

# Measurement uncertainty evaluation by kurtosis method at calibration of electrical resistance standards using a comparator

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## Abstract

The article provides examples of the application of electrical resistance standards in metrological practice. Existing methods for their calibration are analyzed. It is established that calibration using a comparator is the most accurate and common method for calibrating electrical resistance standards.

A model for transferring the size of the resistance unit in calibration of electrical resistance standards using a comparator is considered. An expression is given for the evaluation of the value of measurand. The procedure for estimating the expanded measurement uncertainty based on the kurtosis method is described, and the uncertainty budget is drawn up. An example of evaluation of the measurement uncertainty in calibration of the resistance coil P321 using a resistance comparator P3015 is given. The coincidence of the obtained results with those got using the Monte Carlo method is shown.

**Keywords:** electrical resistance standard; resistance comparator; calibration; measurement uncertainty; uncertainty budget; kurtosis method; Monte-Carlo method.

Received: 24.02.2020

Edited: 16.03.2020

Approved for publication: 18.03.2020

## Introduction

Electrical resistance standards (ERS) are widely used as precision resistors that are built into instruments and measuring systems, working and reference standards of electrical resistance used to verify (calibrate) digital ohmmeters, precision shunts and additional resistances, designed to expand the limits of measurement of electrical measuring instruments by current and voltage in direct and alternating current circuits up to a frequency of 1 MHz.

There are various methods for calibrating ERS: direct measurement using a digital ohmmeter, measurement using a DC bridge (direct measurement, measurement by substitution method and method of permutation), using a resistance comparator, using a voltage comparator or DC potentiometer [1]. Resistance comparator measurement is the most accurate and common method for calibrating ERS.

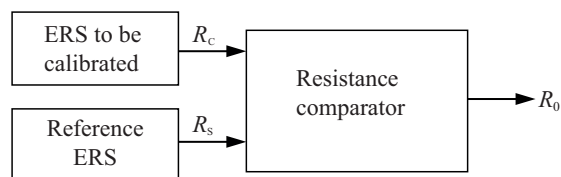
Clause 7.8.6 of ISO/IEC 17025:2017 [2] provides for the possibility of including a statement of conformity in the calibration certificate of a measuring instrument. In this case, the conclusion of conformity should be taken taking into account the expanded measurement uncertainty indicated in the certificate. Therefore, the level of risk associated with the applicable rule for deciding on compliance (clause 7.8.6.1 [1]) will depend on the reliability of the expanded uncertainty evaluation.

It was shown in [3] that it is possible to ensure a high reliability of the expanded uncertainty evaluation using the kurtosis method.

**The purpose** of the article is to create a procedure for measurement uncertainty evaluation by the kurtosis method at calibration of an ERS using a comparator.

## Statement of the main material

Calibration of ERS using a comparator is performed in accordance with the block scheme shown in Fig. [4].



Block scheme of comparison of ERS to be calibrated and reference ERS using a comparator

The resistance value of the ERS to be calibrated  $R_c$  is obtained as a result of calibration on the basis of the measurement model (equation) [4]:

$$R_c = (R_s + \Delta_s) + (R_0 + \Delta_0), \quad (1)$$

where  $R_s$  is resistance of the reference ERS;  $\Delta_s$  is correction for the instability of the reference ERS during the inter-calibration interval;  $R_0$  is quantity indicated by

the comparator;  $\Delta_0$  is correction for the temperature error of the comparator.

The resistance value of the reference measure  $\bar{R}_s$  is taken from its calibration certificate. The value  $\bar{R}_0$  is determined by the results  $n$  of the comparator  $\delta_{0i}$ , %:

$$\bar{R}_0 = \frac{\hat{R}_s}{n \cdot 100} \sum_{i=1}^n \delta_{0i}. \quad (2)$$

$$u(\hat{R}_c) = \sqrt{u_B^2(\hat{R}_s) + u_B^2(\hat{\Delta}_s) + u_A^2(\bar{R}_0) + u_B^2(\hat{R}_0) + u_B^2(\hat{\Delta}_0)}, \quad (4)$$

