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The method of identifying the distribution laws in estimating the results of multiple measurements

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Abstract

The article is devoted to the development of a new method for identifying the distribution laws in the evaluation of the results of multiple measurements. Identification of the distribution laws is today an actual metrological task, because the adopted limitations on the number of measurements and assumptions about the law of distribution of a random error can introduce additional uncertainty into the evaluation of the result of measurements.

The use of well-known classical approaches to the identification of distribution laws involves a number of difficulties that are related to the need to use the completeness of the considered set of models and the correctness of the application of the corresponding statistical methods.

The information approach used in the evaluation of measurement uncertainty allows to express the relationship between the error information characteristic — the entropy value of the error and the probabilistic error characteristic — the root-mean-square deviation. Since the form of the distribution law is characterized by antikurtosis, the classification of distribution laws was considered in the two-dimensional space of the entropy coefficient of the distribution law and its antikurtosis. This approach formed the basis for the developed method of identifying distribution laws.

A model of the method for identifying distribution laws using the entropy coefficient of the distribution and antikurtosis law is obtained. A comparative analysis of the laws of distribution of measurement errors using software is made, which allows simulating the noise effect that adheres to the distributions in question.

Keywords: entropy, error, uncertainty, distribution law, histogram.

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1. Introduction

Identification of the law of distribution of measurement results by histogram is an important metrological task, since the value of the root-mean-square deviation of the measurement error depends on the law of distribution of the measurement results. On the basis of the central limit theorem, it is considered that the law of distributions of random errors is normal [1]. Thus, the adopted limitations on the number of measurements and assumptions about the distribution law of the random error can introduce additional uncertainty into the evaluation of the measurement result. This estimate is difficult to formulate unambiguously because of the variety of laws for the distribution of random variables [2].

2. Analysis of the literature data and statement of the problem

Classical approaches to solving the problems of identification of the distribution law are based on the verification of statistical hypotheses using various criteria of agreement, for example, Pearson, Kolmogorov-Smirnov, and others [3]. This approach to the identification of the distribution law consists in the sequential implementation of the next two-stage procedure for each type of parametric model from the set of laws under consideration. At the first stage of the procedure, based on the sample data, a model of a certain type of law is constructed (from the set of models under consideration) and the parameters of this model are estimated. At the second stage, the degree of adequacy of the obtained model to the experimental observations is estimated [4].

The second approach used to identify distribution laws is to approximate the empirical law of distribution by least-squares method. This approach for different distribution laws differs in the efficiency of calculations and in the complexity of comparison [5].

A grapho-analytical method for identifying distribution laws is also known [6]. This method allows to estimate the form of the distribution law for samples of a small volume. The essence of the method is as follows. On the grid, which axes are encoded in the appropriate scale for a particular distribution law, experimental points are plotted. If these points "fall" on one straight line, then their distribution is consistent with this particular distribution law.

The use of classical approaches involves a number of difficulties, which are connected with the need to use the completeness of the considered set of models and the correctness of the application of the corresponding statistical methods [7]. The use of statistical methods, the need for an informed choice of the criterion of consent when solving the problem of identifying the distribution requires a high qualification of the specialist.

At the same time, the information approach to the estimation of measurement uncertainty allows to express the relation between the error information characteristic – the entropy value of the error and the probabilistic error characteristic – the root-mean-square deviation. Taking into account that the form of the distribution law is characterized by *antikurtosis* coefficient, it is convenient to classify the distribution laws in the two-dimensional space of the entropy coefficient of the distribution law and its *antikurtosis* coefficient [8], which can be used to identify distribution laws.

The development of a method for identifying the distribution laws when evaluating the measurement results of which would characterized by simplicity, convenience and visibility, but would not require high qualification of a specialist and expensive software is an actual scientific and applied problem.

3. Purpose and objectives of the study

The aim of this paper is to develop a method for identifying the law of distribution of a random variable based on comparing empirical laws of distribution of a random variable in a multidimensional vector space by determining the proximity of distributions by correlation coefficients.

