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EXTENDED METHOD OF ESTIMATION UNCERTAINTIES OF INDIRECT MULTIVARIABLE MEASUREMENTS ON THE TWOPORT EXAMPLE

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Abstract

The new extended mathematical model for evaluation uncertainties of indirect multivariable measurements, which upgrades the method given in Supplement 2 of guide GUM, is presented. In this model the uncertainties and correlations of parameters of the processing function are taken also into account. This model can be used for multivariable measurements and to describe the accuracy of instruments and systems that perform such measurements. The estimation of uncertainties of voltage and current on the output of a twoport network from indirect measurements on its input with considering influences the uncertainties of twoport elements is included.

Key words: indirect multivariable measurements, measurand, uncertainty, processing function, correlation coefficient, covariance matrix.

1. Introduction

In the basic and technical research, in monitoring and technical diagnostics, many physical quantities and parameters have to be measured for characterize the object under the test. In many cases there is no possibilities to carry out direct measurements. Then indirect methods must be applied. The international recommendations for application of the method of determination estimators of values, uncertainties and correlations in multivariable measurements are described in Supplement 2 to the GUM.

Directly measured on input the *n*–element measurand $\boldsymbol{X}$ is l processed to the output *m*- measurand $\boldsymbol{Y}$ by relation

$$\boldsymbol{Y}=\boldsymbol{F}(\boldsymbol{X},\boldsymbol{P}) \tag{1}$$

In this paper are used such designations: $\boldsymbol{X}, \boldsymbol{Y}$ -input and output measurands; $\boldsymbol{X_0},\boldsymbol{Y_0}$ - their initial values; $\overline{\boldsymbol{X}}, \overline{\boldsymbol{Y}}$ – vectors of estimators of *n* values $\overline{x}_i$ and of *m* values $\overline{y}_j$; $\boldsymbol{u_x}$, $\boldsymbol{u_y}$ and $\boldsymbol{u_{\delta x}}$, $\boldsymbol{u_{\delta y}}$ – their absolute and relative standard uncertainties; $\boldsymbol{F}(\boldsymbol{X})$, $\boldsymbol{F}(\boldsymbol{X},\boldsymbol{P})$ – ideal and real multivariable functional of processing $\boldsymbol{X}$ to $\boldsymbol{Y}$; $\boldsymbol{U_F}$, $\boldsymbol{U_{\delta F}}$ - its covariance matrices; $\boldsymbol{S}=\partial\boldsymbol{Y}/\partial\boldsymbol{X}$, $\boldsymbol{S_\delta}$ –sensitivity matrices for absolute and relative uncertainties; $\boldsymbol{U_X}$, $\boldsymbol{U_Y}$, $\boldsymbol{U_{Y0}}$, $\boldsymbol{U_{\delta X}}$, $\boldsymbol{U_{\delta Y}}$, $\boldsymbol{U_{YP}}$ – covariance matrices of $\boldsymbol{X}$, $\boldsymbol{Y}$; $\boldsymbol{U_P}$, $\boldsymbol{U}(\boldsymbol{P},\boldsymbol{X})$ – covariance and correlation matrices of *k* parameters $\boldsymbol{P}$ of the processing functional $\boldsymbol{F}$ of the measurement circuit

For the ideal functional $\boldsymbol{F}(\cdot)$, i.e. when uncertainties of its parameters $\boldsymbol{P}$ are negligible, the propagation of absolute and relative variances of the t measurand $\boldsymbol{X}$ to $\boldsymbol{Y}$ ones are:

$$\boldsymbol{U_Y}=\boldsymbol{S}\,\boldsymbol{U_X}\boldsymbol{S}^{\mathrm{T}} \tag{2a}$$

$$\boldsymbol{U_{\delta Y}}=\boldsymbol{U_{Y0}}+\boldsymbol{S_\delta}\,\boldsymbol{U_{\delta(X-X_0)}}\,\boldsymbol{S_\delta^{\mathrm{T}}} \tag{2b}$$

.Even in the case when the basic multivariable relation (1) is nonlinear, in the most cases for uncertainties of **X** and $\boldsymbol{Y}$ as small deviations, their scatter regions can be defined by a model of joint multidimensional normal probability distributions. Then, for a given probability density p_0 the distribution region for $p \geq p_0$ takes the form of a *n*- and *m*-dimensional hyper-ellipsoids, if covariance matrix is positive definite, with centers at the ends of their averages.

Matrices $\boldsymbol{U_X}$ and $\boldsymbol{U_Y}$ describe the cover regions of *n* and *m* dimension probabilities if their det($\boldsymbol{U_i}$) > 0 or det($\boldsymbol{R_i}$) > 0 as equal condition for matrix of the correlation coefficients, called the correlator $\boldsymbol{R}$. For example, the relation between

matrix $\boldsymbol{U_X}$ and correlator $\boldsymbol{R_x}$ of the size 3D (3x3) is

$$\boldsymbol{U_X}=\boldsymbol{u}(\boldsymbol{X})\,\boldsymbol{R_x}\,\boldsymbol{u^T}(\boldsymbol{X})=$$

$$=\begin{bmatrix}u_{x_1}&0&0\\0&u_{x_2}&0\\0&0&u_{x_3}\end{bmatrix}\begin{bmatrix}1&\rho_{12}&\rho_{13}\\\rho_{12}&1&\rho_{23}\\\rho_{13}&\rho_{23}&1\end{bmatrix}\begin{bmatrix}u_{x_1}&0&0\\0&u_{x_2}&0\\0&0&u_{x_3}\end{bmatrix}^{\boldsymbol{T}}$$

$$=\begin{bmatrix}u_{x_1}^2&\rho_{12}u_{x_1}u_{x_2}&\rho_{13}u_{x_1}u_{x_3}\\\rho_{12}u_{x_1}u_{x_2}&u_{x_2}^2&\rho_{23}u_{x_2}u_{x_3}\\\rho_{13}u_{x_1}u_{x_3}&\rho_{23}u_{x_2}u_{x_3}&u_{x_3}^2\end{bmatrix} \tag{2c}$$

All, or some components of the vector $\overline{\boldsymbol{Y}}$ of output results can be next used separately or jointly. In the latter case it is necessary to find and take in considerations also the correlations between pairs of variables y_i.

Relations (2a,b) for covariance matrices $\boldsymbol{U_Y}$, $\boldsymbol{U_\delta}$ of absolute and relative uncertainties of estimators $\overline{y}_j$ of results, obtained for indirectly observed *m*-dimensional measurand $\boldsymbol{Y}$, are the same whether the measurement functional $\boldsymbol{F}(\boldsymbol{X},\boldsymbol{P})$ is linear or linearized by the first derivative. All, or some components of measurement results $\overline{\boldsymbol{Y}}$ can be next used separately or jointly. In the latter case it is also necessary to take in considerations the correlations between variables y_i of output measurand $\boldsymbol{Y}$.

