# QUBITS AND THEIR APPLICATIONS IN METROLOGY

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## a. THE PROBLEM

Now there is a great interest in the field of information technologies cause development a quantum computer that make direct use of quantum-mechanical phenomena, such as superposition and entanglement, to perform operations on data. Qubit - is a unit of quantum information—the quantum analogue of the classical bit. As a bit, a qubit admits two of its own state, but it can be in superposition. Following the development of a quantum computer will need to establish a standard unit of quantum information. The search for a candidate for the role of the qubit (and, stable) can give perspectives to the development of quantum metrology.

The development of nanoelectronics and spintronics[1–3] during the last years has attracted considerable attention to novel types of heterostructures where the evolution of charge and spin degrees of freedom for the carriers have more favorable characteristics for possible device applications than in conventional semiconductors. A promising candidate has been recently discovered in the field of new materials called topological insulators (TI). In these materials the bulk material is insulating but the edge states having the energies within the bulk gap are of helical nature and are protected from the backscattering by the time reversal symmetry, creating efficient channels of spin and charge transport.[4, 5]

## b. INTRODUCTION AND RECENT RESEARCH AND PUBLICATION

One of the first examples of TI-based structures were the HgTe/CdTe 2D quantum wells[5] where the tuning of the well width may create the phase where topologically protected edge states exist. The applications of TI in nanoelectronic devices require the fabrication of localized small-to-medium size object like quantum dots (QD). Several models of QD formation at the edge of TI where the symmetry protected state exist have been proposed during the last years.[6, 7] Most of them relevant to 1D QD on the edge of 2D TI deal with simplified assumptions of non-transparent magnetic barriers which are required to confine the electrons with massless Dirac (or Weyl) spectrum.[4] Under such assumptions the spectrum of discrete energy levels inside QD forms a set of equidistant levels located in two ladders above and below the Dirac point of TI where two linear dispersion branches cross.[6] For each level the corresponding eigenstate is a two-component spinor with certain spin polarization, which makes this system a promising candidate for studying there a driven dynamics excited by external electric field tuned to match the interlevel resonance splitting.

## c. WAYS TO SOLVE PROBLEM

Requirements for performing a volumetric calculations and upgrade the cryptographic mechanisms causing the need for the creation of quantum computers. Scientists are engaged in the topic since the 80s of the 20th century. Need to get a heterostructure, providing a stable living condition at adequate temperatures. To this end, research is needed and new types of materials called topological insulators. Studies of their characteristics can give a significant contribution to the development of information technologies and quantum metrology, respectively.

## d. WORK PURPOSES

Here we derive a model of a 1D-quantum dot formed at the edge of 2D TI based on the HgTe/CdTe quantum well bounded by magnetic barriers on both ends of the QD, which are described by a realistic model of finite barrier height. We discuss various mechanisms leading to the barrier formation, including the exchange interaction and Zeeman term. In particular, the two-level dot representing a model for the qubit is studied, and its Rabi frequency is found. The qubit operating time defined by the Rabi frequency is compared with relaxation times created by several mechanisms that are briefly discussed.

e. RESEARCH

We firstly describe an unperturbed Hamiltonian HQD for the 1D electron in a quantum dot (QD) confining the edge states in 2D HgTe/CdTe topological insulator (TI). The 3D layout of our structure is shown schematically in Fig.1. The HgTe quantum well in the center (lightcolored) region is formed by two neighboring CdTe layers, and the edge of the structure where the localized and topologically protected states are formed is described by the line x = 0. The edge states are localized transversally to the edge along the x-direction, decaying into the bulk volume of the sample, and they are freely propagating parallel to the edge along the y-direction. Such states, as it has been shown previously,[4], can be described by the effective Weyl Hamiltonian

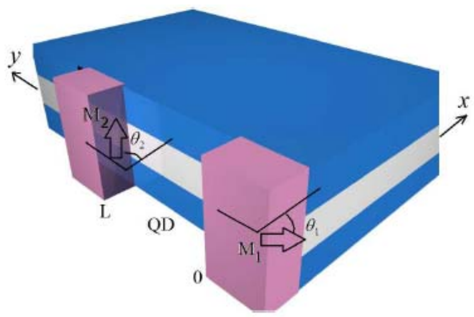


FIG. 1. (Color online) Schematic view of a 1D-quantum dot with length L formed at the edge of 2D topological insulator in HgTe/CdTe quantum well by two magnetic barriers (pink) with magnetization amplitudes $M_1$ and $M_2$ and polarization angles in the xy-plane