To achieve this goal, it is necessary to solve the following tasks:

- to develop a model of a method for identifying distribution laws using the entropy coefficient of the distribution and *antikurtosis* law;
- to perform a comparative analysis of the laws of distribution of measurement errors using software that simulates a noise effect that adheres to the distributions in question.

4. Construction of a mathematical model of the method for identifying the laws of error distribution using the entropy coefficient of the distribution and antikurtosis coefficient law

The laws for the distribution of errors of individual measuring instruments must be classified in order to assess the accuracy of the measurement results.

From the position of the probability theory, the form of the distribution law is numerically characterized by its antikurtosis $\varkappa = \sigma^2 / \sqrt{\mu_4}$, and from the position of information theory – by the value of the entropy coefficient k=Δ/σ. For all possible distribution laws ϰ varies from 0 to 1, and k – from 0 to 2,076, so the classification of the distribution laws is conveniently considered in the (ϰ, k) – plane in which each law can be characterized by a certain point. For example, for a uniform law, for which the fourth moment

$$\mu_4 = \frac{\Delta^4}{5} \text{ and the antikurtosis}$$

$$\varkappa = \frac{\sigma^2}{\sqrt{\mu_4}} = \frac{\Delta^2}{3}\frac{1}{\sqrt{\mu_4}} = \frac{\Delta^2}{3}\frac{\sqrt{5}}{\Delta^2} = \frac{\sqrt{5}}{3} \approx 0,74,$$

such a point has the coordinates (ϰ=0,74; k=1,73); for a normal law, for which the fourth moment $\mu_4 = 3\sigma^4$ and the antikurtosis,

$$\varkappa = \frac{\sigma^2}{\sqrt{\mu_4}} = \frac{\sigma^2}{\sqrt{3\sigma^4}} = \frac{1}{\sqrt{3}} \approx 0,58,$$

the point has coordinates ($\varkappa$=0,58; k=2,07).

To estimate the accuracy of the classification of the laws of distribution of a random variable by classical and proposed identification methods, 40 series of experiments were carried out for 100 test samples of a random variable distributed over the logistic and gamma distributions. To calculate the entropy coefficient and the correlation coefficients from the final sample, the probability density function of the random variable was evaluated using the corresponding histogram at 12 intervals of the grouping. An even number of intervals is chosen to smooth the peaks of the histogram centers and complicate in this experiment the problem of classifying the distribution laws of random variables solved by the method with the use of correlation coefficients on the set of logistic, triangular, normal and other distributions.

We will classify some laws of distribution of a random variable from the experimental data of the model sample and calculate the distances between the mappings of the distribution laws in the ($\varkappa$, k)-plane. To calculate the probabilistic and information characteristics of the distribution laws from the experimental data of simulated samples with certain probability distribution laws, we use the program, the algorithm of which is shown in Fig. 1 (Annex 1).

Comparative analysis and identification of the laws of distribution of measurement errors was carried out by means of Python. Today Python is the ideal language for quickly writing various applications running on most common platforms [9]. Python is a freely available software product, which makes it possible to widely use the results of development.

The results of calculating the probability and information characteristics of the distribution laws are summarized in Table 1. The histograms of the generated samples of random variables with the corresponding distribution laws are shown in Fig. 2.

Calculation of distances between the laws of probability distribution in the ($\varkappa$, k)-plane. The result of the analysis is shown in Fig. 3. The graph below shows the division of the most common laws of distribution of measurement errors into two groups. On the plane in the lower left corner is the Pareto law, and in the upper right — a group of laws that are close in information indicators to Gaussian.

The results of the calculation of the probability and information characteristics of the distribution laws from the experimental data of the simulated samples (Tables 1, 2) show that the closest to the test sample 7 will be the normal distribution law. This does not correspond to the initial model of the test sample 7. The closest to test sample 6 will be the gamma distribution law, which corresponds to the initial model of the test sample 6.

