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A proposed new definition of measurement uncertainty

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Abstract

A new definition of the measurement uncertainty (MU) was proposed by the Working Group 1 of the Joint Committee for Guides in Metrology, JCGM-WG1. This definition avoids any quantitative aspect of the measurement uncertainty and focuses on its subjective nature.

The proposed new definition reads as follows:

doubt about the true value of the measurand that remains after making a measurement.

Here, MU is no longer quantitative; it is rather a (subjective) *state of mind*. MU is the concept, and its quantitative measures, such as the standard measurement uncertainty, are different in nature. This separation greatly contributes to clarity.

Being defined as a state of mind, MU is subjective and reflects the belief of the experimenter in the result. There is no "true uncertainty" in nature to be estimated. There exists the measurand, and the uncertainty about its true value is a personal matter. Of course, the state of belief is based on objective data, and a good experiment is conceived in such a way as to minimise subjectivity. Yet, the hope to eliminate subjectivity from a measurement or from science at large is just a hope.

The proposed new definition explicitly uses the term "true value". Perhaps, at the philosophical level, the concept can be questioned, whereas in the context of parameter estimation, the mathematics behind calculations needs a unique true value, which is ideally represented by a unique real number.

The doubt is about the unknown value of the measurand, not about the estimate. The estimate is viewed as a realization of a random variable describing the state of knowledge about the measurand. As such, the estimate is fixed and has no uncertainty. Randomness is in the variable, not in its realizations.

Keywords: measurement uncertainty; subjective interpretation of probability; GUM; JCGM-WG1.

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Introduction

Measurement uncertainty, hereafter MU, is still a controversial concept despite the existence of internationally agreed documents dealing with its evaluation, propagation and expression. The reason for the permanence of the controversies is that the measurement uncertainty is just an aspect, although fundamental, of the wider field of measurement and estimation. This broader field is common to epistemology on the one hand, and mathematics and probability on the other hand. These two branches are represented in the framework of international metrology by two Working Groups of the Joint Committee for Guides in Metrology, JCGM (see https://www.bipm.org/en/committees/jc/jcgm). WG1 deals with the measurement uncertainty, and WG2 with the vocabulary of metrology, VIM, that is to say with concepts and definitions of terms.

The two Working Groups, of course, aim at the maximum possible harmonisation of concepts, yet some discrepancies remain, especially concerning the common topic of measurement uncertainty. In this paper, I will outline the joint efforts to fill the remaining gaps between the viewpoints of the two WGs.

Current definition(s) of measurement uncertainty

The current VIM3 (JCGM 200:2012) definition of the measurement uncertainty reads as follows:

measurement uncertainty
uncertainty of measurement
uncertainty
non-negative parameter characterizing the dispersion of the quantity values being attributed to a measurand, based on the information used ([1], def 2.26).

The JCGM 100:2008 (the legacy GUM) definition is that of the previous edition of the VIM of 1993, VIM2, reads as follows:

***uncertainty* (*of measurement*)**

parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand ([2], 2.2.3).

The two definitions are very similar, apart from some specifications added to the VIM3 definition. The first, "non-negative", was added in the (vain) attempt to discourage the use of expressions such as u=+/−10 mV or similar. It just added perplexity, both among knowledgeable scientists and in practitioners.

The important term here is "parameter". It clearly shows the quantitative nature of MU as defined. A note in both definitions specifies that the parameter can be a standard deviation (the standard measurement uncertainty) or something else.

The JCGM-WG2 is currently working on the fourth edition of the VIM. A first Committee Draft (VIM4 CD) was circulated in early 2021 among the JCGM Member Organisations and National Metrology Institutes. Some criticisms were formulated by the readership against the current definition of MU. Other criticisms came from JCGM-WG1. Most of them concerned the term "parameter", which was considered too restrictive. More generally, the very nature of MU as a quantitative concept (thus expressed as a number) was questioned [3]. JCGM-WG1 thus endeavoured an effort, jointly with WG2, to come to a set of agreed definitions concerning MU-related terms, such as the standard uncertainty, expanded uncertainty and so on. Eventually, WG1 elaborated an own definition of MU and offered it to WG2 for consideration.

Proposed new definition of measurement uncertainty

The proposed new definition reads as follows:

doubt about the true value of the measurand that remains after making a measurement.

This definition, taken from that of NIST [4], neatly illustrates most of the current views of JCGM-WG1 about MU and its nature.

