UDC 006.91:053.088 DOI:

Accounting for the kurtoses of input quantities in the procedure of evaluating measurement uncertainty using the example of weight calibration

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Abstract

Example 9.3.1.1 of JCGM-S1 "Mass calibration" is analyzed, which describes the comparison in air of reference and calibrated weights having the same nominal mass. JCGM-S1 compares uncertainty evaluation procedures based on the GUM uncertainty framework and the Monte Carlo method.

The article uses the procedure developed by the authors and consists in decomposing the measurement model into a Taylor series of the second order taking into account the kurtoses of the distributions of input quantities. To facilitate the calculations, the finite increment method is used. To find expanded uncertainty, the kurtosis method is used. Good agreement between the results obtained by the proposed method and the result obtained by the Monte Carlo method is shown.

Keywords: measurement uncertainty, calibration, weight, nonlinear model equation, kurtosis method.

Received: 01.06.2020 Edited: 19.07.2020 Approved for publication: 28.07.2020

Introduction

Weight calibration is carried out by comparing its mass with a reference weight having the same mass using balance working in the air. Measurement uncertainty evaluating for this example is discussed in regulatory document JCGM-S1, 9.3.1.1. [1].

In [1], the measurement uncertainty evaluation using the methods of GUM [2] and Monte Carlo [3] is performed. At this, a significant bias in the measurement uncertainty evaluation obtained using GUM is discovered.

To implement the Monte Carlo method, one can apply, for example, the "Uncertainty machine" program developed by NIST [4]. However, one of the drawbacks of such programs is the lack of a total measurement uncertainty budget. Elimination of the above disadvantages is possible using the methods proposed by the authors in [5–7].

The purpose of this article is to implement the methods [5–7] for measurement uncertainty evaluation during calibration of weights with verification of their adequacy.

1. Development of a measurement model

Mass of calibrated weight m_W with mass density ρ_W is compared with a reference weight with mass density ρ_R , having nominally the same mass using balance working in air with mass density ρ_a. Since ρ_W and ρ_R are generally different, buoyancy of air should be taken into account. Therefore, based on the Archimedes law, we can write the following formula [1]:

$$m_W(1-\rho_a/\rho_W) = (m_R + \delta m_R)(1-\rho_a/\rho_R), \qquad (1)$$

where m_R – actual mass of the reference weight; δm_R – actual mass of small weight with density ρ_R, added to equalize balance.

Usually the conventional mass of the calibrated weight $m_{W,c}$ – mass of hypothetical weight with density ρ_0 = 8000 kg/m^3, which balances the calibrated weight in the air with density ρ_{a0} = 1.2 kg/m^3, is determined. Therefore, taking into account the notation of the conventional masses of reference weights $m_{R,c}$, δm_R, model (1) takes the form [1]:

$$m_{W,c}\frac{(1-\rho_a/\rho_W)}{(1-\rho_{a0}/\rho_W)}=(m_{R,c}+\delta m_{R,c})\frac{(1-\rho_a/\rho_R)}{(1-\rho_{a0}/\rho_R)}. \quad (2)$$

Ultimately, for deviation δm of conventional mass of calibrated weight from its nominal value m_{nom}

$$\delta m = m_{\text{W},c} - m_{\text{nom}}, \quad (3)$$

the final model takes the form [1]:

$$\delta m = (m_{R,c}+\delta m_{R,c})\frac{(1-\rho_a/\rho_R)(1-\rho_{a0}/\rho_W)}{(1-\rho_{a0}/\rho_R)(1-\rho_a/\rho_W)} - m_{\text{nom}}. \quad (4)$$

Insofar as the equation (4) is nonlinear, in numerical value evaluation of the measurand and its uncertainty it is necessary to consider higher terms in the expansion (4) in the Taylor series of the second order considering kurtoses of input quantity distributions.