The developed method for identifying the distribution law can be used in the control systems of parameters when selecting a data filtering algorithm [10, 11].

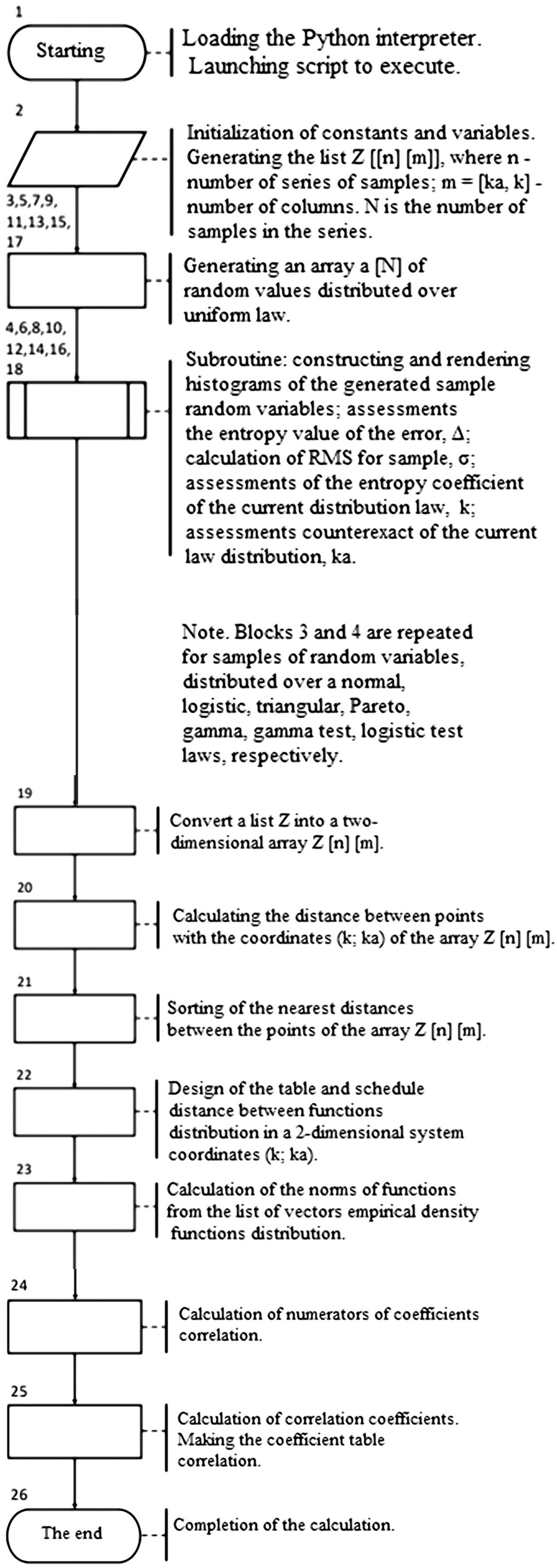

Fig.1. Algorithm of the program for calculating the probability and information characteristics of distribution laws from the experimental data of simulated samples

