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Considering of the input quantities distributions in the procedure for measurement uncertainty evaluating on the example of resistance box calibration

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Abstract

The controversy over estimates of measurement uncertainty in the Guide to the Expression of Uncertainty in Measurement and Supplement 1 to it is considered. It is shown that possible ways to overcome these disagreements are to use the methods developed by the authors. Using the example of resistance calibration on a direct current, the features of taking into account the distribution of input values in the procedure for uncertainty evaluation when using the kurtosis method and low of propagation of expanded uncertainty are shown. A model of direct measurement of the resistance value of a resistance measure using a reference ohmmeter is written, the procedures for measurement uncertainty evaluation are described, and the uncertainty budgets are given. An example of measurement uncertainty evaluation at calibrating a resistance box P33 class 0.2 using a Fluke 8508 A digital multimeter is described. The expanded uncertainty of measurement for this example was estimated based on the NIST Uncertainty Machine web application, which showed good agreement with the estimates obtained by the methods considered.

Keywords: calibration; resistance box; kurtosis method; expanded uncertainty; law of propagation of expanded uncertainty; uncertainty budget.

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Introduction

The measurement uncertainty evaluation in accredited testing and calibration laboratories is regulated by the international standard ISO 17025:2017 [1]. At the same time [1] prescribes to use the Guide on the expression of uncertainty in measurement (GUM) [2] as a normative document. However, the use of the GUM is associated with a number of disadvantages, the main of which is the independence of the obtained expanded uncertainty estimates from the probability distribution functions (PDF) of the input values and the presence of a bias in the numerical values of the measured value and its standard and expanded uncertainties under nonlinear model equations. That is why the Working Group 1 (WG-1) of the Joint Committee for Guides in Metrology (JCGM) has developed Appendix 1 to the GUM based on the Monte Carlo method (MCM) [3], which eliminates these disadvantages. However, it should be noted that even with linear model equations and Gaussian distributions of input values, the uncertainty estimates obtained using [2] and [3] differ from each other [4]. The reason for this is the different approaches to evaluation the characteristics of type A uncertainty in both documents [4]. Therefore, when developing procedures for uncertainty evaluation, it is advisable to rely on approaches that lead to results consistent with the results obtained by the Monte Carlo method. Such approaches are described in the Guide [5], as well as in articles [6–7]. The Guide [5] does not contain examples of the use of the proposed approaches. Compensate for this gap, this article discusses the features of taking into account the PDF of input values when evaluating the uncertainty of measurements using the example of the resistance box calibration.

Basic theoretical relations

Calibration of resistance box is, as well as their verification [8], created in one of two ways: by the method of element-by-element or complete calibration. The method of element-by-element calibration consists in the separate determination of the resistances of all stages of the box decades. By the method of complete calibration, the actual values of the resistances of each decade are determined for all indications (or the smallest) indications of all other decades.

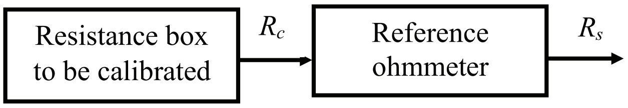

Fig. 1. Scheme of direct measurement with a reference ohmmeter of the value reproduced by the calibrated resistance box

The DC resistance box calibration scheme carried out by direct measurement of the value reproduced by the resistance box with a reference ohmmeter, is shown in Fig. 1.

In this case, the model equation has the form:

$$R_c = R_s + \varepsilon_s + R_0 \alpha \Delta_t, \qquad (1)$$

where R_c – quantity reproduced by the resistance box; R_s – quantity measured with a reference ohmmeter; ε_s – correction for random variability of readings of a reference ohmmeter; R_0 – nominal resistance of the resistance box at the calibrated point; α – temperature coefficient of resistance of the resistance measure (resistance box); Δ_t – correction for the error of temperature measuring of resistance box.

The value $\bar{R}_s$ is determined from the results of repeated measurements R_{si} with a reference ohmmeter:

$$\bar{R}_s = \frac{1}{n}\sum_{i=1}^{n} R_{si}. \qquad (2)$$

A normal distribution law with zero mathematical expectation is assigned to the correction ε_s for random variability of the readings of the reference ohmmeter. A uniform distribution law with zero mathematical expectation is assigned to the correction $\hat{\Delta}_t$ for the error in temperature measuring of resistance box.

