UDC 621.317.732
DOI: 10.24027/2306-7039.2.2021.236078

# Method of kurtosis in estimating the measurement uncertainty during calibration of the electrical resistance measures using a potentiometer

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**Abstract**

Calibration of electrical resistance measures is considered by the indirect method, which is realized through measuring the voltage drop across the series-connected reference and calibrated resistors. The biases of the estimates of the measurand and the combined standard uncertainty due to the nonlinearity of the measurement model were calculated. The distribution laws of the input quantities in the calculation of the expanded uncertainty were taken into account by the kurtosis method. An example of measurement uncertainty evaluating during calibration of single-size electrical resistance measure R331 with a nominal resistance of 1000 Ω by comparing its value using a potentiometer R345 with the value of the calibrated reference standard is considered. Estimates of the measurand and its standard and expanded uncertainties obtained using the proposed method showed good agreement with the estimates obtained using the Monte Carlo method.

**Keywords:** electrical resistance measure; calibration; potentiometer; measurement uncertainty; kurtosis method.

Received: 12.05.2021 Edited: 09.06.2021 Approved for publication: 14.06.2021

## Introduction

Various measurement methods are applicable to calibrate electrical resistance measures (ERM): direct measurement using a digital ohmmeter, measurement using a DC bridge, zero method using a resistance comparator, indirect method through measuring voltage drops across the reference and calibrated resistors [1]. Indirect resistance measurement is realized through the measurement of the ratio of voltage drop across the measured and reference resistors. The calibrated and reference ERM are connected in series in the current circuit, which must be stable during measurements, and the voltage drop across them is measured using a potentiometer. The article [2] gives an example of the estimation of the measurement uncertainty when calibrating a single-size ERM R331 with a nominal resistance of 1000 Ω using a calibrated single-size ERM with a real value and expanded uncertainty specified in its calibration certificate. In this case, the contributions of uncertainty due to the instability of the reference resistance during the time elapsed since the previous calibration and the change in the ambient temperature were not taken into account, and the traditional calculation method was used [3], which does not take into account the nonlinearity of the measurement model and approximately takes into account the distribution laws of input quantities at estimation of expanded uncertainty. The authors of the article have developed a number of approaches [4], allowing to take into account the indicated disadvantages. This article is devoted to the application of these approaches to the estimation of measurement uncertainty when calibrating resistance measures using a potentiometer.

## 1. Development of measurement model

Indirect resistance measurement is realized through the measurement of the ratio of voltage drop across the measured and reference resistors. The calibrated $R_c$ and reference $R_s$ ERM are connected in series in the current circuit, which must be stable during measurements, and the voltage drop across them $V_c$ and $V_s$ is measured using a voltage comparator or potentiometer.

The measurement equation in this case has the form:

$$R_C = R_S \frac{V_C}{V_S}. \tag{1}$$

The value of the resistance of the reference ERM is determined with corrections for the instability of the resistance of the reference ERM for the time elapsed since its previous calibration ($\Delta_s$) and the deviation of the ambient temperature during calibration $\Delta_t$.

In this case, the measurement model will take the form:

$$R_C = (R_S + \Delta_s + \alpha R_{\text{nom}} \Delta_t) \frac{V_c}{V_s}, \quad (2)$$

if $\alpha$ =20×10$^{-6}$ 1/°C – temperature coefficient of manganin resistance; $R_{\text{nom}}$=1000 Ω – nominal resistance of ERM.

**2. Calculation of the numerical value of the measurand**

In the first approximation, the calculation of the numerical value of the measurand is carried out according to the formula:

$$\hat{R}_C = (\hat{R}_S + \hat{\Delta}_s + \alpha R_{\text{nom}} \hat{\Delta}_t) \frac{\hat{V}_C}{\hat{V}_S}, \quad (3)$$

in which the values of the input quantities are replaced by their numeric values, marked with caps. In the example [2] value $\hat{R}_S$ made up 1000.006 Ω; value $\hat{V}_C$ and $\hat{V}_S$ made up 1 V and 1.000005 V, respectively; and the average value of all corrections: $\hat{\Delta}_S = \hat{\Delta}_t = 0$. In this case, we get $\hat{R}_C =$ 1000.001 Ω.