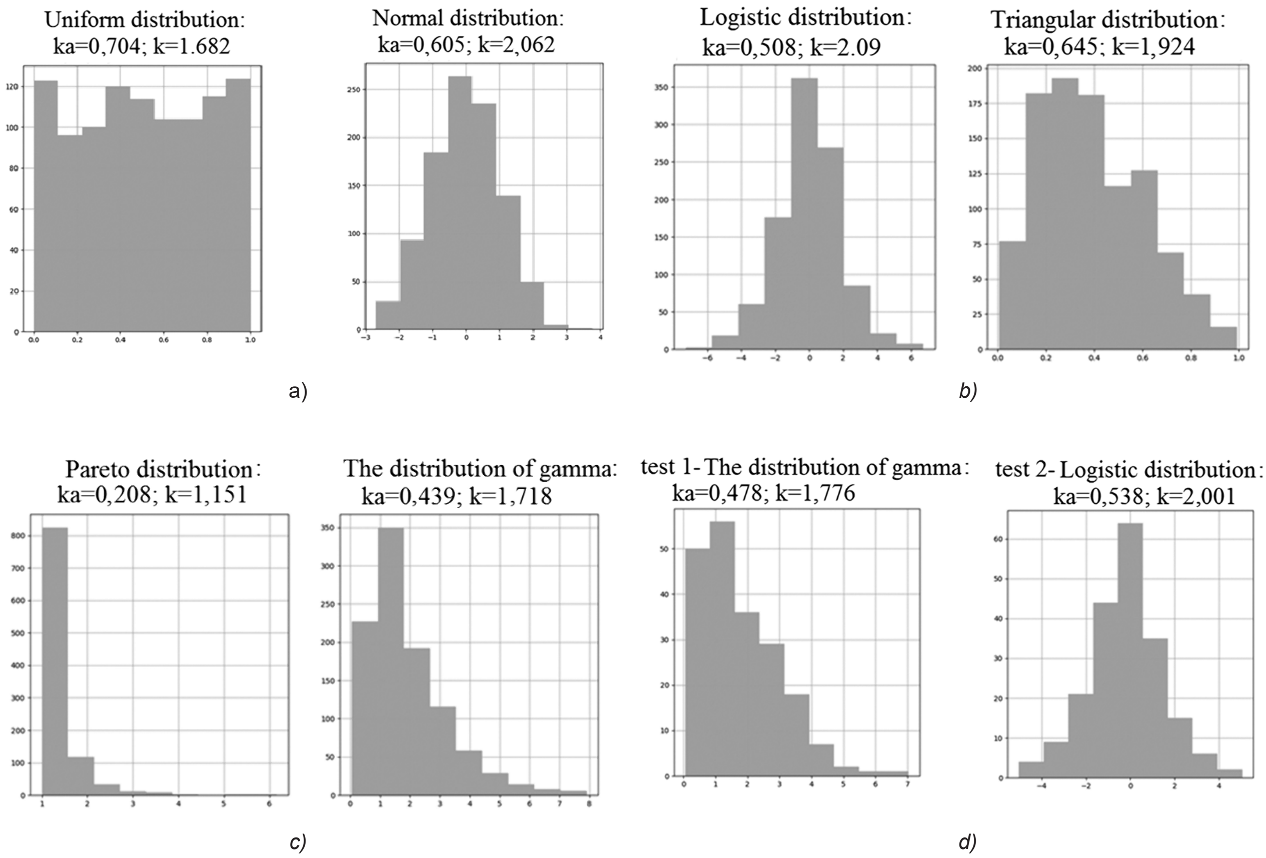

Fig. 2. Histograms of the generated samples of random variables with different distribution laws, where *ka* is the antikurtosis; *k* is the entropy coefficient of the distribution law

Table 1

Results of calculating the probability and information characteristics of distribution laws from the experimental data of simulated samples with some probability distribution laws (Annex 1)

Error distribution law	Mean square error, σ	Entropy value of error, $\Delta = \pm \frac{1}{2} e^{H_\Delta}$	Entropy coefficient of uniform distribution law, $k=\Delta/\sigma$	Antikurtosis, $\varkappa = \sigma^2 / \sqrt{\mu_4}$
Uniform	0,30	0,50	0,74351312	1,68185207
Normal	1,04	2,13	0,60477625	2,06180561
Logistic	1,85	3,87	0,50800657	2,08984593
Triangular	0,22	0,41	0,64530733	1,92406677
Pareto	0,47	0,54	0,20830584	1,15149634
Gamma	1,35	2,31	0,43895244	1,71772379
Gamma (test1)	1,22	2,17	0,47783109	1,77634945
Logistic (test2)	1,70	3,40	0,5383763	2,00119226

Table 2

Results of calculating distances between the laws of probability distribution in the ($\varkappa$, k)-plane (Annex 1)

