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# MEASUREMENT UNCERTAINTY EVALUATION AT MICROMETER CALIBRATION

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**Abstract**

The procedure for measurement uncertainty evaluation at micrometer calibration by the kurtosis method is considered. An expression for the shift of the micrometer readings from the length of the reference gage block is recorded. The measurement model takes into account the corrections for the micrometer resolution to be calibrated, the absence of flatness and deviation from parallelism of its measuring surfaces, as well as for the temperature difference between the gage block and the micrometer. The input quantities and their standard uncertainties are evaluated. The calculation of the combined standard and expanded uncertainties when calibrating the micrometer is performed, taking into account the kurtosis of the input quantities for the number of measurements more than five. When evaluating expanded uncertainty with a small number of measurements, it is proposed to use the law of propagation of expanded uncertainty. An uncertainty budgets have been drawn up, which can serve as a basis for creating a software tool that facilitates routine calculations. The proposed procedures were validated by the Monte Carlo method, which showed that they are likely for an intended use.

**Keywords:** calibration, micrometer, measurement uncertainty, kurtosis method, expanded uncertainty propagation law

## Introduction

The micrometer is a universal measuring instrument that is used to measure the geometric parameters of an object with high accuracy. Like any measuring instrument, a micrometer needs in periodic calibration. In laboratories accredited to compliance the requirements of the ISO/IEC 17025:2017 standard, it is necessary to have a calibration method with a procedure for the measurement uncertainty evaluating. A feature of this procedure is the presence of the expanded uncertainty evaluation operation. The most reliable way to estimate expanded uncertainty is the Monte Carlo method (MCM), which, in contrast to the GUM [1], takes into account the laws of distribution of input quantities. The estimates of measurement uncertainty obtained using the MCM correspond to Bayesian estimates and differ numerically from the estimates obtained using the GUM approach. However, the direct use of MCM for measurement uncertainty evaluation in testing and calibration laboratories accredited for compliance with the requirements of the ISO/ IEC 17025:2017 standard is hampered by the following factors: lack of specialized certified software for estimating measurement uncertainty based on MCM; the impossibility of the existing programs implementing MCM to receive the full budget of the measurement uncertainty; impossibility of documenting a step-by-step procedure for measurement uncertainty evaluation based on MCM.

This article describes a procedure for the measurement uncertainty evaluation at micrometer calibration, based on the kurtosis method and the law of propagation of expanded uncertainty, which make it possible to obtain estimates of the numerical values and the uncertainties of the measurement result close to the estimates obtained using MCM.

## Theoretical justification

**Measurement model substantiation**

The measured value at micrometer calibrating a is the deviation of its readings $l_c$ from the length $l_s$ of the reference gauge block (RGB):

$$\Delta = (l_c + \varepsilon_c + \Delta_c + \Delta_{fl} + \Delta_{pr}) - (l_s + \varepsilon_s) + \alpha l_s \Delta_t \quad (1)$$

where $\varepsilon_c$ is the correction for random variation of the micrometer readings; $\Delta_c$ is the correction, taking into account the resolution of the calibrated micrometer; $\Delta_{fl}$, $\Delta_{pr}$ are the corrections for non-flatness and non-parallelism of the measuring surfaces of the micrometer, respectively; $\varepsilon_s$ is the correction for random variation in the length of the RGB during its periodic calibrations; $\Delta_t$ is a correction that takes into account the temperature difference between the RGB and the micrometer to be calibrated; $\alpha$ is the average coefficient of thermal expansion of the materials of the micrometer and RGB.

**Input quantities evaluation**

Evaluation of multiple micrometer readings $l_c$ is the arithmetic mean of the results of multiple measurements $\hat{l}_{ck}$

$$\bar{l}_c = \frac{1}{n_c}\sum_{r=1}^{n_c}\hat{l}_{cr} \quad (2)$$

where $n_c$ is the number of multiple measurements during calibration.

The evaluated values of all corrections, due to the symmetry of their boundaries, are taken to be zero: $\bar{\varepsilon}_c$=0; $\hat{\Delta}_c$=0; $\hat{\Delta}_{fl}$=0; $\hat{\Delta}_{pr}$=0; $\hat{\varepsilon}_s$=0; $\hat{\Delta}_t$=0. The estimate of the length of the RGB $\hat{l}_s$ is taken from the last certificate of its calibration.

