ланцюга.

Запропоновано рекомендації щодо практичного застосування розробленої математичної моделі ланцюга метрологічної простежуваності, яку може бути використано для ланцюгів будь-яких вимірювань. Вони можуть бути використані для встановлення необхідних метрологічних характеристик для певного рівня ланцюга метрологічної простежуваності. Така модель і рекомендації щодо її практичного застосування можуть бути застосовані в національних метрологічних інститутах і калібрувальних лабораторіях.

Ключові слова: моделювання; метрологічна простежуваність; невизначеність вимірювання; стандарт вимірювання; засіб вимірювання; системний підхід.

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