Error distribution law		Identifier							
Identifier	Name	further							
0	Uniform	0	3	6	5	7	1	2	4
1	Normal	1	7	2	3	6	5	0	4
2	Logistic	2	7	1	3	6	5	0	4
3	Triangular	3	7	1	2	6	0	5	4
4	Pareto	4	5	6	0	3	7	2	1
5	Gamma	5	6	3	7	0	2	1	4
6	Gamma (test 1)	***6***	***5***	3	7	0	1	2	4
7	Logistic (test 2)	***7***	***1***	2	3	6	5	0	4

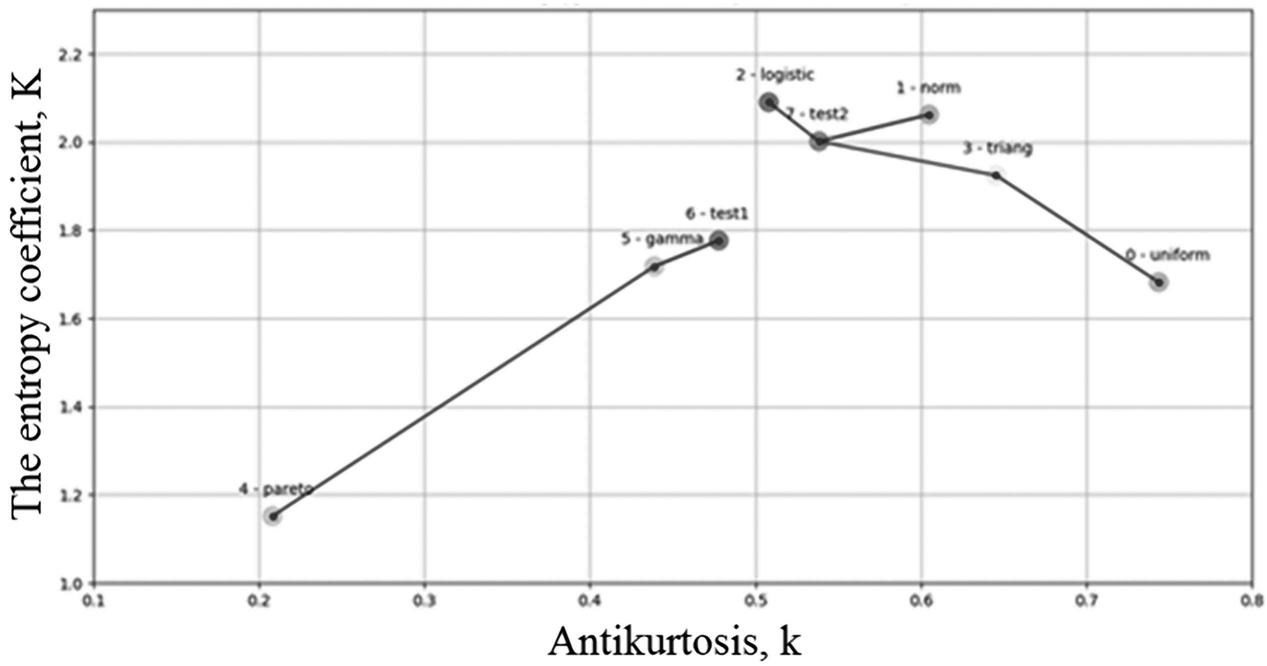

Fig. 3. Distances between the laws of probability distribution in the (ϰ, *k*)-plane (Annex 1)

5. Identification of distribution laws from experimental data of simulated samples using the correlation coefficient

To determine the degree of independence or similarity of one set of data with another, or one process with another, a correlation analysis of variables is often used, which gives additional information about their relationship. There are several methods for estimating the correlation dependence of quantitative indices [5], among which the general linear correlation coefficient developed by Karl Pierson, Francis Edgeworth and Raphael Weldon:

$$r_{XY} = \frac{cov_{XY}}{\sigma_X \sigma_Y} = \frac{\sum (X - \overline{X})(Y - \overline{Y})}{\sqrt{\sum (X - \overline{X})^2 \sum (Y - \overline{Y})^2}}, \qquad (1)$$

where $\overline{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$, $\overline{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$ — the average arithmetic values of the samples; N — the number of sample values; σ_X, σ_Y — standard deviations.