where  $u^2(\hat{R}_s)$  is standard uncertainty of type B of the reference ERS, which is calculated through the value of the expanded uncertainty  $U(\hat{R}_s)$ , and the coverage factor taken from its calibration certificate:

$$u(\hat{R}_s) = U(\hat{R}_s)/k_s; \quad (5)$$

$u_B(\hat{\Delta}_s)$  is standard uncertainty of type B due to the instability of the reference ERS during the calibration interval, which is found through the value of the relative boundaries of this instability  $\delta_s$  under the assumption of a uniform distribution of instability within the boundaries:

$$u(\hat{\Delta}_s) = \delta_s \frac{\hat{R}_s}{\sqrt{3} \cdot 100}; \quad (6)$$

$u_A(\bar{R}_0)$  is type A standard uncertainty due to variability of comparator readings  $\delta_{0i}$ :

$$u_A(\bar{R}_0) = \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^n (\delta_{0i} \frac{\hat{R}_s}{100} - \bar{R}_0)^2}; \quad (7)$$

$u_B(\hat{R}_0)$  is the standard uncertainty of type B, due to the correction for the main error of the comparator, is determined from the expression for the relative boundaries of this error  $\gamma_0$ , taken from the technical description of the comparator under the assumption of a uniform distribution within the boundaries:

$$u_B(\hat{R}_0) = \gamma_0 \frac{\bar{R}_0}{\sqrt{3} \cdot 100}; \quad (8)$$

$u_B(\hat{\Delta}_0)$  is standard uncertainty of type B, due to the correction for the temperature error of the comparator, is determined  $u_B(\hat{R}_0)$  by the deviation of the ambient temperature  $t_{amb}$  from 20 °C by the formula:

$$u_B(\hat{\Delta}_0) = \frac{|t_{amb} - 20^\circ\text{C}|}{10^\circ\text{C}} u_B(\hat{R}_0). \quad (9)$$

Since corrections  $\Delta_s, \Delta_0$  are centered quantities, therefore their estimates  $\hat{\Delta}_s, \hat{\Delta}_0$  are zero.

Measured value  $\hat{R}_c$  are found by substituting in (1) estimates of input quantities:

$$\hat{R}_c = \hat{R}_s + \bar{R}_0. \quad (3)$$

Standard uncertainty of the measured quantity  $u(R_c)$  (combined standard uncertainty) will be determined through standard uncertainties of the input quantities included in (1) from the expression:

Expanded uncertainty is calculated by the formula:

$$U(\hat{R}_c) = k \cdot u_c(\hat{R}_c), \quad (10)$$

where  $k$  is coverage factor determined by the kurtosis method according to the formula [3]:

$$k = \begin{cases} 0,12\eta^3 + 0,1\eta + 2, & \text{at } \eta < 0; \\ 2, & \text{at } \eta \geq 0. \end{cases} \quad (11)$$

Here  $\eta$  is the kurtosis of the measurand, which is calculated as:

$$\eta = \left( \sum_{j=1}^m \eta_j u_j^4(y) \right) / u_c^4(y), \quad (12)$$

where  $u_j(y)$  and  $\eta_j$  are contribution of uncertainty of  $j$ -th input quantity into the uncertainty of the measurand and its kurtosis, respectively. The values of kurtosis for different distribution laws are given in [3].

The uncertainty budget for this case is given in Table 1.

As an example, let's consider the calibration of an ERS of 1  $\Omega$  resistance, type P321, class 0,01 by comparison with the working standard P321 of 1,000020  $\Omega$  and an expanded uncertainty of 0,00001  $\Omega$  taken from a calibration certificate using a resistance comparator P3015 at an ambient temperature of 23 °C.

The comparator readings  $\delta_{0i}$ , % [4] are:  
0,00295; 0,00315; 0,00323; 0,00356; 0,00319;  
0,00282; 0,00298; 0,00304; 0,00298; 0,00295.