2. Calculation of the numerical value of the measurand

In the first approximation, the calculation of the numerical value of the measurand is carried out according to the formula:

$$\widehat{\delta m} = (\widehat{m}_{R,c}+\widehat{\delta m}_{R,c})\frac{(1-\widehat{\rho}_a/\widehat{\rho}_R)(1-\widehat{\rho}_{a0}/\widehat{\rho}_W)}{(1-\widehat{\rho}_{a0}/\widehat{\rho}_R)(1-\widehat{\rho}_a/\widehat{\rho}_W)} - \widehat{m}_{\text{nom}}, \quad (5)$$

in which the values of input quantities are replaced by their numerical values marked with hats.

For given in [1] estimates of the numerical values of input quantities: $\widehat{m}_{R,\text{c}}$ = 100 g; $\widehat{\delta m}_R$ = 1.234 mg; $\widehat{\rho}_a$ = 1.2 kg/m^3, $\widehat{\rho}_W = \widehat{\rho}_R$ = 8000 kg/m^3 and $\widehat{m}_{\text{nom}}$ = 100 g, we obtain $\widehat{\delta m}$ = 1.234 mg.

Formula (2) gives an unbiased estimate of the numerical value of the measurand only in the absence of uncertainties of input quantities.

The bias Δ_y of the numerical value of the measurand can be estimated taking into account the partial derivatives of the second order of the measurand with respect to the corresponding input quantities $c(x_i)_2$ [5]:

$$\Delta_y = \frac{1}{2}\sum_{i=1}^{N} c(x_i)_2 u^2(x_i), \quad (6)$$

where $u(x_i)$, i = 1,2,...,N – standard uncertainties of input quantities X_1, X_2,...,X_N.

For equation (4) the calculation of the bias for $\widehat{\delta m}$ is performed by the formula:

$$\Delta_c = -\frac{1}{2}[c(m_{R,c})_2 u^2(m_{R,c}) + c(\delta m_{R,c})_2 u^2(\delta m_{R,c}) + \\ + c(\rho_a)_2 u^2(\rho_a) + c(\rho_R)_2 u^2(\rho_R) + c(\rho_W)_2 u^2(\rho_W)]. \quad (7)$$

The expressions for $c(x_i)_2$ are given in Table 1.

Since the second-order partial derivatives of the measurand with respect to all input quantities turned out to be equal to zero, the bias of the measurand numerical value will also be zero.

3. Calculation of the standard uncertainty of the measurand

In the first approximation, the calculation of the standard uncertainty of the measurand is carried out based on the equation:

$$u(\delta m) = \sqrt{c^2(m_{R,c})u^2(m_{R,c}) + c^2(\delta m_{R,c})u^2(\delta m_{R,c}) + c^2(\rho_a)u^2(\rho_a) + c^2(\rho_R)u^2(\rho_R) + c^2(\rho_W)u^2(\rho_W)}, \quad (8)$$

where $u(x_i)$ and $c(x_i)$ – the standard uncertainty of the input quantity X_i and the partial derivative of the measurand with respect to this input quantity (sensitivity coefficient), respectively. Expressions for $c(x_i)$ and their values are given in Table 2.

Table 1

Second-order partial differential expressions

$c(x_i)_2$	**Expressions for** $c(x_i)_2$	**Values** $c(x_i)_2$
$c(m_{R,c})_2$	0	0
$c(\delta m_{R,c})_2$	0	0
$c(\rho_a)_2$	$2(m_{R,c}+\delta m_{R,c})\frac{(\rho_W-\rho_{a0})}{(\rho_R-\rho_{a0})}\cdot\frac{(\rho_R-\rho_W)}{(\rho_W-\rho_a)^3}$	0 m^6/kg
$c(\rho_R)_2$	$2(m_{R,c}+\delta m_{R,c})\frac{(\rho_W-\rho_{a0})}{(\rho_W-\rho_a)}\cdot\frac{(\rho_a-\rho_{a0})}{(\rho_R-\rho_{a0})^3}$	0 m^6/kg
$c(\rho_W)_2$	$-2(m_{R,c}+\delta m_{R,c})\frac{(\rho_R-\rho_a)}{(\rho_R-\rho_{a0})}\cdot\frac{(\rho_a-\rho_{a0})}{(\rho_W-\rho_a)^3}$	0 m^6/kg