The linear correlation coefficient takes values in the range [−1; +1]. The boundary values −1, +1 of the linear correlation coefficient mean the total correlation, and the value 0 means no correlation.

In this paper, we use the correlation coefficient to estimate the proximity of the probability density functions, and calculate for pairs of different probability densities represented by histograms in a multidimensional vector space, on the orthonormal basis of unit sampling intervals:

$$r_{XY} = \frac{\sum_{i=1}^{N} X_i Y_i}{\sqrt{\sum_{i=1}^{N} X_i^2} \sqrt{\sum_{i}^{N} Y_i^2}}. \qquad (2)$$

When calculating the correlation coefficients according to formula (2), we do not take into account the normalization of the distribution laws represented by histograms, since in this case this does not affect the value of the correlation coefficients

Table 3

Coefficients of correlation of distribution laws from experimental data of simulated samples

Error distribution law		Correlation coefficient							
Identifier	Name	0	1	2	3	4	5	6	7
0	Uniform	1	0,752	0,666	0,847	0,426	0,671	0,723	0,738
1	Normal	0,752	1	0,86	0,909	0,127	0,592	0,667	0,948
2	Logistic	0,666	0,86	1	0,687	0,027	0,29	0,379	0,969
3	Triangular	0,847	0,909	0,687	1	0,298	0,828	0,855	0,793
4	Pareto	0,426	0,127	0,027	0,298	1	0,594	0,658	0,082
5	Gamma	0,671	0,592	0,29	0,828	0,594	1	0,984	0,424
6	Gamma (test 1)	0,723	0,667	0,379	0,855	0,658	**0,984**	1	0,515
7	Logistic (test 2)	0,738	0,948	**0,969**	0,793	0,082	0,424	0,515	1

and, consequently, the calculation by formula (1) is simplified.

Based on the obtained matrix of values of correlation coefficients, we carry out a classification evaluation of unknown distribution laws from the experimental data of simulated samples, Table 3.

The results of calculating the correlation coefficients of the distribution laws from the experimental data of the simulated samples, Table 4, show that the closest to the test sample 7 will be the logistic distribution law, which corresponds to the initial model of the test sample 7. The closest to test sample 6 will be the gamma distribution law, which corresponds to the initial model of the test sample 6.

Conclusions

From the standpoint of information theory, the accuracy of measurements is characterized by the value of the entropy error of the measurement. Therefore, if the error with an arbitrary probability distribution law has the entropy H_Δ, then it can be replaced by an error with a uniform, in general, any other probability distribution law and entropy H_Δ.

Comparing the methods for identifying the laws of distribution of a random variable: the first is based on the probability and information characteristics of a random variable in the ($\varkappa$, k)-plane, and the second is based on comparing empirical laws of the distribution of a random variable in a multidimensional vector space, determining the proximity of distributions by the correlation coefficients and based on the calculations, it can be stated that the latter method is more accurate.

By combining the method of identifying the distribution law, based on the calculation of the correlation coefficients and the method for estimating the entropy interval of uncertainty, it is possible to more accurately estimate the results of multiple measurements.