**An estimate of the measurand** is obtained by substituting the values of the input quantities into equation (1), thus obtaining:

$$\hat{\Delta} = \bar{l}_c - \hat{l}_s. \quad (3)$$

**Evaluation of standard uncertainties of input quantities**

The standard uncertainty of the random variation of the micrometer readings will be determined by the formula [3]:

$$u(\bar{\varepsilon}_c) = \sqrt{\frac{1}{n_c(n_c-3)}\sum_{r=1}^{n_c}(l_{cr}-\bar{l}_c)^2} \,. \qquad (4)$$

The standard uncertainty of the correction, which takes into account the resolution of the calibrated micrometer, is expressed through the division value of its vernier scale $d$ on assuming a uniform law of distribution of the reading error as:

$$u(\widehat{\Delta}_c) = \frac{d}{2\sqrt{3}} \,. \qquad (5)$$

The standard uncertainties of the corrections $\Delta_{fl}$, $\Delta_{pr}$ for non-flatness and non-parallelism of the measuring surfaces of a micrometer are determined based on the assumptions of a uniform distribution law of these corrections within the limits of their variability $\left[-\theta_{fl};\theta_{fl}\right]$, $\left[-\theta_{pr};\theta_{pr}\right]$:

$$u(\widehat{\Delta}_{fl}) = \frac{\theta_{fl}}{\sqrt{3}} \,; \qquad (6)$$

$$u(\widehat{\Delta}_{pr}) = \frac{\theta_{pr}}{\sqrt{3}} \,. \qquad (7)$$

The standard uncertainty of the length of the RGB during its calibration is found by the formula:

$$u(\widehat{l}_s) = \frac{U_s}{k_s} \,, \qquad (8)$$

where $U_s$, $k_s$ are the expanded instrumental uncertainty and the coverage factor from the RGB calibration certificate, respectively.

The standard uncertainty is a correction for the random variation in the length of the RGB $u(\varepsilon_s)$ calculate based on information about the length of the RGB $l_{sr}$ for $n_s$ its periodic calibrations as

$$u(\widehat{\varepsilon}_s) = \sqrt{\frac{1}{(n_s-3)}\sum_{r=1}^{n_c}(l_{sr}-\bar{l}_s)^2} \,. \qquad (9)$$

The standard uncertainty of the correction $\Delta_t$, taking into account the temperature difference between the RGB and the calibrated micrometer, is determined based on the assumption of a uniform distribution law of this correction within the limits of its variability $\left[-\theta_t;\theta_t\right]$:

$$u(\widehat{\Delta}_t) = \frac{\theta_t}{\sqrt{3}} \,. \qquad (10)$$

**Calculation of the standard uncertainty of the measurand (combined standard uncertainty)**

The estimation of the standard uncertainty of the measurand is carried out in accordance with the law of propagation of uncertainty, taking into account the absence of correlation between the estimates of the input quantities:

$$u(\widehat{\Delta}) = [u^2(\widehat{\varepsilon}_c) + u^2(\widehat{\Delta}_c) + u^2(\widehat{\Delta}_{fl}) + u^2(\widehat{\Delta}_{pr}) + $$
$$+ u^2(\widehat{l}_s) + u^2(\widehat{\varepsilon}_s) + \alpha^2 \widehat{l}_s^2 u^2(\Delta_t)]^{0,5} \qquad (11)$$

# Evaluation of expanded uncertainty

**Evaluation of expanded uncertainty by the kurtosis method**

The expanded uncertainty will be estimated by the kurtosis method [4] according to the formula:

$$U = ku(\widehat{\Delta}) \,, \qquad (12)$$

where the coverage factor $k$ for a confidence level of 0,95 is calculated by the formula [4]:

$$k_{0,95} = \begin{cases} 0{,}1085\eta^3 + 0{,}1\eta + 1{,}96, & \text{when } \eta < 0; \\ t_{0.95;(6/\eta+4)} \cdot \sqrt{\dfrac{3+\eta}{3+2\eta}}, & \text{when } \eta \ge 0, \end{cases} \qquad (13)$$

at that $\eta$ is the kurtosis of distribution of the measured quantity, defined as:

$$\eta = [\eta(\varepsilon_c)u^4(\widehat{\varepsilon}_c) + \eta(\Delta_c)u^4(\widehat{\Delta}_c) + \eta(\Delta_{fl})u^4(\widehat{\Delta}_{fl}) + \eta(\Delta_{pr})u^4(\widehat{\Delta}_{pr}) +$$
$$+ \eta(l_s)u^4(\widehat{l}_s) + \eta(\varepsilon_s)u^4(\widehat{\varepsilon}_s) + \eta(\Delta_t)\alpha^4 \widehat{l}_s^4 u^4(\widehat{\Delta}_t)] / u^4(\Delta) \,, \qquad (14)$$

where the values of the kurtosis of the input quantities $\eta(\varepsilon_c)$, $\eta(\Delta_c)$, $\eta(\Delta_{пл})$, $\eta(\Delta_{пр})$, $\eta(l_s)$, $\eta(\varepsilon_s)$, $\eta(\Delta_t)$ are taken from the Table 1 in accordance with the probability distribution function (PDF) attributed to them.

Table 1

Kurtosis values for different pdf of input quantities

| PDF | Arcsine | Uniform | Triangular | Gaussian | t-pdf with ν * |
|---|---|---|---|---|---|
| Kurtosis | -1,5 | -1,2 | -0,6 | 0 | 6/(ν-4) |

* ν is the number degrees of freedom.

Obviously $\eta(\varepsilon_c) = 6/(n_c-5)$, $\eta(\Delta_c) = \eta(\Delta_{fl}) = \eta(\Delta_{pr}) =$ $= \eta(\Delta_t) =$ - 1,2; $\eta(\varepsilon_s) = 6/(n_s-5)$. In addition, for $k_s$ = 2 (for the Gaussian PDF) $\eta(\widehat{l}_s) = 0$.

The deviation of the expanded uncertainty estimates obtained by the kurtosis method from the estimates obtained using the MCM does not exceed does not exceed 2,5%.

**Uncertainty budget**

All information about the input and measured values obtained above are summarized in Table 2, which is the uncertainty budget.

Uncertainty budget presents the standard uncertainties of the input quantities calculated by formulas (4)-(11), the kurtosis of the input quantities, the sensitivity coefficients, the contributions of the uncertainty of the input quantities to the measured quantity, as well as information on measurand: its value, standard uncertainty, kurtosis, coverage factor and expanded uncertainty.

It is convenient to use the uncertainty budget as a basis for building a software tool for automating the process of measurement uncertainty evaluation.

**EXAMPLE 1**

10 measurements were made during the calibration of the first-class micrometer MK-25 at the point 15,36 mm.

The measurement results are shown in Table 3.

Table 2

Uncertainty budget for micrometer calibration based on the kurtosis method

| Input quantity | Estimate of input quantity | Standard uncertainty of input quantity | Kurtosis of input quantity | Sensitivity coefficient | Uncertainty contribution |
|---|---|---|---|---|---|
| $l_c$ | $\overline{l}_c$ | – | – | – | – |
| $\varepsilon_c$ | 0 | $u(\overline{\varepsilon}_c)$ | $6/(n_c-5)$ | 1 | $u(\overline{\varepsilon}_c)$ |
| $\Delta_c$ | 0 | $u(\hat{\Delta}_c)$ | -1,2 | 1 | $-u(\hat{\Delta}_c)$ |
| $\Delta_{fl}$ | 0 | $u(\hat{\Delta}_{fl})$ | -1,2 | 1 | $-u(\hat{\Delta}_s)$ |
| $\Delta_{pr}$ | 0 | $u(\hat{\Delta}_{pr})$ | -1,2 | 1 | $u(\hat{\Delta}_{pr})$ |
| $l_s$ | $\hat{l}_s$ | $u(\hat{l}_s)$ | 0 | -1 | $-u(\hat{l}_s)$ |
| $\varepsilon_s$ | 0 | $u(\hat{\varepsilon}_s)$ | $6/(n_s-5)$ | -1 | $-u(\hat{\varepsilon}_s)$ |
| $\Delta_t$ | 0 | $u(\hat{\Delta}_t)$ | -1,2 | $\alpha l_s$ | $\alpha l_s u(\hat{\Delta}_t)$ |
| Measurand | Estimate of measurand | Combined standard uncertainty | Kurtosis of measurand | Coverage factor | Expanded uncertainty |
| $\Delta$ | $\hat{\Delta}$ | $u(\Delta)$ | $\eta$ | $k$ | $U$ |