In many cases the indirect measurements of *m* – components of the measurand $\boldsymbol{Y}$ are made now by automatic measurement systems. If not, then a collection of individual quantities of $\boldsymbol{X}$ should be synchronically measured, and from above data both, of $\overline{\boldsymbol{Y}}$ and covariance matrices $\boldsymbol{U_X}$, $\boldsymbol{U_Y}$ externally calculated.

2. Basic formulas of the extended method

The actual GUM-S2 method [1] and earler literature, e,g. [2 -5] do not cover situations of the not accurate multivariable functional $\boldsymbol{F}(\boldsymbol{X},\boldsymbol{P})$, for example due to approximation, the limited frequency range of transfer function, uncertainties of passive and active elements, AC/DC converters and analogue multipliers, and also when measurements are possible only indirectly via other nonideal internal parameters of the tested object. In precise measurements, the rounding of calculations also becomes essential, including ones resulting from the precision of the digital part of the circuit. In the instrumental measuring systems, the real multivariable processing function is $\boldsymbol{Y} = \boldsymbol{F}(\boldsymbol{X}, \boldsymbol{P})$ – eq. (3) given in Table 1 [8,11]. The accuracy of indirect measurements of the multivariable measurand $\boldsymbol{Y}$ depends on the uncertainty and correlations of $\boldsymbol{X}$ and also on uncertainties and correlations of its parameters $\boldsymbol{P}$ - formula (5a). The relative uncertainty propagation is also given. Developed is the extended formula (5) for the covariance matrix $\boldsymbol{U_Y}$, which includes all influences on uncertainties $\boldsymbol{u_y}$ and its simpler cases (6) – (9a-c) given also in Table 1.

Table.1. Formulas of the extended method of estimate uncertainties of indirect multivariable measurements.

General formula		$\boldsymbol{Y} = \boldsymbol{F}(\boldsymbol{X}, \boldsymbol{P}) \qquad (\boldsymbol{u_F} \neq \boldsymbol{0})$ (3) where: $\boldsymbol{Y} = [\boldsymbol{y_1}, \ , .. \boldsymbol{y_m}]^{\boldsymbol{T}}, \quad \boldsymbol{X} = [\boldsymbol{x_1}, \dots \boldsymbol{x_n}]^{\boldsymbol{T}}, \quad \boldsymbol{P} = [\boldsymbol{p_1}, \boldsymbol{p_2}, \ \dots \ \boldsymbol{p_k}]^{\boldsymbol{T}}$ (3a,b,c)
Deviations (absolute errors)		$\Delta \boldsymbol{Y} = \ \Delta \boldsymbol{F}(\boldsymbol{X}, \boldsymbol{P}) = \boldsymbol{S_{X,P}}[\boldsymbol{\Delta X}, \boldsymbol{\Delta P}]^{\mathrm{T}} = \boldsymbol{S_X}\, \boldsymbol{\Delta X} \ + \boldsymbol{S_P} \boldsymbol{\Delta P}$ (4) where: $\boldsymbol{S_{X,P}}$ - sensitivity matrix of function $\boldsymbol{F}(\boldsymbol{X}, \boldsymbol{P})$ of dimensions $[(n+k)\times m]$, $\boldsymbol{S_X} \equiv \boldsymbol{S}, \ \boldsymbol{S_P}, \ \boldsymbol{S_\delta}$ - sensitivity matrices of influence of deviations $\boldsymbol{\Delta X}, \boldsymbol{\Delta P}$ or $\boldsymbol{\delta_X} = \frac{\boldsymbol{\Delta X}}{\boldsymbol{X}}, \ \boldsymbol{\delta_P} = \frac{\boldsymbol{\Delta P}}{\boldsymbol{P}}$
Sensitivity matrices of ΔX, δX, ΔP		$\boldsymbol{S_X} = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \dots & \frac{\partial y_1}{\partial x_n} \\ \dots & \dots & \dots \\ \frac{\partial y_m}{\partial x_1} & \dots & \frac{\partial y_m}{\partial x_n} \end{bmatrix}, \quad \boldsymbol{S_p} = \begin{bmatrix} \frac{\partial y_1}{\partial p_1} & \dots & \frac{\partial y_1}{\partial p_k} \\ \dots & \dots & \dots \\ \frac{\partial y_m}{\partial p_1} & \dots & \frac{\partial y_m}{\partial p_k} \end{bmatrix}, \quad \boldsymbol{S_\delta} = \begin{bmatrix} \frac{x_1}{y_1}\frac{\partial y_1}{\partial x_1} & \dots & \frac{x_n}{y_1}\frac{\partial y_1}{\partial x_n} \\ \dots & \dots & \dots \\ \frac{x_1}{y_m}\frac{\partial y_m}{\partial x_1} & \dots & \frac{x_n}{y_m}\frac{\partial y_m}{\partial x_n} \end{bmatrix}$ (4a,b,c)
Propagation of variances general case: $V = SUS_P^T \neq 0$		$\boldsymbol{U_Y}(\boldsymbol{X}, \boldsymbol{P}) = \ \boldsymbol{S_{X,P}}\, \boldsymbol{U_{X,P}} \ = [\boldsymbol{S}, \boldsymbol{S_p}] \begin{bmatrix} \boldsymbol{U_X} & \boldsymbol{U} \\ \boldsymbol{U^T} & \boldsymbol{U_p} \end{bmatrix} \begin{bmatrix} \boldsymbol{S^T} \\ \boldsymbol{S_P^T} \end{bmatrix}$ (5) $\boldsymbol{U_Y} = \boldsymbol{U_{YX}} + \boldsymbol{U_{YF}} = \boldsymbol{S_X} \boldsymbol{U_X} \boldsymbol{S_X^T} + \boldsymbol{U_{YF}}$ (5a) $\boldsymbol{U_{YF}} = \boldsymbol{S_p} \boldsymbol{U_p} \boldsymbol{S_P^T} + \boldsymbol{S_X} \boldsymbol{U} \boldsymbol{S_P^T} + (\boldsymbol{S_X} \boldsymbol{U} \boldsymbol{S_P^T})^{\boldsymbol{T}} = \boldsymbol{S_P} \boldsymbol{U_P} \boldsymbol{S_P^T} + \boldsymbol{V} + \boldsymbol{V^T}$ (5b)
Covariance matrices: U_Y, U_X, U_P; and matrix U of X, P correlation		$\boldsymbol{U_Y} = \begin{bmatrix} u_{y1}^2 & \dots & \rho_{y1m} u_{y1} u_{ym} \\ \dots & \dots & \dots \\ \rho_{y1m} u_{ym} u_{y1} & \dots & u_{ym}^2 \end{bmatrix}, \quad \boldsymbol{U_X} = \begin{bmatrix} u_{x1}^2 & \dots & \rho_{x1n} u_{x1} u_{xn} \\ \dots & \dots & \dots \\ \rho_{x1n} u_{xn} u_{x1} & \dots & u_{xn}^2 \end{bmatrix}$ (5c, d) $\boldsymbol{U_p} = \begin{bmatrix} u_{p1}^2 & \dots & \rho_{p1k} u_{p1} u_{pk} \\ \dots & \dots & \dots \\ \rho_{p1k} u_{pk} u_{p1} & \dots & u_{pk}^2 \end{bmatrix}, \quad \boldsymbol{U} = \begin{bmatrix} \rho_{x1p1} u_{x1} u_{p1} & \dots & \rho_{x1pk} u_{x1} u_{pk} \\ \dots & \dots & \dots \\ \rho_{xnp1} u_{xn} u_{p1} & \dots & \rho_{xnpk} u_{xn} u_{pk} \end{bmatrix}$ (5e, f)
Propagation of variances for $V = 0$	absolute uncertainty	$\boldsymbol{U_Y} = \boldsymbol{S} \cdot \boldsymbol{U_X} \cdot \boldsymbol{S^T} + \boldsymbol{S_P} \cdot \boldsymbol{U_P} \cdot \boldsymbol{S_P^T}$ (6)
	relative uncertainty	$\boldsymbol{U_{\delta(Y-Y_0)}} = \ \boldsymbol{S_\delta} \boldsymbol{U_{\delta(X-X_0)}} \boldsymbol{S_\delta^T} + \boldsymbol{S_{\delta P}} \boldsymbol{U_{\delta P}} \boldsymbol{S_{\delta P}^T}$ (7)
	type A and type B uncertainty components	$\boldsymbol{U_Y} = \boldsymbol{U_{YA}} + \boldsymbol{U_{YB}} = (\boldsymbol{S} \boldsymbol{U_{XA}} \boldsymbol{S^T} + \boldsymbol{S_P} \boldsymbol{U_{PA}} \boldsymbol{S_P^T}) + (\boldsymbol{S} \boldsymbol{U_{XB}} \boldsymbol{S^T} + \boldsymbol{S_P} \boldsymbol{U_{PB}} \boldsymbol{S_P^T})$ (8) $\boldsymbol{U_{XA}} = \begin{bmatrix} u_{x1A}^2 & \dots & \rho_{Ax1,n} u_{x1A} u_{xnA} \\ \dots & \dots & \dots \\ \rho_{Ax1,n} u_{xnA} u_{x1A} & \dots & u_{xnA}^2 \end{bmatrix}, \ \boldsymbol{U_{XB}} = \begin{bmatrix} u_{x1B}^2 & \dots & \rho_{Bx1,n} u_{x1B} u_{xnB} \\ \dots & \dots & \dots \\ \rho_{Bx1,n} u_{xnB} u_{x1B} & \dots & u_{xnB}^2 \end{bmatrix}$ $\boldsymbol{U_{PA}} = \begin{bmatrix} u_{p1A}^2 & \dots & \rho_{Ap1k} u_{p1A} u_{pkA} \\ \dots & \dots & \dots \\ \rho_{Ap1k} u_{pkA} u_{p1A} & \dots & u_{pkA}^2 \end{bmatrix}, \ \boldsymbol{U_{PB}} = \begin{bmatrix} u_{p1B}^2 & \dots & \rho_{Ap1k} u_{p1B} u_{pkB} \\ \dots & \dots & \dots \\ \rho_{Bp1k} u_{pkB} u_{p1B} & \dots & u_{pkB}^2 \end{bmatrix}$ (8a-d) Uncertainties and correlation coefficient of two variables: $u_1^2 = u_{1A}^2 + u_{1B}^2; \quad u_2^2 = u_{2A}^2 + u_{2B}^2; \quad \rho_{1,2} = \frac{\rho_A u_{1A} u_{2A} + \rho_B u_{1B} u_{2B}}{\sqrt{u_{1A}^2 + u_{1B}^2}\sqrt{u_{2A}^2 + u_{2B}^2}}$ (8e, f, g)