Therefore, the measured value of the resistance of the box will be equal to:

$$\hat{R}_c = \bar{R}_s. \qquad (3)$$

The standard measurement uncertainty at resistance box calibration is determined by the formula:

$$u(\hat{R}_c) = \sqrt{u^2(\hat{R}_s) + u^2(\bar{\varepsilon}_s) + R_0^2 \alpha^2 u^2(\hat{\Delta}_t)}, \qquad (4)$$

where $u(\hat{R}_s)$ – standard uncertainty of resistance measurement with an ohmmeter, expressed of the expanded instrumental uncertainty of an ohmmeter U_s assuming a normal distribution (coverage factor $k_s = 2$), using the formula:

$$u(\hat{R}_s) = \frac{U_s}{k_s}; \qquad (5)$$

$u(\bar{\varepsilon}_s)$ – the standard uncertainty of the random variability of the readings of the reference ohmmeter is equal to [6]:

$$u(\bar{\varepsilon}_s) = \frac{s(\varepsilon_{si})}{\sqrt{n}}\sqrt{\frac{n-1}{n-3}}, \qquad (6)$$

moreover, the standard deviation of the random variability of the readings of the reference ohmmeter $s(\varepsilon_{si})$ is found by the formula:

$$s(\varepsilon_{si}) = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(\varepsilon_{si} - \bar{\varepsilon}_s)^2}; \qquad (7)$$

$u^2(\hat{\Delta}_t)$ – standard uncertainty of the temperature setting in the laboratory, expressed through its boundaries $\pm\theta_t$, assuming a uniform distribution law Δ_t within these boundaries as follows:

$$u(\hat{\Delta}_t) = \frac{\theta_t}{\sqrt{3}}. \qquad (8)$$

Expanded uncertainty computation

The expanded uncertainty in accordance with [5] can be calculated in two ways: by the kurtosis method [6] and using the law of propagation of expanded uncertainty [7].

Kurtosis method [6] involves the calculation of expanded uncertainty by the formula:

$$U(\hat{R}_c) = k(\eta) u(\hat{R}_c), \qquad (9)$$

where $k(\eta)$ – coverage factor depending on the kurtosis η of the measurand PDF.

For a confidence level of 0.95:

$$k(\eta) = \begin{cases} 0.1085 \cdot \eta^3 + 0.1 \cdot \eta + 1.96 \\ 1.96 \end{cases}, \qquad (10)$$

where the kurtosis of the measurand is calculated by the formula:

$$\eta = \frac{\eta_s u^4(R_s) + \eta_\varepsilon u^4(\bar{\varepsilon}_s) + \eta_t R_0^4 \alpha^4 u^4(\Delta_t)}{u^4(R_c)}, \qquad (11)$$

in which the kurtosis of the input quantities are taken in accordance with their PDF's and are equal, respectively, $\eta_s = 0$; $\eta_\varepsilon = \frac{6}{n-5}$; $\eta_t = -1.2$.

The uncertainty budget for this case is shown in Table 1.

The law of propagation of expanded uncertainty [7] assumes a separate calculation of expanded uncertainty for non-random (basic) U_B and random U_R input values, followed by their combination by the formula:

$$U = \sqrt{U_B^2 + U_R^2}. \qquad (12)$$

The expanded uncertainty U_B is calculated by the kurtosis method using the formulas:

$$U_B = k(\eta_B) u_B(\hat{R}_c); \qquad (13)$$

$$k(\eta_B) = 0.1085 \cdot \eta_B^3 + 0.1 \cdot \eta_B + 1.96, \qquad (14)$$

moreover, the process of the main components of the measurand is calculated as

Table 1

Measurement uncertainty budget at resistance box calibrating when implementing the kurtosis method