Table 3

The measuring results of the RGB length

| Measurement number | Measured value, mm |
|---|---|
| 1 | 15,359 |
| 2 | 15,359 |
| 3 | 15,359 |
| 4 | 15,358 |
| 5 | 15,359 |
| 6 | 15,358 |
| 7 | 15,359 |
| 8 | 15,359 |
| 9 | 15,359 |
| 10 | 15,359 |

The actual values of the RGB with a nominal value of 15,36 mm, obtained from the results of its calibration for 6 years, are given in Table 4.

Table 4

RGB calibration results

| Calibration year | RGB length deviation from the nominal value, µm |
|---|---|
| 2015 | 0,36 |
| 2016 | 0,34 |
| 2017 | 0,35 |
| 2018 | 0,35 |
| 2019 | 0,34 |
| 2020 | 0,36 |

For this type of micrometer $d$ = 1 µm, $\theta_{fl}$ = 0,6 µm; $\theta_{pr}$ = 1,5 µm; $\theta_t$ = 2 $^0$C; $\alpha$ = 11,5 $10^{-6}$ $K^{-1}$. From RGB certificate: $U_s$ = 0,02 µm and $k_s$ = 2.

Based on this information of these measurements, an uncertainty budget was drawn up (Table 5).

Table 5

Example of uncertainty Budget for Micrometer Calibration based on the kurtosis method

| Input quantity | Estimate of input quantity | Standard uncertainty of input quantity | Kurtosis of input quantity | Sensitivity coefficient | Uncertainty contribution, µm |
|---|---|---|---|---|---|
| $l_c$ | 15,3588 mm | – | – | – | – |
| $\varepsilon_c$ | 0 mm | 0,151 µm | 1,2 | 1 | 0,151 |
| $\Delta_c$ | 0 mm | 0,289 µm | -1,2 | 1 | 0,289 |
| $\Delta_{fl}$ | 0 mm | 0,866 µm | -1,2 | 1 | 0,866 |
| $\Delta_{pr}$ | 0 mm | 0,346 µm | -1,2 | 1 | 0,346 |
| $l_s$ | 15,36035 mm | 0,01 µm | 0 | -1 | -0,01 |
| $\varepsilon_s$ | 0 mm | 0,1155 µm | 6 | -1 | -0,1155 |
| $\Delta_t$ | 0 $^0$C | 1,155 $^0$C | -1,2 | 0,177 µm/$^0$C | 0,204 |
| Measurand | Estimate of measurand | Combined standard uncertainty | Kurtosis of measurand | Coverage factor | Expanded uncertainty |
| $\Delta$ | -0,00155 mm | 1,009 µm | -0,68 | 1,86 | 1,88 µm |

Simulation of these measurements by the MCM using the program [7] made it possible to obtain the following results: $\hat{\Delta}$ = 0,00155 mm; $u(\hat{\Delta})$ = 1,01 μm; $k$ = 1,85; $U$ = 1,87 μm (Fig. 1).

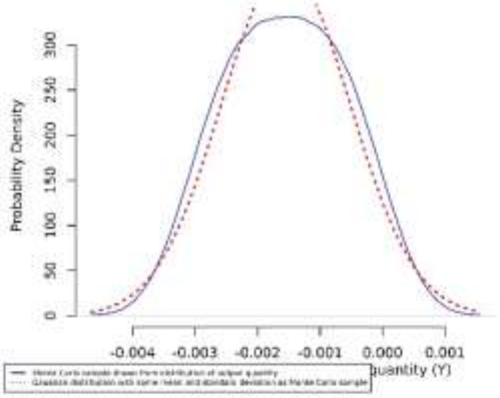


Fig.1. The results of modeling MCM of Example 1

**Evaluation of expanded uncertainty based on the law of propagation of expanded uncertainty**

It is known that the kurtosis method is applicable for the number of repeated measurements of input quantities more than 5 [4]. More universal, but less accurate, is the law of propagation of expanded uncertainty [6], which can be used to calculate expanded uncertainty when the number of repeated measurements is greater than 3.