The relationships between small deviations of the values of n-elements of the input measurand $\boldsymbol{X}$ and m-elements of the indirectly measured measurand $\boldsymbol{Y}$ are described by the formula (4). The deviations of the measuring system $\boldsymbol{P}$ parameters are determined from their nominal values on the basis of the maximum permissible errors (MPE) known from their technical data or as deviations from the estimators of their values determined in measurements. Sensitivity matrices $\boldsymbol{S_X}$, $\boldsymbol{S_\delta}$ and $\boldsymbol{S_P}$ (4a-c) express the influence of deviations $\boldsymbol{\Delta X}, \boldsymbol{\delta_X}$ and $\boldsymbol{\Delta P}$ on the output deviations $\boldsymbol{\Delta Y}$, $\boldsymbol{\delta_Y}$ of the initial quantities. The deviations of known values or their course during the period of measurement experiment, are removed from the results by corrections. Other, which are not known and not determinate, are randomized. For the single-parameter measurand, the statistical properties of a set of deviations of each quantity are described in GUM [1] by the standard uncertainty u as the geometric sum of its components u_A and u_B. For multidimensional measurands, the equivalent of the variance of single variable, are their covariance matrices, e.g. symmetrical matrices $\boldsymbol{U_Y}$, $\boldsymbol{U_X}$ and $\boldsymbol{U_P}$ (5c-e). They contain on the main diagonal squares of standard uncertainties (variances) of individual quantities, and on other places, products of the corresponding one from $n(n-1)/2$ correlation coefficients and uncertainties of both type correlated quantities.

Sets of random deviations from estimators of the output measurand $\boldsymbol{Y}$ variables are the result of multi-parameter distributions of the deviations of the input measurand $\boldsymbol{X}$ variables and deviations of parameters $\boldsymbol{P}$ of the measurement system performing the multivariable functional $\boldsymbol{F(X, P)}$. When linearizing each of its functions for small deviations, the general formula (5) for the $\boldsymbol{U_Y}$ covariance matrix in multi-parameter measurements and its subsequent developed forms (5a), (5b) is obtained from the propagation law of variance. Uncertainties and correlation coefficients of n variables of the measurand $\boldsymbol{X}$ and of k system parameters $\boldsymbol{P}$ are included in the $\boldsymbol{U_X}$ and $\boldsymbol{U_P}$ covariance matrices (5c,d),

In general case, variables $\boldsymbol{X}$ can be also correlated with parameters $\boldsymbol{P}$ of measuring system. This is described by the matrix $\boldsymbol{U}$ with the size [n x k], given in the formula (5f). Such relationship may appear under the influence of a common external random effect on $\boldsymbol{X}$ and $\boldsymbol{P}$, e.g. a variable outside temperature.