$$H_0 = Ak_y\sigma_z \quad (1)$$

Here the parameter A is determined by the HgTe/CdTe quantum well geometry where the twodimensional electron gas in confined, and for our model we take the value A = 0.36 eV · nm and consider the band gap in HgTe/CdTe to be around 40 meV which corresponds to the quantum well width in the range of 7 . . . 8 nm.[4] In order to confine the states along the TI edge in the y-direction on our Fig.1, one needs to insert a mass term into the Hamiltonian, at least in the barrier area bounding the 1D region of length L at the edge of TI where the 1D quantum dot can be formed. We take two mesoscopic barriers separated by the distance L, as it is shown in Fig.1. Each of these barriers may be viewed as a mesoscopic magnet having a specific magnetization of amplitude $M_{1,2}$ and orientation $\Theta_{1,2}$ in the xy-plane. By generalizing the idea of non-transparent magnetic barriers,[6] we consider the model of barriers with finite transparency reflected in their finite height in units of energy M1,2, leading to the Hamiltonian of the form:

$$H_{QD} = Ak_y\sigma_z - M_1\Theta(-y)(\sigma_x\cos\theta_1 + \sigma_y\sin\theta_1) - M_2\Theta(y-L)(\sigma_x\cos\theta_2 + \sigma_y\sin\theta_2) \quad (2)$$

Here the first term $H_0 = Ak_y\sigma_z$ is the effective Weyl Hamiltonian (1) for massless edge states propagating on the boundaries of the HgTe/CdTe TI. The second and third term in (2) describe the magnetic barriers located along the TI edge at y = 0 and y = L forming a confining QD potential, as it is shown schematically in Fig.1.

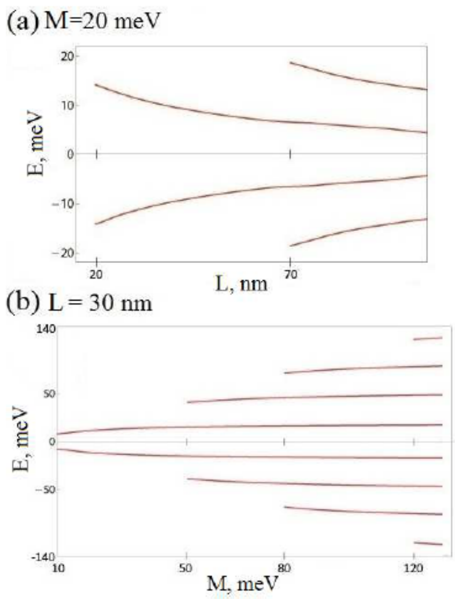


FIG. 2. (Color online) (a) Energy level dependence on the QD size L, (b) Dependence of the energy levels on the magnetization energy M. The number of levels is always even.

In our model where the Hamiltonian (2) describes the barriers with finite transparency the spectrum cannot be found analytically, and has to be obtained from a transcendental equation which can be derived from the continuity of wavefunction at the boundaries between the QD and the corresponding barrier. After solving it numerically we obtain a non-equidistant spectrum with non-uniform level spacing E. The spectrum and the associated properties of the eigenfunctions, including the spin polarization, strongly depend both on the QD size L and on the relative orientation and magnitude of the barrier magnetizations defined by the parameters $M_{1,2}$ and $\Theta_{1,2}$, respectively. Below we will present several typical examples.

Let us first consider the case of parallel magnetizations $\Theta_1$=$\Theta_2$=0, for the magnetization energies of both barriers $M_1$ = $M_2$ = 20 meV which is equal to the one half of the band gap in HgTe/CdTe quantum well. The first plot to be shown is the energy level dependence on the QD size L shown in Fig.2(a). On can see that the number of levels in the QD grows with the growing QD size L, and the interlevel distance decreases in general accordance with the expression for the non-transparent barriers. It should be specially noted that for the sufficiently narrow QD when L ≤ 70 nm there are only two levels available which turns the QD into an effective twolevel system suitable for the qubit applications. For low magnetization energies $M_{1,2}$ ≤ 50 meV and for fixed L = 30 nm there are only two levels available inside QD, which again is a sign of a stable two-level system suitable for qubit applications. It turns out from the specific form of the wavefunction that the z component of the spin density is always zero. In Fig.3(a),(b) we show the examples of level structure (top) and two-dimensional spin density vector field (Sx(y), Sy(y)) inside the QD and in the neighboring barrier regions (bottom). The results in Fig.3(a) are shown for the narrow QD with L = 30 nm where only two levels are available, and in Fig.3(b) a wider QD with L = 110 nm is considered where four levels are formed. It can be seen that different levels have different spin textures, which should be taken into account during the Fermi level variations leaving certain level being filled.