The expression for calculating the expanded uncertainty for a probability of 0,95 in this case has the form [6]:

$$U = \sqrt{U_B^2 + U_R^2}\,, \qquad (15)$$

where $U_B(y)$, $U_R(y)$ are estimates of the basic expanded uncertainty and random expanded uncertainty of the measurand, respectively.

The estimate of the basic expanded uncertainty is calculated by the formula:

$$U_B = k_B \cdot u_B(\Delta)\,, \qquad (16)$$

where $u_B(\Delta)$ is the main (excluding random corrections) standard uncertainty of the measurand:

$$u_B(\hat{\Delta}) = \sqrt{u^2(\hat{\Delta}_c) + u^2(\hat{\Delta}_{пл}) + u^2(\hat{\Delta}_{пр}) + u^2(\hat{l}_s) + \alpha^2 \hat{l}_s^2 u^2(\Delta_t)}\,; \qquad (17)$$

$k_B$ is the coverage factor, calculated by the kurtosis method according to the formula (13).

Here $\eta_B$ is the kurtosis of the distribution of the composition of the basic input quantities of the measurand, equal to:

$$\eta_B = [\eta(\hat{\Delta}_c)u^4(\hat{\Delta}_c) + \eta(\hat{\Delta}_{fl})u^4(\hat{\Delta}_{fl}) + \eta(\hat{\Delta}_{pr})u^4(\hat{\Delta}_{pr}) +$$
$$+ \eta(\hat{l}_s)u^4(\hat{l}_s) + \eta(\hat{\Delta}_t)\alpha^4 \hat{l}_s^4 u^4(\hat{\Delta}_t)] / u_B^4(\Delta)\,, \qquad (18)$$

where $\eta_B(x_i)$ is the kurtosis of distribution of the basic $i$-th input quantity.

Expanded uncertainty of random corrections for $n_i \geq 4$ is calculated by the formula:

$$U_R = \sqrt{t^2_{(0,95;\nu_c)} \frac{n_c - 3}{n_c - 1} u^2(\bar{\varepsilon}_c) + t^2_{(0,95;\nu_s)} \frac{n_s - 3}{n_s - 1} u^2(\hat{\varepsilon}_s)}\,, \qquad (19)$$

where $t_{(0,95;\nu_i)}$ is the t-coefficient for the probability 0,95 and the number of degrees of freedom $\nu_i = n_i - 1$.

In this case, the combined standard uncertainty of random corrections is found by the formula:

$$u_R(\varepsilon) = \sqrt{u^2(\hat{\varepsilon}_c) + u^2(\hat{\varepsilon}_s)}\,, \qquad (20)$$

and the coverage factor for the expanded uncertainty of random corrections will be:

$$k_R = U_R \Bigg/ \sqrt{\frac{n_c - 3}{n_c - 1} u^2(\hat{\varepsilon}_c) + \frac{n_s - 3}{n_s - 1} u^2(\hat{\varepsilon}_s)}\,, \qquad (21)$$

then the equivalent number of degrees of freedom is found by the formula [7]:

$$\nu_{eq} = \frac{1}{0,822}\left(\frac{1,96}{k_R - 1,96} + 0,87\right). \qquad (22)$$

The standard uncertainty of the measured quantity is determined by the formula:

$$u(\Delta) = \sqrt{u_B^2(\Delta) + u_R^2(\varepsilon)}\,, \qquad (23)$$

and the coverage factor is calculated as:

$$k = U / u(y) \qquad (24)$$

The deviation of the expanded uncertainty estimates obtained by this method from the estimates obtained using the MCM does not exceed ± 4,5%.

When implementing the law of propagation of expanded uncertainty, it is necessary to draw up two uncertainty budgets: for the basic components (Table 6) and for random corrections (Table 7).