Input quantities	Values of input quantities	Standard uncertainty of input quantities	Kurtosis of input quantities	Sensitivity coefficient	Uncertainty contributions
R_s	$\overline{R}_s$	$u(\widehat{R}_s)$	η_s	1	$u(\widehat{R}_s)$
ε_s	0	$u(\overline{\varepsilon}_s)$	η_ε	1	$u(\overline{\varepsilon}_s)$
Δ_t	0	$u(\widehat{\Delta}_t)$	η_t	$R_0\alpha$	$\alpha R_0 u(\widehat{\Delta}_t)$
Measurand	Measurand value	Standard uncertainty of the measurand	Measurand kurtosis	Coverage factor	Expanded uncertainty
R_c	$\widehat{R}_c$	$u(\widehat{R}_c)$	η	$k(\eta)$	$U(\widehat{R}_c)$

$$\eta_B = \frac{\eta_s u^4(R_s) + \eta_t R_0^4 \alpha^4 u^4(\Delta_t)}{u_B^4(R_c)}, \quad (15)$$

where $u_B(\widehat{R}_c)$ – basic (excluding random corrections) standard uncertainty of the measurand:

$$u_B(\widehat{R}_c) = \sqrt{u^2(\widehat{R}_s) + R_0^2\alpha^2 u^2(\widehat{\Delta}_t)}. \quad (16)$$

The expanded uncertainty U_R is calculated by the formula:

$$U_R = t_{0.95;(n-1)} \frac{s(\varepsilon_{si})}{\sqrt{n}}, \quad (17)$$

where $s(\varepsilon_{si})$ is calculated by the formula (7), a $t_{0.95;(n-1)}$ – Student's coefficient for the probability of 0.95 and the number of degrees of freedom $n-1$.

After calculating the total expanded uncertainty U, the combined standard uncertainty is determined by the formula (4), followed by the calculation of the coverage factor by the formula:

$$k = \frac{U}{u(\widehat{R}_c)}. \quad (18)$$

The resulting value of the coverage factor allows you to determine the kurtosis of the measurand by the formula:

$$\eta = 17.071 \cdot k^3 - 81.944 \cdot k^2 + 132.31 \cdot k - 73.109. \quad (19)$$

This kurtosis can be used for further measurement uncertainty evaluates, in which the value of the resistance box $\widehat{R}_c$ will be the input value.

The uncertainty budget for this case is shown in Table 2.

Example.

Let's consider the calibration of a resistance box type P33 class 0.2 having a range from 0.1 to 99999.9 Ω, using a working standard – a Fluke 8508 A digital multimeter, using the complete calibration method at the 9 kΩ point.Temperature coefficient of resistance of a measure of resistance, $\alpha = 10^{-5}\,K^{-1}$. The error limits for measuring the temperature of the measure are 0.5 °C.

Table 2

Measurement uncertainty budget at resistance box calibrating when implementing the law of propagation of expanded uncertainty

Input quantities	Values of input quantities	Standard uncertainty of input quantities	Kurtosis of input quantities	Sensitivity coefficient	Uncertainty contributions
R_s	$\overline{R}_s$	$u(\widehat{R}_s)$	η_s	1	$u(\widehat{R}_s)$
Δ_t	0	$u(\widehat{\Delta}_t)$	η_t	$R_0\alpha$	$\alpha R_0 u(\widehat{\Delta}_t)$
Measurand	Measurand value	Standard uncertainty of measurand	Measurand kurtosis	Coverage factor	Expanded uncertainty
ε_s	0	$u(\overline{\varepsilon}_s)$	–	–	$U_R(\overline{\varepsilon}_s)$
R_c	$\widehat{R}_c$	$u_B(\widehat{R}_c)$	η_B	$k(\eta_B)$	$U_B(\widehat{R}_c)$
		$u(\widehat{R}_c)$	η	$k(\eta)$	$U(R_c)$

The expanded uncertainty of the multimeter in the resistance measurement mode in this point is 2.2×10^{-5} kΩ.

The standard uncertainty of measuring resistance with an ohmmeter is calculated by formula (5) and is 1.1×10^{-5} kΩ.

The results of 6-fold measurement of the resistance of the resistance box at the point 9 kΩ are given in Table 3.

Table 3

The results of measuring the resistance of the box, kΩ

9.00075	9.00074	9.00073
9.00073	9.00074	9.00075

Using the formula (2), we calculate the estimate of the resistance measurement result $\overline{R}_s$ = 9.00074 kΩ. The standard deviation of the variation of the readings of the reference ohmmeter $s(\varepsilon_{si})$, calculated by the formula (7), was 8.94×10^{-6} kΩ. The standard uncertainty of the variation of the readings of the reference ohmmeter $u(\overline{\varepsilon}_s)$, calculated by the formula (6), was 4.71×10^{-6} kΩ. The standard uncertainty of measuring the temperature of the measure, calculated by formula (8), is $u(\widehat{\Delta}_t)=0.289$ °C.