The number of independent correlation coefficients in the $\boldsymbol{U}$ matrix is smaller by the number of m equations elements of measurand $\boldsymbol{Y}$. In the measurement practice, including electrical measurement systems, there is usually a simpler case when the directly measured quantities $\boldsymbol{X}$ and deviations of parameters $\boldsymbol{P}$ of the measuring system are not correlated (e.g. $\boldsymbol{X}$ is differently located then $\boldsymbol{P}$ and they do not affect themselves and their external influences are also not related). Then the covariance matrix $\boldsymbol{U}$ does not occur and $\boldsymbol{V} = \boldsymbol{V^T} = \boldsymbol{0}$. The propagation equation of variance (5b) has then a simpler two-component form (6). The first component depends on the uncertainties and correlations of elements of the input measurand $\boldsymbol{X}$, similar as in the classic approach according to GUM-S2 [1]. The second component, depending on the uncertainty of the processing function, appeared in the extended method and constitutes its essence. It expresses the influence of uncertainties and correlation coefficients ρ_p of $\boldsymbol{P}$ parameters of the system processing function $\boldsymbol{F}(\boldsymbol{X},\boldsymbol{P})$, analog or digital.

From (6) follows the covariance matrix (7) for the relative uncertainties of $\boldsymbol{Y} - \boldsymbol{Y_0}$ increments, or of $\boldsymbol{Y}$, if uncertainties of the initial input quantities are negligible, i.e. $||\boldsymbol{u_X}(\boldsymbol{X})|| \gg ||\boldsymbol{u_X}(\boldsymbol{X_0})|| \approx \boldsymbol{0}$ (measurement of values close to the beginning of the range are usually avoided) and the covariance matrix $||\boldsymbol{U_X}(\boldsymbol{X} - \boldsymbol{X_0})|| \approx ||\boldsymbol{U_X}(\boldsymbol{X})||$. Relation (7) was not included in Supplement 2 of GUM.

In the papers [6-11], the authors stated that only sets of deviations with uncertainties of the same type, i.e. only of u_A or only of u_B, can be correlated with each other, for variables of the same or of different multi-measurands. Covariance matrices of multi-measurands, similarly as the variance $u^2 = u_A^2 + u_B^2$ of each single measured variable, can be presented also as the sum of two component matrices of type $\boldsymbol{A}$ and $\boldsymbol{B}$, i.e. $\boldsymbol{U_X} = \boldsymbol{U_{XA}}+\boldsymbol{U_{XB}}$, $\boldsymbol{U_Y} = \boldsymbol{U_{YA}}+\boldsymbol{U_{YB}}$ - formula (8). The elements of component matrices type A and B are given in (8a)-(8d) and method of their calculations in (8e,f,g).

For the measurand $\boldsymbol{X}$, only the correlation coefficients ρ_{xA} in the $\boldsymbol{U_{XA}}$ matrix can be experimentally determined by synchronous measurements of variables of $\boldsymbol{X}$. On the other hand, the coefficients ρ_B of the $\boldsymbol{U_{XB}}$ matrix, similarly as the uncertainties of type B, have to be estimated heuristically. If two quantities are measured with the same or similar instrument and under the same conditions, then the correlation coefficient ρ_{xB} equal to 1 [4], [5] can be assumed. For different instruments and in different operating conditions this coefficient is closer to 0. The correlation coefficient –1 is rather rare. It occurs e.g. when changes of both correlated variables from common interactions, have the opposite signs.

Type A and type B uncertainties for individual quantities of the output measurand $\boldsymbol{Y}$ should be carried out separately from the component $\boldsymbol{U_{YA}}$, $\boldsymbol{U_{YB}}$ of the covariance matrix $\boldsymbol{U_Y}$, according to formulas (8), (8a-g). If during the measurements the values of $\boldsymbol{P}$ system parameters are constant, the $\boldsymbol{U_{PA}}$ matrix does not occur, and the $\boldsymbol{U_P} = \boldsymbol{U_{PB}}$ matrix is estimated heuristically for the long period changes of deviations $\Delta\boldsymbol{P}$. However, if $\Delta\boldsymbol{P}$ is changing randomly in the duration of the measurement experiment, then the elements of their $\boldsymbol{U_{PA}}$ component matrix must also be estimated heuristically based on technical data and own knowledge. It is also valuable to perform additional measurements in the specially created influencing conditions to estimate the level of the short time random changes in $\boldsymbol{P}$ parameters.

In several papers, authors described how to use this new method. Few examples of implementation this model to indirect measurement of a two-terminal circuit parameters through a four-terminal T type network, considering the uncertainties and correlation of its impedances in general case and for $\boldsymbol{U} = \boldsymbol{0}$ are presented in detail in [8-11]. One of these examples, the indirect measurements of voltage and current at the output of a loaded four-terminal circuit is considered in detail below.

3. Uncertainties of indirect measurements of the twoport output variables

Let us consider the indirect measurements of the voltage and current of an inaccessible branch on the output of a linear passive twoport network based on measurements of these variables on its input terminals. It was assumed that this twoport has the T-type structure given in Fig. 1. Then observed is the 2D measurand $\boldsymbol{Y} = [U_{out}, I_{out}]^{\boldsymbol{T}}$. The accuracy of $\boldsymbol{Y}$ are determined from measurements of the input voltage and current, i.e.: $\boldsymbol{X} = [U_{in}, I_{in}]^T$. Accuracy of results depends also from values, uncertainties and correlations of twoport impedances Z_1, Z_2, Z_3.