Table 6

Uncertainty budget for basic components in micrometer calibration

| Input quantity | Estimate of input quantity | Standard uncertainty of input quantity | Kurtosis of input quantity | Sensitivity coefficient | Uncertainty contribution |
|---|---|---|---|---|---|
| $l_c$ | $\bar{l}_c$ | – | – | – | – |
| $\Delta_c$ | 0 | $u(\hat{\Delta}_c)$ | -1,2 | 1 | $-u(\bar{T}_s)$ |
| $\Delta_{fl}$ | 0 | $u(\hat{\Delta}_{fl})$ | -1,2 | 1 | $-u(\hat{\Delta}_{fl})$ |
| $\Delta_{pr}$ | 0 | $u(\hat{\Delta}_{pr})$ | -1,2 | 1 | $u(\hat{\Delta}_{pr})$ |
| $l_s$ | $\hat{l}_s$ | $u(\hat{\Delta}_s)$ | 0 | -1 | $-u(\hat{\Delta}_s)$ |
| $\Delta_t$ | 0 | $u(\hat{\Delta}_t)$ | -1,2 | $\alpha l_s$ | $\alpha l_s u(\hat{\Delta}_t)$ |
| Measurand | Estimate of measurand | Combined standard uncertainty | Kurtosis of measurand | Coverage factor | Basic expanded uncertainty |
| $\Delta$ | $\hat{\Delta}$ | $u_B(\hat{\Delta})$ | $\eta$ | $k$ | $U_B$ |

Table 7

Uncertainty budget for random corrections at micrometer calibration

| Input quantity | Estimate of input quantity | Standard uncertainty of input quantity | Number degrees of freedom | Sensitivity coefficient | Expanded uncertainty contribution |
|---|---|---|---|---|---|
| $\varepsilon_c$ | 0 | $u(\varepsilon_c)$ | $\nu_c$ | 1 | $U_c(\varepsilon)$ |
| $\varepsilon_s$ | 0 | $u(\varepsilon_s)$ | $\nu_s$ | -1 | $-U_s(\hat{\varepsilon})$ |
| Measurand | Estimate of measurand | Combined standard uncertainty | Equivalent number degrees of freedom | Coverage factor | Random expanded uncertainty |
| $\varepsilon$ | 0 | $u(\varepsilon)$ | $\nu_{eq}$ | $k_R$ | $U_R$ |

**EXAMPLE 2**

5 measurements were made during the calibration of the micrometer MK-25 at the point 15,36 mm (measured values 1-5 in Table 3). The actual values of the RGB with a nominal value of 15,36 mm, obtained from the results of its calibration for 4 years, are shown (values of 2017-2020, Table 4).

Based on the results of these measurements, an uncertainty budget was drawn up for the main components (Table 8) and for random corrections (Table 9) to implement the law of expanded uncertainty propagation.

According to the results of Tables 8, 9 the characteristics of the measured value are calculated: $\hat{\Delta} = -0{,}00155$ mm; $u(\hat{\Delta}) = 1{,}037$ μm; $k = 1{,}859$; $U = 1{,}93$ μm.

The MCM modeling of these measurements using the program [5] made it possible to obtain the following results: = -0.00155 mm; = 1.04 μm; = 1.85; = 1.93 μm (Fig. 2).

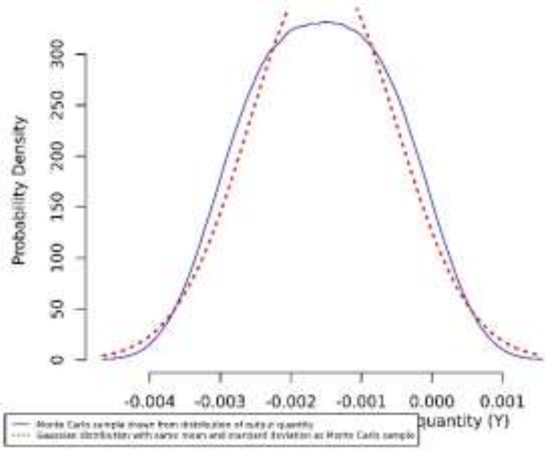


Fig.1. The results of modeling MCM of Example 2

Table 8

Example of uncertainty budget for basic components in micrometer calibration

| Input quantity | Estimate of input quantity | Standard uncertainty of input quantity | Kurtosis of input quantity | Sensitivity coefficient | Uncertainty contribution |
|---|---|---|---|---|---|
| $l_c$ | 15,3588 mm | – | – | – | – |
| $\Delta_c$ | 0 мм | 0,289 мкм | -1,2 | 1 | 0,289 |
| $\Delta_{fl}$ | 0 мм | 0,866 мкм | -1,2 | 1 | 0,866 |
| $\Delta_{pr}$ | 0 мм | 0,346 мкм | -1,2 | 1 | 0,346 |
| $l_s$ | 15,36035 mm | 0,01 мкм | 0 | -1 | -0,01 |
| $\Delta_t$ | 0 $^0$C | 1,155 $^0$C | -1,2 | 0,177 μm/$^0$C | 0,204 |
| Measurand | Estimate of measurand | Combined standard uncertainty | Kurtosis of measurand | Coverage factor | Expanded uncertainty |
| $\Delta$ | -0,00155 mm | 0,9975 мкм | -0,71 | 1,85 | 1,846 мкм |