The budget of the measurement uncertainty when calibrating the resistance box when implementing the method of excesses is presented in Table 4.

The uncertainty budget of the measurement at calibrating the resistance box when implementing the law of propagation of expanded uncertainty is presented in Table 5. It contains expanded uncertainties U, U_B and U_R, which are calculated by formulas (12), (13) and (17), respectively. In it, after calculating the combined expanded uncertainty *U,* the combined standard uncertainty was determined by the formula (4), followed by the calculation of the coverage factor by the formula (18). The resulting value of the coverage factor 1.809 made it possible to determine the kurtosis of the measured value using the formula (19):

$$\eta = 17.071\cdot1.809^3 - 81.944\cdot1.809^2 + \\ +132.31\cdot1.809 - 73.109 = -0.816.$$

According to the results of Table 4, the Monte Carlo simulation of the considered measurements was carried out using the program [9], as a result of which the following results were obtained: $\widehat{R}_c=9.00074$ kΩ; $u(\widehat{R}_c)=0.0000286$ kΩ; $k=1.8$; $U=0.00005$ kΩ (Fig. 2). Thus, the relative deviation of the obtained estimate of the expanded uncertainty from the results obtained by both methods proposed in [5] does not exceed 4%.

Table 4

Uncertainty budget at resistance box calibrating when implementing the kurtosis method

Input quantities	Values of input quantities	Standard uncertainty of input quantities	Kurtosis of input quantities	Sensitivity coefficient	Uncertainty contributions, kΩ
R_s	9.00074 kΩ	0.000011 kΩ	0	1	0.000011
ε_s	0 kΩ	4.714×10^{-6} kΩ	6	1	4.714×10^{-6}
Δ_t	0 °C	0.289 °C	-1.2	0.00009 kΩ/K	0.00002601
Measurand	Measurand value	Standard uncertainty of measurand	Measurand kurtosis	Coverage factor	Expanded uncertainty
R_c	9.00074 kΩ	0.0000286 kΩ	-0.816	1.82	0.000052 kΩ

Table 5

Measurement uncertainty budget at resistance box calibrating the when implementing the law of propagation of expanded uncertainty

Input quantities	Values of input quantities	Standard uncertainty of input quantities	Kurtosis of input quantities	Sensitivity coefficient	Uncertainty contributions, kΩ
R_s	9.00074 kΩ	0.000011 kΩ	0	1	0.000011
Δ_t	0 °C	0.289 °C	-1.2	0.00009 kΩ/K	0.00002601
Measurand	Measurand value	Standard uncertainty of measurand	Measurand kurtosis	Coverage factor	Expanded uncertainty
ε_s	0	0.00000471 kΩ	–	–	0.0000094 kΩ
R_c	9.00074 kΩ	0.00002824 kΩ	-0.863	1.804	0.0000509 kΩ
		0,00002863 kΩ	-0.816	1.809	0.0000519 kΩ