Annex 1

Program 1. Calculation of the probabilistic and information characteristics of distribution laws from the experimental data of simulated samples with certain probability distribution laws.

```
import matplotlib.pyplot as plt
import numpy as np
import math
from scipy.stats import uniform, norm, logistic, triang, pareto, gamma
from itertools import product

yy=[]
def diagram(a, nr):
  a.sort() # sort the list
  n=len(a) # number of items in the list
  #m= int(10+np.sqrt(n)) # (was) the number of intervals of the partition
  m= int(9) # number of break intervals
  d=(max(a)-min(a))/m # length of the intervals
  x=[];y=[]
  avery = 0
  for i in np.arange(0,m,1):
     x.append(min(a)+d*i) # Adding the boundaries of intervals to the list x
     k=0
     for j in a:
        if min(a)+d*i <= j <= min(a)+d*(i+1):
              k=k+1
     y.append(k)# adding frequencies to the interval in the list y
  yy.append(y) # list: consists of function vectors
  # an estimate of the entropy value of the error (23)
  #delta=0.5*d*n*10**(-sum([w*np.log10(w) for w in y if w!=0])/n) # formula (23)
  # or formulas (21)+(15)
  delta=0.5*np.exp(sum([w*np.log(n/w)/n for w in y if w!=0]) + np.log(d))
  # estimation of the entropy coefficient of the current distribution law
  k=delta/np.std(a) # formula (16)
  sigma = np.std(a) # calculation of the standard deviation of the sample
  print('sigma = %.2f'%sigma)
  mu4=sum([(w-np.mean(a))**4 for w in a])/n
  # evaluation of the antikurtosis of the current distribution law
  ka=(np.std(a))**2/np.sqrt(mu4)
  # Drawing the histogram
  plt.title("%s : ka = %s;  k = %s"%(nr,str(round(ka,3)),str(round(k,3))))
  plt.bar(x, y, d, align='edge', alpha=0.5, color='g')
  return [ka, k]
```

```
# list generation
n = 8 # rows (number of series of samples)
m = 2 # of columns [ka k]
Z = [[0] * m for i in range(n)]
N = 1000 # number of samples in the series
# inscription list
snr = ['0 — uniform', '1 — norm', '2 — logistic', '3 — triang', '4 — pareto', '5 — gamma', '6 — test1', '7 — test2']
nr="Uniform distribution "
a=uniform.rvs(size=N)
plt.subplot(121)
Z[0] = diagram(a, nr)
plt.grid(True)
nr="Normal distribution"
b=norm.rvs(size=N)
plt.subplot(122)
Z[1] = diagram(b, nr)
plt.grid(True)
plt.show()
nr="Logistic distribution"
c=logistic.rvs(size=N)
plt.subplot(121)
Z[2] = diagram(c, nr)
plt.grid(True)
nr="Triangular distribution"
cd = 0.158
d = triang.rvs(cd, size=N)
plt.subplot(122)
Z[3] = diagram(d, nr)
plt.grid(True)
plt.show()
nr=" Pareto distribution "
e = pareto.rvs(4, size=N)
plt.subplot(121)
Z[4] = diagram(e, nr)
plt.grid(True)
nr= Gamma distribution"
cf = 1.99
f = gamma.rvs(cf, size=N)
plt.subplot(122)
Z[5] = diagram(f, nr)
plt.grid(True)
plt.show()
# test sample 1
nr=" test1-Gamma distribution "
cf_t = 1.99
t1 = gamma.rvs(cf_t, size=200)
plt.subplot(121)
Z[6] = diagram(t1, nr)
plt.grid(True)
# test sample 2
nr="test2-Logistic distribution"
t2 = logistic.rvs(size=200)
plt.subplot(122)
Z[7] = diagram(t2, nr)
plt.grid(True)
plt.show()
plt.grid(True)
# converting the list to an array
Z = np.array(Z)
# Array Formatting
Z = Z.reshape((n, m))
print("\n Z = ")
print(Z)
# calculating the distance between points
```

```
dist_sq = np.sum((Z[:, np.newaxis,:] — Z[np.newaxis,:,:]) ** 2, axis = –1)
# sorting of the nearest distances between points
nearest = np.argsort(dist_sq, axis = 1)
print("\n nearest = ")
print(nearest)
# Draw lines between the nearest to points
K = 1
nearest_partition = np.argpartition(dist_sq, K + 1, axis = 1)
#draw points
colors = np.random.rand(n) #n=8 — number of series of samples
area = 144 # yfdcrblre
plt.scatter(Z[:, 0], Z[:, 1], s=area, c=colors, alpha=0.5)
#Drawing text near points
for i in range(Z.shape[0]):
    plt.text(Z[i, 0]-0.02, Z[i, 1]+0.05, snr[i])
#Drawing lines
for i in range(Z.shape[0]):
    for j in nearest_partition[i, :K+1]:
        plt.plot(*zip(Z[j], Z[i]), color=(0.1, 0.2, 0.5))
plt.plot(Z[:, 0], Z[:, 1], "bo", markersize = 4)
plt.axis([0.1, 0.8, 1, 2.3])
plt.title("Distance between functions in 2-D coordinate system")
plt.xlabel(r'$ antikurtosis, \kappa$')
plt.ylabel(r'$ entropy coefficient, k$')
plt.show()
plt.grid(True)
```