Let's determine the standard uncertainty of type B of the reference ERS  $u_B(\hat{R}_s)$  through its expanded uncertainty  $U(\hat{R}_s) = 0,00001 \Omega$  and coverage factor  $k_s = 2$  by the formula (5):

$$u_B(\hat{R}_s) = \frac{0,00001\Omega}{2} = 0,000005 \Omega.$$

Standard uncertainty due to instability of the reference ERS during the calibration interval  $u_B(\hat{\Delta}_s)$

Table 1

Measurement uncertainty budget for ERS calibration using a comparator

Input quantity	Input quantity estimate	Standard uncertainty	Kurtosis of input quantity	Sensitivity coefficient	Uncertainty contribution
$R_s$	$\widehat{R}_s$	(5)	0	1	(5)
$\Delta_s$	0	(6)	-1,2	1	(6)
$R_0$	(2)	(7)	$6/(n-5)$	1	(7)
		(8)	-1,2	1	(8)
$\Delta_0$	0	(9)	-1,2	1	(9)
Measurand	Measurand estimate	Combined standard uncertainty	Kurtosis of measurand	Coverage factor	Expanded uncertainty
$R_c$	(3)	(4)	(12)	(11)	(10)

we find through the value of the relative boundaries of this instability  $\delta_H = 0,002 \%$  to the formula (6):

$$u_B(\Delta_s) = 0,002 \frac{1,00002}{\sqrt{3} \cdot 100} = 0,0000115 \Omega.$$

Value  $\overline{R}_0$ , calculated by comparator (Table 2) to the formula (2), amounted to  $0,0000309 \Omega$ , and its standard uncertainty of type A, calculated by the formula (7) is equal to  $u_A(\overline{R}_0) = 0,00000066 \Omega$ .

The standard uncertainty of type B due to the correction for the main error of the comparator  $u_B(R_0)$  is determined by the formula (8) through the expression for the relative boundaries of this error  $\gamma_0 = 0,003 + 0,001 \cdot \overline{\delta}_0$ , taken from the technical description on the comparator, in which  $\overline{\delta}_0 = 0,00309 \%$ . In this case  $\gamma_0 = 0,003003 \%$  and  $u_B(R_0) = 0,0000173 \Omega$ .

Standard uncertainty of type B, due to the correction for the temperature non-excluded systematic error of the comparator  $u_B(\widehat{\Delta}_0)$  will be determined through

$u_B(\widehat{R}_0)$  and the deviation of the ambient temperature of  $23 \text{ }^\circ\text{C}$  from  $20 \text{ }^\circ\text{C}$  according to the formula (9):

$$u_B(\widehat{\Delta}_0) = \frac{3}{10} 0,0000173 = 0,0000052 \Omega.$$

The uncertainty budget for this case is given in Table 2.

The combined standard uncertainty of the measurement of the resistance of the ERS to be calibrated, calculated by the formula (2), will be equal to  $0,000022 \Omega$ .

The kurtosis of the measurand is calculated by the formula (12):

$$\eta = \left( \sum_{j=1}^m \eta_j u_j^4(y) \right) / u_c^4(y) = -0,555.$$

Coverage factor, corresponding to this kurtosis for a coverage level of 0,9545 will be 1,92 based on the formula (11).

Table 2

Measurement uncertainty budget for calibrating ERS P321 using comparator P3015

Input quantity	Input quantity estimate, $\Omega$	Standard uncertainty, $\Omega$	Kurtosis of input quantity	Sensitivity coefficient	Uncertainty contribution, $\Omega$
$R_s$	1,00002	0,000005	0	1	0,000005
$\Delta_s$	0	0,0000115	-1,2	1	0,0000115
$R_0$	0,0000309	0,00000066	1,2	1	0,00000066
		0,0000173	-1,2	1	0,0000173
$\Delta_0$	0	0,0000052	-1,2	1	0,0000052
Measurand	Measurand estimate, $\Omega$	Combined standard uncertainty, $\Omega$	Kurtosis of measurand	Coverage factor	Expanded uncertainty, $\Omega$
$R_c$	1,0000509	0,000022	-0,555	1,92	0,0000423

The expanded uncertainty calculated by formula (11) is:

$$U(R_c) = 1,92 \cdot 0,000022 = 0,0000423 \Omega.$$

The study of the results obtained by the Monte-Carlo method [6] is carried out. The estimates of the measurand  $R_c = 1,0000509 \Omega$ , expanded uncertainty  $0,0000422 \Omega$  and coverage factor  $1,91$  are obtained.