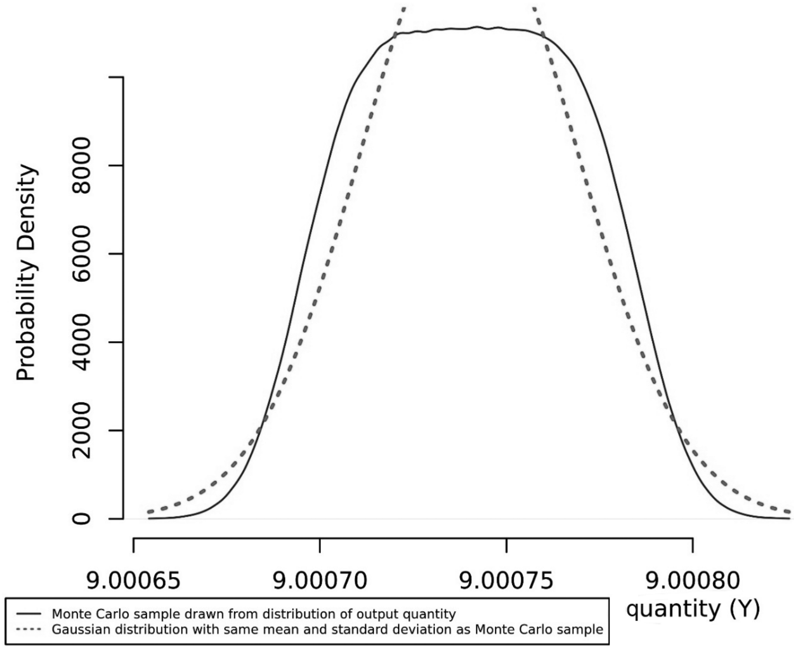

Fig. 2. Monte Carlo Method Simulation Results

Conclusions

1. At uncertainty evaluation in calibration laboratories, the Guide to the Expression of Uncertainty in Measurement and the Supplement 1 to it based on the Monte Carlo method are used, which give different values of the uncertainty for linear model equations and Gaussian distributions of the input quantities.

2. In order to eliminate discrepancies in uncertainty estimates, it is proposed to use the kurtosis method and the law of propagation of expanded uncertainty, developed at the NSC "Institute of Metrology".

3. The procedures for uncertainty evaluation at DC resistance box calibrating are considered, uncertainty budgets are compiled, which can serve as a basis for creating software for automating the evaluation of measurement uncertainty during calibration.

4. Investigation of the uncertainty of measurements carried out during the verification of the P33 resistance box using a digital multimeter Fluke 8508 A showed good agreement of the results obtained with the expanded uncertainty estimates received by the Monte Carlo method.

Урахування розподілів вхідних величин у процедурі оцінювання невизначеності вимірювань на прикладі калібрування магазину опору

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Анотація

Розглянуто основні недоліки використання Настанови з подання невизначеності вимірювання: незалежність одержуваних оцінок розширення невизначеності від закону розподілення вхідних величин і наявності зміщення числових значень вимірюваної величини та її стандартної й розширеної невизначеностей при нелінійних модельних рівняннях. Показано розбіжності в оцінках невизначеності вимірювань в Настанові з подання невизначеності вимірювань та Додатку 1 до нього, який реалізує метод Монте-Карло, навіть при лінійних модельних рівняннях і гаусівських розподілах вхідних величин. Показано, що можливими шляхами подолання цих розбіжностей є застосування методу ексцесів і закону поширення розширеної невизначеності, розроблених авторами. На прикладі калібрування магазину опору на постійному струмі показано особливості застосування цих методів у процедурі оцінювання невизначеності вимірювань. Записано модель прямого вимірювання значення опору міри опору за допомогою зразкового омметра, описані процедури оцінювання невизначеності вимірювань, наводяться бюджети невизначеності для кожного із методів. Описано приклад оцінювання невизначеності вимірювань при калібруванні магазину опорів Р33 класу 0,2 за допомогою цифрового мультиметра Fluke 8508 A. Моделювання методом Монте-Карло продемонструвало хороший збіг його результатів з оцінками розширеної невизначеності, які були отримані розглянутими методами.

Ключові слова: калібрування; магазин опору; розширена невизначеність; метод ексцесів; закон поширення розширеної невизначеності; бюджет невизначеності.

Учет распределений входных величин в процедуре оценивания неопределенности измерений на примере калибровки магазина сопротивления

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Аннотация

Рассмотрены разногласия в оценках неопределенности измерений в Руководстве по выражению неопределенности измерений и Дополнении 1 к нему. Показано, что возможными путями преодоления этих разногласий является применение метода эксцессов и закона распространения расширенной неопределенности, разработанных авторами. На примере калибровки магазина сопротивления на постоянном токе показаны особенности применении этих подходов в процедуре оценивания неопределенности измерений. Записана модель прямого измерения значения сопротивления меры сопротивления с помощью образцового омметра, описаны процедуры оценивания неопределенности измерений, приводятся бюджеты неопределенности для каждого из методов. Описан пример оценивания неопределенности измерений при калибровке магазина сопротивлений Р33 класса 0,2 с помощью цифрового мультиметра Fluke 8508 A. Произведена оценка расширенной неопределенности измерений для этого примера на основе веб-приложения NIST Uncertainty Machine, которая показала хорошее совпадение с оценками, полученными рассмотренными методами.

Ключевые слова: калибровка; магазин сопротивления; расширенная неопределенность; метод эксцессов; закон распространения расширенной неопределенности; бюджет неопределенности.

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