Table 2

Expressions for sensitivity coefficients

$c(x_i)$	**Expressions for** $c(x_i)$	**Values** $c(x_i)$
$c(m_{R,c})$	$\dfrac{(\rho_R-\rho_a)(\rho_W-\rho_{a0})}{(\rho_R-\rho_{a0})(\rho_W-\rho_a)}$	1
$c(\delta m_{R,c})$	$\dfrac{(\widehat{\rho}_R-\widehat{\rho}_a)(\widehat{\rho}_W-\widehat{\rho}_{a0})}{(\widehat{\rho}_R-\widehat{\rho}_{a0})(\widehat{\rho}_W-\widehat{\rho}_a)}$	1
$c(\rho_a)$	$(m_{R,c}+\delta m_{R,c})\dfrac{(\rho_W-\rho_{a0})(\rho_R-\rho_W)}{(\rho_R-\rho_{a0})(\rho_W-\rho_a)^2}$	0 m^3
$c(\rho_R)$	$-(m_{R,c}+\delta m_{R,c})\dfrac{(\rho_W-\rho_{a0})(\rho_a-\rho_{a0})}{(\rho_W-\rho_a)(\rho_R-\rho_{a0})^2}$	0 m^3
$c(\rho_W)$	$(m_{R,c}+\delta m_{R,c})\dfrac{(\rho_R-\rho_a)(\rho_{a0}-\rho_a)}{(\rho_R-\rho_{a0})(\rho_W-\rho_a)^2}$	0 m^3

For the above evaluations of input quantities, as well as for the standard uncertainty of the input quantities specified in [1]: $u(m_{R,c}) = 0.05$ mg; $u(\delta m_{R,c}) = 0.02$ mg; $u(\rho_a) = 0.05774$ kg/m^3; $u(\rho_W) = 577.35$ kg/m^3; $u(\rho_R) = 28.87$ kg/m^3, we obtain $u(\delta m) = 0.05385$ mg.

Formula (6) provides an unbiased estimate of the standard uncertainty of the measurand only in the absence of uncertainties of the input quantities.

The bias of the variance of the measurand is calculated by the formula [6]:

$$\Delta(u^2)=\frac{1}{4}\sum_{i=1}^{N}c^2(x_i)_2\cdot[\eta(x_i)+2]\cdot u^4(x_i)+\sum_{i=2}^{N}\sum_{j=1}^{i-1}c^2(x_i,x_j)u^2(x_i)u^2(x_j), \tag{9}$$

where $\eta(x_i)$ – kurtosis of the distribution of the i-th input quantity, which is taken from Table 3 [5], $c(x_i,x_j)$ – a mixed second-order partial derivative of the measurand with respect to the i-th and j-th input quantities, which is estimated for known values of the input quantities.

The calculation the bias for $u(\delta m)$ is made by the formula:

$$\begin{aligned}\Delta[u^2(\delta m)]=&\frac{1}{4}\{[c(\rho_a)_2u^2(\rho_a)]^2[\eta(\rho_a)+2]+[c(\rho_R)_2u^2(\rho_R)]^2[\eta(\rho_R)+2]+[c(\rho_W)_2u^2(\rho_W)]^2[\eta(\rho_W)+2]\}+\\&+\{[c(\delta m_{R,c},\rho_R)u(m_{R,c})u(\rho_R)]^2+[c(m_{R,c},\rho_a)u(m_{R,c})u(\rho_a)]^2+[c(m_{R,c},\rho_W)u(m_{R,c})u(\rho_W)]^2+\\&+\{[c(\delta m_{R,c},\rho_R)u(\delta m_{R,c})u(\rho_R)]^2+[c(\delta m_{R,c},\rho_a)u(\delta m_{R,c})u(\rho_a)]^2+[c(\delta m_{R,c},\rho_W)u(\delta m_{R,c})u(\rho_W)]^2+\\&+\{[c(\rho_a,\rho_R)u(\rho_a)u(\rho_R)]^2+[c(\rho_a,\rho_W)u(\rho_a)u(\rho_W)]^2+[c(\rho_R,\rho_W)u(\rho_R)u(\rho_W)]^2,\end{aligned} \tag{10}$$