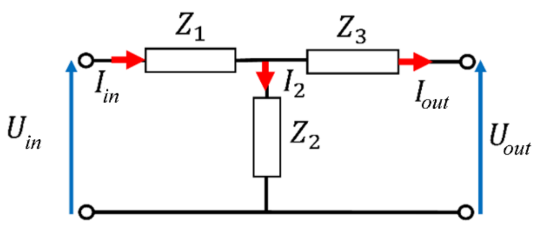

Fig 1. Diagram of a passive twoport T-type circuit

For the T-type passive twoport from Kirchhoff laws: $I_{in} = I_2 + I_{out}$; $U_{in} = Z_2 I_2 + Z_1 I_{in}$; $Z_2 I_2 = U_{out} + Z_3 I_{out}$ the following relations for the output variables are obtained

$$U_{out} = \left(1 + \frac{Z_3}{Z_2}\right) U_{in} - \left(Z_1 + Z_3 + \frac{Z_3 Z_1}{Z_2}\right) I_{in}$$

$$-I_{out} = \frac{1}{Z_2} U_{in} - \left(1 + \frac{Z_1}{Z_2}\right) I_{in}$$

The relation between U_{out}, I_{out} and directly measured U_{in}, I_{in} of the T-type twoport circuit has the form of matrix function $\boldsymbol{Y} = \boldsymbol{B} \cdot \boldsymbol{X}$, i.e.:

$$\begin{bmatrix} U_{out} \\ -I_{out} \end{bmatrix} = \boldsymbol{B} \cdot \begin{bmatrix} U_{in} \\ -I_{in} \end{bmatrix} = \begin{bmatrix} B_{11} U_{in} - B_{12} I_{in} \\ B_{21} U_{in} - B_{22} I_{in} \end{bmatrix} \quad (10a)$$

where

$$\boldsymbol{B} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} 1 + \frac{Z_3}{Z_2} & Z_1 + Z_3 + \frac{Z_1 Z_3}{Z_2} \\ \frac{1}{Z_2} & 1 + \frac{Z_1}{Z_2} \end{bmatrix} (10b)$$

If the twoport is passive and reversible, then the determinant of the matrix $\boldsymbol{B}$ satisfies the equation

$$\det(\boldsymbol{B}) = B_{11} B_{22} - B_{12} B_{21} = 1 \qquad (11)$$

and only three of the matrix $\boldsymbol{B}$ elements are independent, the fourth one follows from (11).

In the opposite situation, when tested are variables on input of the twoport and measured are on its output, the matrix $\boldsymbol{A}$ is used, which for the twoport T is like $\boldsymbol{B}$ with replaced impedances Z_3 and Z_1.

3.1 Matrix $\boldsymbol{U_Y}$ when $\boldsymbol{U_P} = 0$ (GUM-S2 case)

If uncertainties u_{Z_i} of impedances Z_i (i=1,2,3) are negligible, then the matrix $\boldsymbol{U_P} = \boldsymbol{0}$ and from (6) results that the output covariance matrix $\boldsymbol{U_Y} = \boldsymbol{U_{YX}}$, as it is in GUM-S2 method. If the measured twoport quantities $\boldsymbol{X}$ are uncorrelated, its input covariance and the sensitivity matrices are

$$\boldsymbol{U_X} = \begin{bmatrix} u^2_{U_{in}} & 0 \\ 0 & u^2_{I_{in}} \end{bmatrix}, \quad \boldsymbol{S_X} = \begin{bmatrix} B_{11} & -B_{12} \\ -B_{21} & B_{22} \end{bmatrix}.$$

From (6) with $\boldsymbol{U_{YP}} = 0$ is obtained $\boldsymbol{U_Y} = \boldsymbol{U_{YX}}$, i.e.

$$\boldsymbol{U_{YX}} = \boldsymbol{S_X} \boldsymbol{U_X} \boldsymbol{S_X^T} =$$

$$\begin{bmatrix} B^2_{11} u^2_{U_{in}} + B^2_{12} u^2_{I_{in}} & -B_{11} B_{21} u^2_{U_{in}} - B_{12} B_{22} u^2_{I_{in}} \\ -B_{11} B_{21} u^2_{U_{in}} - B_{12} B_{22} u^2_{I_{in}} & B^2_{21} u^2_{U_{in}} + B^2_{22} u^2_{I_{in}} \end{bmatrix} (12)$$

Then in the case as is consider in GUM-S2, i.e. exact processing function $\boldsymbol{F} = \boldsymbol{B} \cdot \boldsymbol{X}$, variances of the current and voltage at output terminals of T-twoport and their correlation coefficient ρ_{out} are:

$$u^2_{U_{out}} = B^2_{11} u^2_{U_{in}} + B^2_{12} u^2_{I_{in}} = \frac{1}{Z_2^2} [(Z_2 + Z_3)^2 u^2_{U_{in}} + (Z_1 Z_2 + Z_3 Z_2 + Z_1 Z_3)^2 u^2_{I_{in}}] \qquad (13a)$$

$$u^2_{I_{out}} = B^2_{21} u^2_{U_{in}} + B^2_{22} u^2_{I_{in}} = \frac{1}{Z_2^2} [u^2_{U_{in}} + (Z_1 + Z_2)^2 u^2_{I_{in}}]$$

$$\rho_{out} = -\frac{B_{11} B_{21} u^2_{U_{in}} + B_{12} B_{22} u^2_{I_{in}}}{u_{I_{out}} u_{U_{out}}} \qquad (13b, c)$$

Correlation coefficient $\rho_{out} \neq 0$ and is always negative, because $B_{11} B_{21} > 0$ and $B_{12} B_{22} > 0$.
For $Z_3 = 0$ a voltage divider is created, and the variance patterns of output variables simplify, i.e.:

$$u^2_{U_{out}} = B^2_{11} u^2_{U_{in}} + B^2_{12} u^2_{I_{in}} = u^2_{U_{in}} + Z_1^2 u^2_{I_{in}},$$

$$u^2_{I_{out}} = \frac{1}{Z_2^2} [u^2_{U_{in}} + (Z_1 + Z_2)^2 u^2_{I_{in}}] \qquad (14a, b)$$

In general case when the input variables are correlated the input covariance matrix is:

$$\boldsymbol{U_X} = \begin{bmatrix} u^2_{U_{in}} & \rho_{in} u_{U_{in}} u_{U_{in}} \\ \rho_{in} u_{U_{in}} u_{U_{in}} & u^2_{I_{in}} \end{bmatrix} \qquad (15)$$

These uncertainties are modified by correlation:

$$u^2_{U_o} = u^2_{U_{out}} - 2 B_{11} B_{12} \rho_{in} u_{U_{in}} u_{I_{in}}$$

$$u^2_{I_o} = u^2_{I_{out}} - 2 B_{21} B_{22} \rho_{in} u_{U_{in}} u_{I_{in}} \qquad (16a,b)$$

and correlation coefficient:

$$\rho_o = \frac{-B_{11} B_{21} u^2_{U_{in}} - B_{12} B_{22} u^2_{I_{in}} + (B_{11} B_{22} + B_{12} B_{21}) \rho_{in} u_{U_{in}} u_{I_{in}}}{u_{I_o} u_{U_o}} \quad (17)$$