Метод ідентифікації законів розподілу при оцінці результатів багаторазових вимірювань

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Анотація

Статтю присвячено розробці нового методу ідентифікації законів розподілу при оцінці результатів багаторазових вимірювань. На сьогодні ідентифікація законів розподілу є актуальним метрологічним завданням, оскільки прийняті обмеження за кількістю вимірювань і допущення про закон розподілу випадкової похибки можуть внести додаткову невизначеність в оцінці результатів вимірювань.

Використання відомих класичних підходів щодо ідентифікації законів розподілу поєднано з рядом труднощів, які пов'язані з необхідністю використання повноти розглянутої безлічі моделей та коректності застосування відповідних статистичних методів.

Використовуваний у роботі інформаційний підхід щодо оцінки невизначеності вимірювань дозволяє визначитися зі ставленням між інформаційною характеристикою похибки – ентропійним значенням похибки та ймовірнісною характеристикою похибки – середньоквадратичним відхиленням. Позаяк форма закону розподілу характеризується контрексцесом, класифікацію законів розподілу було розглянуто у двовимірному просторі ентропійного коефіцієнта закону розподілу та його контрексцесу. Такий підхід ліг в основу розроблювального методу ідентифікації законів розподілу.

У роботі отримано модель методу ідентифікації законів розподілу із використанням ентропійного коефіцієнта закону розподілу і контрексцесу. Виконано порівняльний аналіз законів розподілу похибок вимірювання із використанням програмного забезпечення, яке дозволяє імітувати вплив шуму, що підкоряється розглянутим розподілам.

Ключові слова: ентропія, похибка, невизначеність, закон розподілу, гістограма.

Метод идентификации законов распределения при оценке результатов многократных измерений

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Аннотация

Статья посвящена разработке нового метода идентификации законов распределения при оценке результатов многократных измерений. Идентификация законов распределения является сегодня актуальной метрологической задачей, поскольку принятые ограничения по количеству измерений и допущения о законе распределения случайной погрешности могут внести дополнительную неопределённость в оценке результатов измерений.

Использование известных классических подходов по идентификации законов распределения сопряжено с рядом трудностей, которые связаны с необходимостью использования полноты рассматриваемого множества моделей и корректности применения соответствующих статистических методов.

Используемый в работе информационный подход к оценке неопределённости измерений позволяет выразить отношение между информационной характеристикой погрешности — энтропийным значением погрешности и вероятностной характеристикой погрешности — среднеквадратичным отклонением. Так как форма закона распределения характеризуется контрэксцессом, классификация законов распределения была рассмотрена в двумерном пространстве энтропийного коэффициента закона распределения и его контрэксцесса. Данный подход лег в основу разрабатываемого метода идентификации законов распределения.

Получена модель метода идентификации законов распределения с использованием энтропийного коэффициента закона распределения и контрэксцесса. Выполнен сравнительный анализ законов распределения погрешностей измерения с использованием программного обеспечения, которое позволяет имитировать шумовое воздействие, подчиняющееся рассматриваемым распределениям.

Ключевые слова: энтропия, погрешность, неопределенность, закон распределения, гистограмма.

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