where $c(x_i,x_j)$ – second-order mixed partial derivatives of the measurand with respect to the corresponding input quantities; $\eta(x_i)$ – kurtoses of input quantities. Expressions for $c(x_i,x_j)$ and their values are given in Table 4.

For the above input quantities estimates and their standard uncertainties, as well as for kurtoses corresponding to the distribution laws of input quantities taken in [1] $\eta(m_{R,c})=\eta(\delta m_{R,c})=0$; $\eta(\rho_a)=\eta(\rho_R)=\eta(\rho_W)=-1.2$, we obtain $\Delta(u^2) = 0.00272$ mg^2.

Table 3

Kurtosis values for various input quantities distributions

Distribution law	η
Arcsine	–1.5
Uniform	–1.2
Triangular	–0.6
Normal	0
Student’s with the number of degrees of freedom v	$6/(v–4)$

Table 4

Expressions for mixed second-order partial derivatives

$c(x_i, x_j)$	**Expressions for** $c(x_i, x_j)$	**Values** $c(x_i, x_j)$
$c(m_{R,c}, \rho_a)$	$\dfrac{(\rho_W - \rho_{a0})(\rho_R - \rho_W)}{(\rho_R - \rho_{a0})(\rho_W - \rho_a)^2}$	0 m^3/kg
$c(m_{R,c}, \rho_W)$	$\dfrac{(\rho_R - \rho_a)(\rho_{a0} - \rho_a)}{(\rho_R - \rho_{a0})(\rho_W - \rho_a)^2}$	0 m^3/kg
$c(m_{R,c}, \rho_R)$	$\dfrac{(\rho_W - \rho_{a0})(\rho_a - \rho_{a0})}{(\rho_W - \rho_a)(\rho_R - \rho_{a0})^2}$	0 m^3/kg
$c(\delta m_{R,c}, \rho_a)$	$\dfrac{(\rho_W - \rho_{a0})(\rho_R - \rho_W)}{(\rho_R - \rho_{a0})(\rho_W - \rho_a)^2}$	0 m^3/kg
$c(\delta m_{R,c}, \rho_W)$	$\dfrac{(\rho_R - \rho_a)(\rho_{a0} - \rho_a)}{(\rho_R - \rho_{a0})(\rho_W - \rho_a)^2}$	0 m^3/kg
$c(\delta m_{R,c}, \rho_R)$	$\dfrac{(\rho_W - \rho_{a0})(\rho_a - \rho_{a0})}{(\rho_W - \rho_a)(\rho_R - \rho_{a0})^2}$	0 m^3/kg
$c(\rho_a, \rho_W)$	$-(m_{R,c} + \delta m_{R,c})\dfrac{(\rho_{a0} + \rho_R - 2\rho_W)\rho_a + (\rho_W - 2\rho_R)\rho_{a0} + \rho_W \rho_R}{(\rho_W - \rho_a)^3(\rho_R - \rho_{a0})}$	-1.56×10^{-9} m^6/kg
$c(\rho_a, \rho_R)$	$(\hat{m}_{R,c} + \hat{\delta} m_{R,c})\dfrac{(\hat{\rho}_W - \hat{\rho}_{a0})^2}{(\hat{\rho}_R - \hat{\rho}_{a0})^2(\hat{\rho}_W - \hat{\rho}_a)^2}$	1.56×10^{-9} m^6/kg
$c(\rho_W, \rho_R)$	$-(\hat{m}_{R,c} + \hat{\delta} m_{R,c})\dfrac{(\hat{\rho}_a - \hat{\rho}_{a0})^2}{(\hat{\rho}_R - \hat{\rho}_{a0})^2(\hat{\rho}_W - \hat{\rho}_a)^2}$	0 m^6/kg