In the case of $\rho_{in} = 1$, these absolute and relative uncertainties are:

$$u_{U_o} = |B_{11} u_{U_{in}} - B_{12} u_{I_{in}}|,$$

$$u_{I_o} = |B_{21} u_{U_{in}} - B_{22} u_{I_{in}}|, \qquad (18a,b)$$

$$\delta_{U_o} = \frac{u_{U_o}}{U_{out}} = \frac{|B_{11} u_{U_{in}} - B_{12} u_{I_{in}}|}{B_{11} U_{in} - B_{12} I_{in}},$$

$$\delta_{I_o} = \frac{u_{I_o}}{I_{out}} = \frac{|B_{21} u_{U_{in}} - B_{22} u_{I_{in}}|}{B_{22} I_{in} - B_{21} U_{in}}, \qquad (19a, b)$$

and correlation coefficient

$$\rho_o = -\frac{(B_{11} u_{U_{in}} - B_{12} u_{I_{in}}) \cdot (B_{21} u_{U_{in}} - B_{22} u_{I_{in}})}{|(B_{11} u_{U_{in}} - B_{12} u_{I_{in}}) \cdot (B_{21} u_{U_{in}} - B_{22} u_{I_{in}})|} = \pm 1 \quad (20)$$

and its sign depend on the sign of expression in module: for plus $\rho_o = -1$ and for minus $\rho_o = 1$.

3.2 Component $\boldsymbol{U_{YP}}$ of covariance matrix $\boldsymbol{U_Y}$ for uncorrelated impedances of twoport T

Covariance matrix $\boldsymbol{U_P}$ for uncorrelated impedances Z_1, Z_2, Z_3 of the twoport T and sensitivity matrix $\boldsymbol{S_P}$ have the following forms

$$\boldsymbol{U_P} = \begin{bmatrix} u^2_{Z_1} & 0 & 0 \\ 0 & u^2_{Z_2} & 0 \\ 0 & 0 & u^2_{Z_3} \end{bmatrix},$$

$$\boldsymbol{S_P} = \begin{bmatrix} -I_{in}\left(1 + \frac{Z_3}{Z_2}\right), & -\frac{Z_3}{Z_2^2}(U_{in} - I_{in} Z_1), & \frac{U_{in}}{Z_2} - I_{in}\left(1 + \frac{Z_1}{Z_2}\right) \\ \frac{I_{in}}{Z_2} & \frac{1}{Z_2^2}(U_{in} - I_{in} Z_1) & 0 \end{bmatrix}$$

(21a,b)

By entering the designation for current in impedance Z_2 as $I_2 = (U_{in} - I_{in} Z_1)/Z_2$, from Kirchhoff's first law is obtained $I_{in} = I_{out} + I_2$ and then the form of sensitivity matrix $\boldsymbol{S_P}$ simplifies:

$$\boldsymbol{S_P} = \frac{1}{Z_2} \begin{bmatrix} -I_{in}(Z_2 + Z_3) & -Z_3 I_2 & -Z_2 I_{out} \\ I_{in} & I_2 & 0 \end{bmatrix} \quad (22)$$

The equation (6) shows that the $\boldsymbol{U_{YP}}$ component of the covariance matrix $\boldsymbol{U_Y} = \boldsymbol{U_{YX}} + \boldsymbol{U_{YP}}$ of the output quantities, depending on the uncertainties of uncorrelated impedances of the T twoport is

$$\boldsymbol{U_{YP}} = \boldsymbol{S_P U_P S_P^T} =$$

$$\frac{1}{Z_2^2}\begin{bmatrix} I_{in}^2(Z_2+Z_3)^2u_{Z_1}^2 + I_2^2Z_3^2u_{Z_2}^2 + I_{out}^2\,Z_2^2u_{Z_3}^2; & -I_{in}^2(Z_2+Z_3)u_{Z_1}^2 - I_2^2Z_3u_{Z_2}^2 \\ -I_{in}^2(Z_2+Z_3)u_{Z_1}^2 - I_2^2Z_3u_{Z_2}^2; & I_{in}^2u_{Z_1}^2 + I_2^2u_{Z_2}^2 \end{bmatrix} \quad (23)$$

Despite the non-correlation of impedance Z_i, when the $\boldsymbol{U_{YP}}$ matrix is diagonal, U_{out}, I_{out} as variables of the output measurand $\boldsymbol{Y}$, will be correlated. From (15b),(19a-c) and (20) for uncorrelated quantities $\boldsymbol{X}$, obtained are the resultant variances for the output voltage and current are:

$$u_{U_{out}}^2 = \frac{1}{Z_2^2}[(Z_2+Z_3)^2u_{U_{in}}^2 + (Z_1Z_2+Z_3Z_2+Z_1Z_3)^2u_{I_{in}}^2 + I_2^2Z_3^2u_{Z_2}^2 + I_{out}^2\,Z_2^2u_{Z_3}^2]$$

$$u_{I_{out}}^2 \frac{1}{Z_2^2}[u_{U_{in}}^2 + (Z_1+Z_2)^2u_{I_{in}}^2 + I_{in}^2u_{Z_1}^2 + I_2^2u_{Z_2}^2] \quad (24a, b)$$

When $Z_3 = 0$, the T-type twoport becomes a voltage divider Z_1, Z_2.

The variations of current and voltage at its output are:

$$u_{I_{out}}^2 = (u_{U_{in}}^2 + (Z_1+Z_2)^2u_{I_{in}}^2 + I_{in}^2u_{Z_1}^2 + I_2^2u_{Z_2}^2)/Z_2^2$$

$$u_{U_{out}}^2 = u_{U_{in}}^2 + Z_1^2u_{I_{in}}^2 + I_{out}^2\,u_{Z_1}^2 \quad (25a, b)$$

3.3. Matrix U_Y for correlated impedances and uncorrelated input variables X

In this case $\boldsymbol{U} = \boldsymbol{0}$, $\boldsymbol{V} = \boldsymbol{0}$ and from (6) and (15), for the accuracy of functional $\boldsymbol{F}$ is responsible the covariance matrix $\boldsymbol{U_{YP}} = \boldsymbol{S_P U_P S_P^T}$, i.e.:

$$\boldsymbol{U_{YP}} = \frac{1}{Z_2^2}\begin{bmatrix} -I_{in}(Z_2+Z_3) & -I_2Z_3 & -I_{out}Z_2 \\ I_{in} & I_2 & 0 \end{bmatrix}\boldsymbol{U_P}\,\cdot$$

$$\cdot\begin{bmatrix} -I_{in}(Z_2+Z_3) & -I_2Z_3 & -I_{out}Z_2 \\ I_{in} & I_2 & 0 \end{bmatrix}^{\boldsymbol{T}} \quad (26)$$

where: the covariance matrix $\boldsymbol{U_P}$ of impedances as system parameters $\boldsymbol{P}$ has the form

$$\boldsymbol{U_P} = \begin{bmatrix} u_{Z_1}^2 & \rho_{Z12}u_{Z_1}u_{Z_2} & \rho_{Z13}u_{Z_1}u_{Z_3} \\ \rho_{Z12}u_{Z_1}u_{Z_2} & u_{Z_2}^2 & \rho_{Z23}u_{Z_2}u_{Z_3} \\ \rho_{Z13}u_{Z_1}u_{Z_3} & \rho_{Z23}u_{Z_2}u_{Z_3} & u_{Z_3}^2 \end{bmatrix} \quad (27)$$