Measurement uncertainty is a (subjective) state of mind

First, measurement uncertainty is no longer a parameter. More generally, it is no longer quantitative at all, but rather a (subjective) *state of mind*. Of course, it is still possible (and necessary) to express a state of mind quantitatively using suitable quantitative measures, such as the standard measurement uncertainty.

The impact of this change is considerable, which nicely separates the concept from its quantitative measures. With the current definition(s), MU is a generic parameter characterising a dispersion and can be any of several specific parameters, standard or expanded uncertainty and possibly more. With the proposed new definition, MU is the concept, and its measures are different in nature, thus greatly contributing to clarity.

Examination uncertainty

Incidentally, this conceptual separation has a further great advantage, as it provides a very simple definition of the uncertainty associated with *examination* of categorical data or nominal properties. Examination is for nominal properties what measurement is for quantities. Both aim at determining the value of the measurand (a quantity) and of the examinand (a property), respectively. Examples of nominal properties are the blood group, the tumour type and a chemical species. Since nominal properties are intrinsically qualitative, it is not possible to define a numerical parameter characterising the dispersion of an examination or attribution. This impossibility makes it impossible to define the examination uncertainty in quantitative terms, whilst the proposed new definition can be easily adapted to define the examination uncertainty as:

doubt about the true value of the examinand that remains after making an examination.

Of course, finding a suitable quantitative measure of the amount of doubt remains an open issue. For example, probability mass functions, describing the degree of belief in each of the possible values of the examinand, can be used.

Although the VIM4 CD contains a chapter devoted to the terms related to the examination, a definition of the examination uncertainty is missing, presumably because of the above-mentioned difficulty in adapting the current definition of MU to the field of examination.

A state of mind is necessarily subjective

There is a further and important implication in the proposed new definition of MU. Being defined as a state of mind, that is, a subjective or individual state, MU is in turn subjective and reflects the confidence of the experimenter in the result. This view of uncertainty is very different from another view, which still has some followers, that is, that for a given measurand there is a sort of corresponding "true uncertainty" in nature to be estimated by the experimenter to the best of his capability. We believe that in nature there exists the measurand with its true value (the subject of the next chapter), and that the uncertainty about the latter is a personal matter. A different experimenter confronted with the measurement of the same measurand would probably obtain a different estimate and a different uncertainty. However, even if the estimate were the same (it may happen with discrete or countable quantities) the uncertainty would most likely be different. Of course, the state of belief is based on objective data, and a good experiment is conceived in such a way as to minimise subjectivity. Yet, the hope to eliminate subjectivity from a measurement or from science at large is just a hope.

True value

A second, fundamental aspect of the proposed new definition is the proud resurfacing of "true value". This term has been demonised for a very long time, and even mentioning it was interpreted as a sign of cultural backwardness. Perhaps at the philosophical level the concept can legitimately be questioned, whereas in the context of parameter estimation there is no doubt that it is indispensable to assume the existence and uniqueness of the parameter to be estimated. Uniqueness is the second key word. The mathematics behind calculations needs a unique true value, ideally represented by a unique real number.

The GUM of 1993 is utterly ambiguous in this respect. On the one hand, it pays a tribute to the views largely widespread at the time it was written. For example, there is a lengthy discussion in Annex D concerning the impossibility of a complete description of the measurand, because an infinite amount of information would be needed. Hence, the concept of definitional uncertainty is introduced (and defined in the VIM, see [1], definition 2.27), which in turn leads to the existence of a whole set of true values since the true value is simply defined as a value consistent with the definition of the quantity (see [1], definition 2.11). This approach, in the GUM of 1993, is contrasted with the "conventional" one, based on the "unknowable values of true value and error" (see [2], E.5). However, in the Scope, it is clearly stated:

This Guide is primarily concerned with the expression of uncertainty in the measurement of a well-defined physical quantity — the measurand — that can be characterized by an essentially unique value.

Furthermore, in [2], D.3.5, it is stated that:

The term "true value of a measurand" or of a quantity (often truncated to "true value") is avoided in this Guide because the word "true" is viewed as redundant.

Therefore, the qualifier "true" is avoided not because of some diffidence against the concept, but simply because it is considered as redundant.

In conclusion, it is clear that, despite the discussions in the Annexes, the GUM of 1993 explicitly adopts the concept of a unique true value.

The GUM of 1993 also offers a solution to the situation when the phenomenon of interest is represented by a distribution of values (a set of true values, in the VIM language).

If the phenomenon of interest can be represented only as a distribution of values or is dependent on one or more parameters, such as time, then the measurands, required for its description, are the set of quantities describing that distribution or that dependence (see [2], 1.2).