The unbiased estimate of the combined standard uncertainty calculated by the formula:

$$u_0(\delta m) = \sqrt{u^2(\delta m) + \Delta(u^2)}. \qquad (11)$$

For the above values $u(\delta m)$ and $\Delta(u^2)$, we obtain $u_0(\delta m) = 0.075$ mg, which is very close to the value calculated in [1] by the Monte Carlo method (0.0754 mg).

4. Calculation of expanded uncertainty

Since there is no bias of the measurand, this indicates that the nonlinearity of the model does not introduce additional asymmetry into the distribution law of the measurand, therefore, to calculate the expanded uncertainty, we can use the kurtosis method proposed by the authors [7].

In this case, the kurtosis of the measurand is calculated by the formula:

$$\eta(\delta m) = \frac{1}{u^4(\delta m)}[\eta(m_{R,c})c^4(m_{R,c})u^4(m_{R,c}) + \\ +\eta(\delta m_{R,c})c^4(\delta m_{R,c})u^4(\delta m_{R,c}) + \eta(\rho_a)c^4(\rho_a)u^4(\rho_a) + \\ +\eta(\rho_W)c^4(\rho_W)u^4(\rho_W) + \eta(\rho_R)c^4(\rho_R)u^4(\rho_R). \qquad (12)$$

For the above estimates of input quantities, standard uncertainties, and their kurtoses we have $\eta(\delta m) = 0$.

The value of the coverage factor for the probability of 0.95 is calculated by the formula [7]:

$$k_{0.95} = \begin{cases} 0.1085\eta^3 + 0.1\eta + 1.96, \text{ at } \eta < 0; \\ 1.96, \text{ at } \eta \geq 0. \end{cases} \qquad (13)$$

Since $\eta(\delta m) = 0$ we take $k_{0.95} = 1.96$. In this case $U(\delta m) = 1.47$ mg, which is very close to the value obtained in [1] by the Monte Carlo method (1.4955 mg).

5. Uncertainty budget

The results are summarized in the uncertainty budget (Table 5).

In contrast to the usual uncertainty budget for the kurtosis method [7], two columns are added to Table 5 that take into account the nonlinearity of the measurement model.

Table 5

Measurement uncertainty budget for weight calibration

X_i	x_i	$u(x_i)$	$\eta(x_i)$	c_i	$u_i(y)$, mg	$\Delta_i(y)$, mg	$\Delta[u_i(y)]$, mg
$m_{R,c}$	100 000 mg	0.05 mg	0	1	0.05	0	0
$\delta m_{R,c}$	1.234 mg	0.02 mg	0	1	0.02	0	0
ρ_a	1.2 kg/m³	0.05774 kg/m³	–1.2	0	0	0	0
ρ_R	8000 kg/m³	577.4 kg/m³	–1.2	0	0	0	–0.0521
ρ_W	8000 kg/m³	28.87 kg/m³	–1.2	0	0	0	0.0026
Y	y_0	$u_0(y)$	$\eta(y)$	$k_{0.95}$	$U(y)$	Δ_y, mg	$\Delta[u(y)]$
δm	1.234 mg	0.077 mg	0	1.96	0.15, mg	0	0.0522, mg

In the penultimate column of the budget $\Delta_i(y)$ – bias of the measurand caused by each i-th input quantity; $\Delta_y = \sum_{i=1}^{N} \Delta_i(y)$ – combined bias of the measurand. In the last column of the budget $\Delta[u_i(y)]$ – bias of the standard uncertainty of the measurand caused by the i-th input quantity and the combined bias $\Delta[u(y)] = \sqrt{\Delta[u^2(y)]}$, which is calculated by the formula:

$$\Delta[u(y)] = \sqrt{\sum_{i=1}^{N} \{\Delta[u_i(y)]\}^2}. \tag{14}$$

The presence of additional columns makes it possible to obtain an unbiased estimate of the measurand and its uncertainty in the nonlinear model equation.