3.3.1 Uncertainty of output voltage

For $\rho_{in} = 0$, from (17a) and after calculation of the $\boldsymbol{U_{YP}}$ elements, the variance of output voltage is

$$u_{U_{out}}^2 = B_{11}^2u_{U_{in}}^2 + B_{12}^2u_{I_{in}}^2 + u_{UF}^2 \quad (28)$$

where: $u_{UF}^2 = W_{UZ_1}^2u_{Z_1}^2 + W_{UZ_2}^2\,u_{Z_2}^2 + W_{UZ_3}^2u_{Z_3}^2 +$

$$+2\begin{bmatrix} \rho_{Z12}W_{UZ_1}W_{UZ_2}u_{Z_1}u_{Z_2} + \rho_{Z23}W_{UZ_2}W_{UZ_3}u_{Z_2}u_{Z_3} \\ +\rho_{Z13}W_{UZ_1}W_{UZ_3}u_{Z_1}u_{Z_3} \end{bmatrix} \quad (28a)$$

$$B_{11} = 1 + \frac{Z_3}{Z_2}; \quad B_{12} = Z_1 + Z_3 + \frac{Z_1Z_3}{Z_2};$$

$$W_{UZ_1} = I_{in}\left(1 + \frac{Z_3}{Z_2}\right); \; W_{UZ_2} = I_2\frac{Z_3}{Z_2}; \; W_{UZ_3} = I_{out} \quad (28b-f)$$

From (2a) and (30a-f) and $u_{UF}^2 \geq 0$ results the condition for the permissible values of correlation coefficients. The determinant of correlator $\boldsymbol{R}$ cannot be negative, i.e.:

$$det(\boldsymbol{R}) = det\begin{bmatrix} 1 & \rho_{Z12} & \rho_{Z13} \\ \rho_{Z12} & 1 & \rho_{Z23} \\ \rho_{Z13} & \rho_{Z23} & 1 \end{bmatrix} =$$

$$= 1 - \rho_{Z12}^2 - \rho_{Z13}^2 - \rho_{Z23}^2 + 2\rho_{Z12}\rho_{Z13}\rho_{Z23} \geq 0 \quad (29)$$

Thus, the minimal correlation coefficients of three impedance pairs $\rho_{Z12} = \rho_{Z13} = \rho_{Z23} = -1$ cannot occur simultaneously, because the $det(\boldsymbol{R}) = -4 \leq 0$. If one of three coefficients, e.g. $\rho_{Z12} = \pm 1$, then others should be $\rho_{Z23} = \pm\rho_{Z13}$. It have to be analyzed when the correlations of the T-type twoport impedances and the determinant of matrix $\boldsymbol{R}$ and matrix $\boldsymbol{U_P}$ are together equal to zero. The boundary values of three correlation coefficients for area det($\boldsymbol{R}$) = 0 result from the equation (29). The correlation coefficient ρ_{Z23} can be obtained from the expression

$$\rho_{Z23} = f(\rho_{Z12}, \rho_{Z13}) = \rho_{Z12}\rho_{Z13} \pm \sqrt{(1-\rho_{Z12}^2)(1-\rho_{Z13}^2)} \quad (30)$$

Condition: $1 - \rho_{Z12}^2 - \rho_{Z13}^2 - \rho_{Z23}^2 + 2\rho_{Z12}\rho_{Z13}\rho_{Z23} \geq 0$ means that the value of ρ_{Z23} belongs to the range

$$\rho_{Z12}\rho_{Z13} - \sqrt{(1-\rho_{Z12}^2)(1-\rho_{Z13}^2)} \;\leq\; \rho_{Z23} \;\leq$$

$$\leq \rho_{Z12}\rho_{Z13} + \sqrt{(1-\rho_{Z12}^2)(1-\rho_{Z13}^2)} \quad (31)$$

It is the interior of a solid resembling a tetrahedron with modified walls (see fig.2).

For full correlation: $\rho_{Z12} = \rho_{Z13} = \rho_{Z23} = 1$ the variance of output voltage is

$$u_{U_{out}}^2 = B_{11}^2u_{U_{in}}^2 + B_{12}^2u_{I_{in}}^2 + \left(W_{UZ_1}u_{Z_1} + W_{UZ_2}u_{Z_2} + W_{UZ_3}u_{Z_3}\right)^2 \quad (32)$$

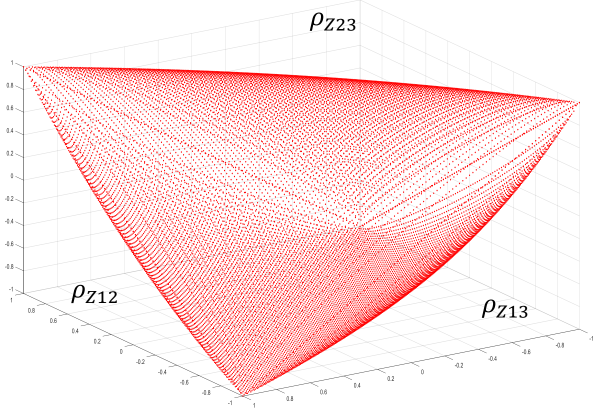

Fig 2. The area of three correlation coefficients correlated by pairs for det ($\boldsymbol{R}$) = 0

In the case of invert suppling of twoport, i.e. $I_{out} < 0$, there is a possibility to obtain the minimum of uncertainty of performance function when

$$W_{UZ_1}u_{Z_1} + W_{UZ_2}u_{Z_2} + W_{UZ_3}u_{Z_3} = 0.$$

3.3.2 Uncertainties and correlation in the output

The formula for uncertainty of current I_{out} if correlated are only impedances, obtained from (28), is:

$$u_{I_{out}}^2 = \frac{1}{Z_2^2}[u_{U_{in}}^2 + (Z_1+Z_2)^2u_{I_{in}}^2 + I_{in}^2u_{Z_1}^2 + I_2^2u_{Z_2}^2 + 2I_{in}I_2\rho_{Z12}u_{Z_1}u_{Z_2}] \quad (33)$$