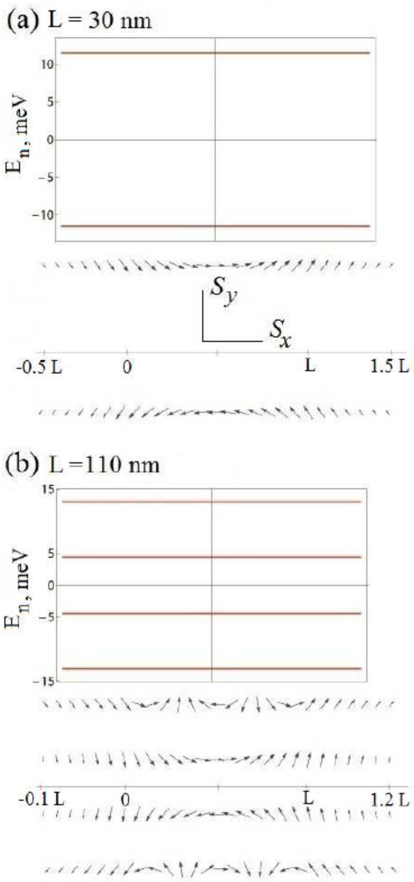


FIG. 3. Level structure (top) and twodimensional spin density vector field ($S_x(y)$,$S_y(y)$) inside the QD and in the neighboring barrier regions (bottom) for (a) the QD with L = 30 nm and (b) the QD with L = 110 nm.

One of possible applications of the tunable QD described by our model can be the scheme for the realization of qubit. It is of interest to find out whether the proposed QD model for sufficiently narrow length L ≤ 40 nm where only two levels are available for the confined electrons may actually work as a potential qubit. The first step, according to the Di Vincenzo criteria for quantum computation schemes [8] is an estimate on how fast the qubit may operate under the control gate pulses. We thus proceed with a simple quantum mechanical calculation of the two-level population swap under the monochromatic electric field on the frequency equal to the level splitting, and extract the resulting Rabi frequency both analytically and numerically. We consider the perturbation for the initial Hamiltonian (3) as a scalar potential V (y, t), in the form of spatially uniform electric field $E_0$ created by electrostatic gates. The field is harmonic with frequency $\omega_0 = (E_2 - E_1)/\hbar$ matching the level splitting:

$$V(y,t) = e\varepsilon_0 y \cdot \cos(\omega_0 t) \quad (3)$$

The transition between the initial states on level $E_1$ to the upper level $E_2$ is characterized by the Rabi frequency $\omega_R = 2\pi/T_{12}$ where $T_{12}$ is the time period during which the level $E_1$ population described by decreases from 1 to 0, and the upper levels population rises from 0 to 1. For two-level system in the rotating wave approximation (RWA) the Rabi frequency can be found analytically. We see that the RWA approximation is in a good agreement with numerical solution. For the narrow two-level QD with L = 30 nm the level spacing $E_2 - E_1$ = 23 meV which corresponds to the driving frequency $\omega_0$ = $3.5 \cdot 10^{13}$ $s^{-1}$.

The maximum achievable Rabi frequency, as it follows from the computational results, can reach as much as $10^{11} \ldots 10^{12}$ $s^{-1}$ which indicates an acceptable range of qubit operation time top ≈ $10^{-11}$ s. Taking into account our results on the qubit operation time top ≈ $10^{-11}$ s and the estimates of the relaxation times discussed above, we may conclude that our two-level QD when being viewed as a qubit can handle up to $10^5 \ldots 10^8$ operations, which is an acceptable value for further studies of quantum computation schemes with this proposal. It should be noted, however, that the experiments on magnetoconductivity in HgTe quantum wells show the relaxation times which are of the order of $10^{-10}$ sec. [9] These rather short relaxation times are applicable to the 2D electron gas inside the QW, i.e. for the bulk states rather than for the edge states which are considered here. One could say with certainty that any possible qubit applications of QD formed at the edge of TI in HgTe quantum well require further experimental studies of relaxation times for these edge states.

## f. CONCLUSIONS

We have studied the quantum states, their spin density patterns, frequency dependence of the optical absorption coefficient, and two-level driven dynamics for the electrons in a quantum dot formed at the edge of two-dimensional HgTe/CdTe topological insulator representing a new class of materials with Weyl massless energy spectra, where the motion of carriers is less sensitive to disorder and impurity potentials. The predicted structure of absorption peaks can be served as a spectroscopic tool for the experimental observation of QD formation where discrete energy levels arise in the band gap of the host material. The dynamical properties of twolevel dot driven by monocrhomatic electric field indicate the possibility of a fast level switch operation suitable for further qubit applications as a new example of two-level system. The predicted properties are interest for future investigation in both fundamental and applied properties of QD formed in topological insulators as future elements of spintronics and nanoelectronics. For metrology is now ongoing studies are of great importance. Since most modern measuring instruments are computerized, the introduction of quantum computing would require the development of fundamentally new programming language for the new architecture of computers. There will be a question of the development of evaluation software protection algorithms related to metrological characteristics of measuring instruments.

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