Table 9

Example of uncertainty budget for random corrections at micrometer calibration

| Input quantity | Estimate of input quantity | Standard uncertainty of input quantity | Number degrees of freedom | Sensitivity coefficient | Expanded uncertainty contribution, μm |
|---|---|---|---|---|---|
| $\varepsilon_c$ | 0 mm | 0,2828 μm | 4 | 1 | 0,5553 |
| $\varepsilon_s$ | 0 mm | 0,0141 μm | 3 | -1 | -0,0260 |
| Measurand | Estimate of measurand | Combined standard uncertainty | Equivalent number degrees of freedom | Coverage factor | Random expanded uncertainty |
| $\varepsilon$ | 0 mm | 0,2829 мкм | 3,98 | 2,78 | 0,5554 мкм |

## CONCLUSIONS

1. The procedure for the measurement uncertainty evaluation at micrometer calibration with the expanded uncertainty evaluation by the kurtosis method is presented. The procedure is suitable for the number of multiple measurements ≥6 and symmetric distribution laws of the input quantities.

2. The procedure for measurement uncertainty evaluation at micrometer calibration with the number of multiple measurements ≥4 is presented. In this case, the estimation of expanded uncertainty was carried out in accordance with the law of propagation of expanded uncertainty.

3. Examples of the implementation of the developed procedures and their modeling by the Monte Carlo method, which showed good agreement of the results, are considered.

**Анотація**

Розглянуто процедуру оцінювання невизначеності вимірювань під час калібрування мікрометра. Записано вираз для відхилень показань мікрометра від довжини еталонної кінцевої міри. Модель вимірювання враховує поправки на роздільну здатність мікрометра, що калібрується, відсутність площинності і відхилення від паралельності його вимірювальних поверхонь, а також на різницю температур між еталонною кінцевою мірою і мікрометром. Оцінюються вхідні величини і їх стандартні невизначеності. Виконується розрахунок сумарної стандартної і розширеної невизначеностей при калібрування мікрометра, з урахуванням ексцесів вхідних величин для числа вимірювань більше п’яти. При оцінюванні розширеної невизначеності при малому числі вимірювань пропонується використовувати закон поширення розширеної невизначеності. Складено бюджети невизначеності, які можуть слугувати основою для створення програмних засобі, які полегшують рутинні обчислення. Запропонована процедура була валідована методом Монте-Карло, який показав, що вона підходить для передбачуваного використання.

**Ключові слова:** калібрування; мікрометр; невизначеність вимірювань; метод ексцесів; закон розповсюдження розширеної невизначеності

**Аннотация**

Рассмотрена процедура оценивания неопределенности измерений при калибровке микрометра. Записано выражение для отклонений показаний микрометра от длины эталонной концевой меры длины. Модель измерения учитывает поправки на разрешение калибруемого микрометра, отсутствие плоскостности и отклонение от параллельности его измерительных поверхностей, а также на разницу температур между измерительным блоком и откалиброванным микрометром. Оцениваются входные величины и их стандартные неопределенности. Выполняется расчет суммарной стандартной и расширенной неопределенностей при калибровке микрометра, с учетом эксцессов входных величин для числа измерений больше пяти. При оценивании расширенной неопределенности при малом числе измерений предлагается использовать закон распространения расширенной неопределенности. Составлен бюджет неопределенности, который может служить основой для создания программного средства, облегчающего рутинные вычисления. Предложенная процедура была валидрована методом Монте-Карло, который показал, что она подходит для предполагаемого использования.

**Ключевые слова:** микрометр, калибровка, метод эксцессов, закон распространения расширенной неопределенности. Метод Монте-Карло.

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