For total correlation $\rho_{Z12} = 1$, this formula simplify to

$$u_{I_{out}}^2 = \frac{1}{Z_2^2}[u_{U_{in}}^2 + (Z_1+Z_2)^2u_{I_{in}}^2\left(I_{in}u_{Z_1} + I_2u_{Z_2}\right)^2] \quad (33a)$$

In the general case, the correlation coefficient between voltage U_{out} and current $-I_{out}$ is:

$$\rho_{out} = \frac{ -B_{11}B_{21}u_{U_{in}}^2 - B_{12}B_{22}u_{I_{in}}^2 + -\frac{1}{Z_2^2}\begin{bmatrix} I_{in}^2(Z_2+Z_3)u_{Z_1}^2 + I_{in}I_2(Z_2+2Z_3)\rho_{Z12}u_{Z_1}u_{Z_2} + \\ +I_{in}I_{out}Z_2\rho_{Z13}u_{Z_1}u_{Z_3} + I_{out}I_2Z_2\rho_{23}u_{Z_2}u_{Z_3} + I_2^2Z_3u_{Z_2}^2 \end{bmatrix} }{u_{U_{out}}u_{I_{out}}}$$

(34)

and for the divider, i.e. when $Z_3 = 0$

$$\rho_{out} = \frac{-u_{U_{in}}^2 - Z_1(Z_1+Z_2)u_{I_{in}}^2 - I_{in}^2 u_{Z_1}^2 - I_{in} I_2 \rho_{Z12} u_{Z_1} u_{Z_2}}{Z_2 u_{U_{out}} u_{I_{out}}} \quad (35)$$

When all correlation are positive: $\rho_{Z12} = \rho_{Z13} = \rho_{Z23}$=1 and output current is negative

$$W_{IZ_1} u_{Z_1} + W_{IZ_2} u_{Z_2} + W_{IZ_3} u_{Z_3} = 0 \quad (36)$$

then the correlation coefficient is:

$$\rho_{out} = \frac{-B_{11}B_{21}u_{U_{in}}^2 - B_{12}B_{22}u_{I_{in}}^2}{u_{U_{out}} u_{I_{out}}} \quad (37)$$

and ρ_{out} is always negative, because for the passive twoport of type T is $B_{11}B_{21}, B_{12}B_{22} > 0$. Then there are no errors of processing the output voltage, i.e.:

$$u_{U_{out}}^2 = B_{11}^2 u_{U_{in}}^2 + B_{12}^2 u_{I_{in}}^2 \quad (38)$$

In the case $u_{Z_1} \longrightarrow 0$ and the impedance of twoport Z_3 =0

$$u_{I_{out}}^2 = B_{21}^2 u_{U_{in}}^2 + B_{22}^2 u_{I_{in}}^2 \quad (39)$$

then the negative correlation coefficient between the output current and voltage is equal to:

$$\rho_{out} = -\frac{B_{11}B_{21}u_{U_{in}}^2 + B_{12}B_{22}u_{I_{in}}^2}{\sqrt{B_{11}^2 u_{U_{in}}^2 + B_{12}^2 u_{I_{in}}^2}\sqrt{B_{21}^2 u_{U_{in}}^2 + B_{22}^2 u_{I_{in}}^2}} \quad (40)$$

3.4. Numerical example

Figure 3 shows examples of the dependence of the uncertainty of the twoport T output voltage as a function of the output current for different values of the correlation coefficients $\rho_{12} = \rho_{13} = \rho_{23} = 0; 0{,}1; 0{,}5; 0{,}7; 1$, U_{in}=25 V; and relative uncertainties $\delta_{U_{we}} = \delta_{I_{we}} = \delta_{Z_{ij}} = 0{,}2\%$.

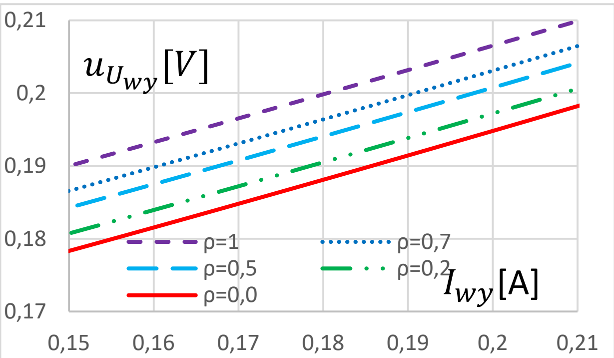

Fig. 3. Uncertainty of the twoport output voltage as a function of the output current I_{out} for different values of the correlation coefficients $\rho = \rho_{12} = \rho_{13} = \rho_{23}$; U_{in} = 25 V, and relative uncertainties $\delta_{U_{we}} = \delta_{I_{we}} = \delta_{Z_{ij}} = 0{,}2\%$

4. Summary and conclusions

Formulas summarized in Table 1, extend the method of GUM Supplement 2 [1] for determining the uncertainty of indirect multi-parameter measurements. The model which considers uncertainties and correlations of system parameters that implement the multivariable processing function, proposed by Z. Warsza, is used.

As an example of the application of this method in indirect 2D measurements of the voltage and current of the two-terminal circuit branch, available only through the twoport T, is presented. Such measurement occur in the identification of voltages and currents of inaccessible directly elements forming the electrical systems, and in multi-sensor measurements and technical diagnostics.

Matrix relationships were derived considering the uncertainty of processing functions performed by twoport. The formulas of increased total uncertainty of the estimated voltage and current due to the impedances uncertainties of this system are find. It did not exceed the sum of uncertainties of input variables and twoport impedances.

In the presented variants of the twoport, the uncertainties at the output also depend on the value and sign of the correlation coefficients of the input quantities and circuit parameters, as well as the current at the output of this circuit. The possibilities of minimizing these uncertainties were also discussed.

The proposed method can be usefully used both for the evaluation of indirect multi-parameter measurements made with a set of instruments, as well as for the assessment of the accuracy of measuring instruments and systems with an integrated measurement system for multivariable measurements. This method may be also the basis for development a new extended version of the Guide GUM Supplement 2 or included in its new version GUM2.

This method will allow the assessment of the accuracy of multivariable instrumental measuring systems by means of uncertainties. Therefore, other interesting partial methods, e.g. given in [5], [9, 10], are not discussed here.

It is possible to analyze by this method the determination of uncertainty of several other multi-parameter measurement systems, e.g. AC networks as in [2], [3], [5], [7], power component measurements in three-phase networks with different waveforms, and then to examine the statistical properties of multi-variable systems with non-Gaussian probability distributions and various processing functions of measurands.

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