### Conclusions

1. Calibration laboratories accredited for compliance with the requirements of the ISO/IEC 17025:2017 standard should evaluate the measurement uncertainty when performing calibrations. For this purpose, they need to develop a procedure for evaluation of the

measurement uncertainty, which should provide reliable estimates in order to reduce risks in conformity assessment.

2. A procedure for evaluating the uncertainty based on the kurtosis method during calibration of the electrical resistance standard has been proposed; an uncertainty budget has been drawn up, which can serve as the basis for creating a software tool for automating the measurement uncertainty evaluation during calibration.

3. The results of calibration of ERS of type P321,  $1 \Omega$  resistance, accuracy class 0,01 using a working measurement standard are considered, the measurement uncertainty evaluation is carried out. The coincidence of the obtained results with those got using the Monte-Carlo method is shown.

## Оцінювання невизначеності вимірювань методом ексцесів під час калібрування мір електричного опору за допомогою компаратора

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### Анотація

Наведено приклади застосування мір електричного опору в метрологічній практиці. Аналізуються існуючі методи їх калібрування. Встановлюється, що калібрування за допомогою компаратора є найбільш точним та поширеним методом калібрування мір електричного опору. Зазначається, що стандарт ISO/IEC 17025:2017 передбачає можливість включення в сертифікат калібрування висновку про відповідність засобу вимірювання метрологічним вимогам. Оскільки висновок про відповідність має прийматися з урахуванням зазначеної в сертифікаті розширеної невизначеності вимірювань, тому від достовірності її оцінювання буде залежати рівень ризику, пов'язаного із застосовуваним правилом прийняття рішення про відповідність. Пропонується оцінювати розширену невизначеність за допомогою методу ексцесів, який враховує закони розподілу вхідних величин.

Розглянуто модель передачі розміру одиниці опору при калібруванні мір електричного опору за допомогою компаратора. Наведено вираз для оцінювання значення вимірюваної величини. Складено модельне рівняння, записані формули для оцінювання стандартних невизначеностей вхідних величин за типом А і В та формула для оцінювання сумарної стандартної невизначеності. Описано процедуру оцінювання розширеної невизначеності вимірювань на основі методу ексцесів, складено бюджет невизначеності.

Наведено приклад оцінювання невизначеності вимірювань при калібруванні котушки електричного опору P321 за допомогою компаратора опору P3015. Оцінено значення опору котушки, що калібрується, сумарна стандартна та розширена невизначеності, коефіцієнт охоплення рівня довіри 0,9545. Показано збіг отриманих результатів із результатами, які отримані за допомогою методу Монте-Карло.

**Ключові слова:** міра опору; компаратор опору; калібрування; невизначеність вимірювання; бюджет невизначеності; метод ексцесів; метод Монте-Карло.

# Оценивание неопределенности измерений методом эксцессов при калибровке мер электрического сопротивления с помощью компаратора

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## Аннотация

Анализируются существующие методы их калибровки. Устанавливается, что калибровка с помощью компаратора является наиболее точным и распространенным методом калибровки мер электрического сопротивления.

Рассмотрена модель передачи размера единицы сопротивления при калибровке мер электрического сопротивления с помощью компаратора. Приведено выражение для оценки значения измеряемой величины. Описана процедура оценивания расширенной неопределенности измерений на основе метода эксцессов, составлен бюджет неопределенности. Приведен пример оценки неопределенности измерений при калибровке катушки сопротивления P321 с помощью компаратора сопротивления P3015. Оценены значения сопротивления калибруемой катушки, суммарная стандартная и расширенная неопределенности, коэффициент охвата для уровня доверия 0,9545. Показано совпадение полученных результатов с результатами, полученными с помощью метода Монте-Карло.

**Ключевые слова:** мера сопротивления; компаратор сопротивления; калибровка; неопределенность измерения; бюджет неопределенности; метод эксцессов; метод Монте-Карло.

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