The doubt is about the true value of the measurand

Another important concept embedded in the proposed new definition is that the doubt concerns the unknown value of the measurand. This view is to be contrasted with an opposed belief, that is, that the doubt is about the estimate (or estimates when, as usually happens, there are more than one input quantities to the measurement model). The belief that the uncertainty is about the true value of the measurand marks a deep conceptual difference of the subjective view of probability as regards the "orthodox" frequentist view. In the latter, (most of) the input estimates and, as a consequence, the measurand estimate, would be different should the measurement be repeated, which is absolutely true and to which nobody would object. We all agree that the indications from a measuring system can be described by random variables. The frequentist view of probability uses this truth to assert that the indications (and thus the input estimates and the measurand estimate) are random so that the uncertainty ultimately concerns them. In the subjective view of probability, on which the proposed new definition is based, emphasis is given to the equally unquestionable fact that a particular indication is a realization of the corresponding random variable and, as such, is fixed. Randomness is in the variable, not in its realisations.

What is a measurement?

The last part of the proposed new definition is "...after making a measurement". The VIM defines the measurement as follows:

process of experimentally obtaining one or more quantity values that can reasonably be attributed to a quantity (see [1], definition 2.1).

The disturbing term here is "experimentally". The reason for the disturbance is that this definition of the measurement, applied to "measurement uncertainty", is too restrictive. The measurement uncertainty is conceivable, and is actually calculated, in a much wider range of processes, including virtual measurements, design, computer simulations and algorithms in general and so on. Therefore, the concept of measurement should be interpreted in the wider sense addressed above. This broader interpretation is explicitly acknowledged in the GUM of 1993:

This Guide is also applicable to evaluating and expressing the uncertainty associated with the conceptual design and theoretical analysis of experiments, methods of measurement, and complex components and systems. Because a measurement result and its uncertainty may be conceptual and based entirely on hypothetical data, the term "result of a measurement" as used in this Guide should be interpreted in this broader context (see [2], 1.3).

Conclusions

The JCGM-WG1 has decided to adopt the proposed new definition of MU in its publications and is working to define a complete set of MU-related definitions besides the principal one of MU itself. Such set would comprise standard measurement uncertainty, coverage interval, expanded uncertainty, covariance matrix and possibly coverage probability and coverage factor.

Disclaimer

The author is the Convener of JCGM-WG1. Although the wording of the definition represents a position of WG1, some interpretations are personal and might not be shared by all members.

Запропоноване нове визначення невизначеності вимірювання

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Анотація

Нове визначення невизначеності вимірювання (НВ) було запропоноване Робочою групою 1 Об'єднаного комітету з настанов у метрології, JCGM-WG1. Це визначення уникає будь-якого кількісного аспекту невизначеності вимірювання та зосереджується на її суб'єктивній природі.

Запропоноване визначення звучить так: "*сумнів щодо істинного значення вимірюваної величини, який залишається після проведення вимірювання*".

Тут НВ більше не є кількісною, а скоріше являє собою суб'єктивний стан розуму. НВ є концепцією, а її кількісні показники, такі як стандартна невизначеність вимірювання, відрізняються за своєю природою. Це розділення значною мірою сприяє ясності.

Будучи визначеною як стан розуму, НВ є суб'єктивною і відображає віру експериментатора в результат. У природі не існує "істинної невизначеності", яку можна оцінити. Вимірювана величина існує, і невизначеність щодо її істинного значення є особистою справою. Звичайно, стан віри базується на об'єктивних даних, і хороший експеримент задумано таким чином, щоб мінімізувати суб'єктивний аспект. Проте надія усунути суб'єктивність вимірювання чи науки в цілому – це лише надія.

У запропонованому новому визначенні явно використовується термін "істинне значення". Можливо, з філософської точки зору це поняття можна поставити під сумнів, тоді як у контексті оцінки параметрів, математика, що стоїть за обчисленнями, потребує унікального істинного значення, ідеально представленого унікальним дійсним числом.

Сумнів стосується невідомого значення вимірюваної величини, а не оцінки. Оцінка розглядається як реалізація випадкової величини, що описує стан знань про вимірювану величину. Таким чином, оцінка є фіксованою та не має жодної невизначеності. Випадковість полягає у змінній, а не в її реалізації.

Ключові слова: невизначеність вимірювання; суб'єктивна інтерпретація ймовірності; GUM; JCGM-WG1.

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