Conclusions

An example of measurement uncertainty evaluation using the methods developed by the authors, at weight calibration as an example, is considered. Unbiased estimates of the measurand and its uncertainty are obtained.

It is shown that when calculating the standard and expanded uncertainty of the measurand, it is necessary to take into account the kurtoses of the input quantities. The obtained estimates of the standard and expanded uncertainties of the measurand showed good agreement with the estimates obtained by the Monte Carlo method [1], which proves the advantage of the proposed approaches in comparison with the GUM method [2].

Врахування ексцесів вхідних величин у процедурі оцінювання невизначеності вимірювань на прикладі калібрування гирі

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Анотація

Аналізується приклад 9.3.1.1 з JCGM-S1 "Калібрування маси", в котрому описуються звірення у повітрі еталонної гирі з гирею, що калібрується, які мають одну і ту ж саму номінальну масу. Здійснюється формування моделі вимірювань. Показано, що отримана модель є нелінійною відносно ряду вхідних величин.

У JCGM-S1 порівнюються процедури оцінювання невизначеності, що виконуються на основі концепції невизначеності GUM і методу Монте-Карло.

У статті використовується процедура, яка розроблена авторами та полягає у розкладанні моделі вимірювання в ряд Тейлора другого порядку з урахуванням ексцесів розподілів вхідних величин.

Оцінюється зміщення результату вимірювань. Отримано вирази для часткових похідних другого порядку. Показано, що для моделі, яка аналізується, їхні значення дорівнюють нулю, тому значення зміщення числового значення вимірюваної величини також дорівнюватиме нулю. Здійснюється обчислення стандартної невизначеності вимірюваної величини з урахуванням часткових похідних другого порядку та ексцесів вхідних величин. Показано, що отримане значення стандартної невизначеності суттєво відрізняється від аналогічного значення, отриманого за процедурою GUM.

Для знаходження розширеної невизначеності застосовується метод ексцесів. Показано хороший збіг результатів, отриманих запропонованим методом, із результатом, отриманим методом Монте-Карло.

Наведено бюджет невизначеності, який відрізняється від звичайного бюджету двома додатковими стовпцями, що враховують нелінійність моделі вимірювань. Присутність додаткових стовпців дозволяє отримувати незміщену оцінку вимірюваної величини та її невизначеності при нелінійному модельному рівнянні.

Ключові слова: невизначеність вимірювань; калібрування; гиря; нелінійне модельне рівняння; метод ексцесів.

Учет эксцессов входных величин в процедуре оценивания неопределенности измерений на примере калибровки гири

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Аннотация

Анализируется пример 9.3.1.1 из JCGM-S1 “Калибровка массы”, в котором описаны сличения в воздухе эталонной и калибруемой гирь, имеющих одну и ту же номинальную массу. В JCGM-S1 сравниваются процедуры оценивания неопределенности, выполняемые на основе концепции неопределенности GUM и метода Монте-Карло.

В статье используется процедура, разработанная авторами и заключающаяся в разложении модели измерения в ряд Тейлора второго порядка с учетом эксцессов распределений входных величин. Для нахождения расширенной неопределенности применяется метод эксцессов. Показано хорошее совпадение результатов, полученных предлагаемым методом, с результатом, полученным методом Монте-Карло.

Ключевые слова: неопределенность измерений; калибровка; гиря; нелинейное модельное уравнение; метод эксцессов.

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