In general, the offset $\Delta_y$ numerical value measured value $Y = f(X_1, X_2, ..., X_N)$ can be estimated, taking into account the partial derivatives of the second order of the measured quantity with respect to the corresponding input $c(x_i)_2$ [4]:

$$\Delta_y = -\frac{1}{2}\sum_{i=1}^{N} c(x_i)_2 u^2(x_i), \quad (4)$$

if $u(x_i)$, $i$=1, 2,...,$N$ – standard uncertainties of input quantities $X_1$, $X_2$,..., $X_N$.

For expression (2), the calculation of the offset for $\hat{R}_S$ according to the formula:

$$\Delta(R_C) = -\frac{1}{2}[c(R_S)_2 u^2(R_S) + c(\Delta_S)_2 u^2(\Delta_S) + \\ + c(\Delta_t)_2 u^2(\Delta_t) + c(V_C)_2 u^2(V_C) + c(V_S)_2 u^2(V_S)]. \quad (5)$$

All quotients $c(x_i)_2$ in expression (5) are equal to zero, except for $c(V_S)_2 = 2R_S V_C / V_S^2$ =2 000.002 A$^{-1}$.

For the standard uncertainties of the input quantities given in [2]: $u(R_S)$ = 0.005 Ω; $u(\Delta_S)$=0.01155 Ω; $u(V_C)$ =5.8 μV; $u(V_S)$=5.8 μV; $u(\Delta_t)$=0.58 °C, the offset value of the numerical value of the measured value will be equal to:

$$\Delta(R_C) = -\frac{1}{2} c(V_S)_2 \cdot u^2(V_S) = -R_c \left[\frac{u(V_s)}{V_s}\right]^2 = \\ = -1000.001 \left[\frac{5.8 \cdot 10^{-6}}{1}\right]^2 = 5.8 \cdot 10^{-9}\,\Omega.$$

The resulting offset value $\Delta(R_c)$ can be taken into account as a correction to $\hat{R}_C$, obtaining an unbiased estimate of the measured value using the formula:

$$R_{\text{co}} = R_c - \Delta(R_C) = 1000.001 - 5.8 \cdot 10^{-9} = \\ = 1000.0009999942\,\Omega, \quad (6)$$

however, due to the negligible value of the correction, it can be neglected.

**3. Calculation of the standard uncertainty of the measured quantity**

In the first approximation, the standard uncertainty of the measured quantity is calculated based on the expression:

$$u(R_c) = \sqrt{\begin{aligned} & c^2(R_S)u^2(R_S) + c^2(\Delta_S)u^2(\Delta_S) + \\ & + c^2(V_C)u^2(V_C) + c^2(V_S)u^2(V_S) \end{aligned}}, \quad (7)$$

in which $u(x_i)$ and $c(x_i)$ – standard uncertainty of the input quantity and the partial derivative of the measured quantity with respect to this input quantity (sensitivity factor), respectively. Expressions for $c(x_i)$ and their values are given in uncertainty budgets (Table 1, 2).

For the standard uncertainties of the input quantities given in [2], we have $u(R_c)$=0.0189 Ω.

Formula (7) gives an unbiased estimate of the standard uncertainty of the measured quantity only in the absence of uncertainties in the input quantities.

The bias of the estimated variance of the measured value is calculated by the formula [4]:

$$\Delta(u^2) = \frac{1}{4}\sum_{i=1}^{N} c^2(x_i)_2 \cdot [\eta(x_i) + 2] \cdot u^4(x_i) + \\ + \sum_{i=2}^{N}\sum_{j=1}^{i-1} c^2(x_i, x_j) u^2(x_i) u^2(x_j), \quad (8)$$

if $\eta(x_i)$ – excess of distribution of the $j$-th input quantity, $c(x_i, x_j)$ – mixed partial derivative of the second order of the measured quantity with respect to the $i$-th and $j$-th input quantities, which is estimated for the known values of the input quantities.

Calculating the offset for $u(R_c)$ is produced (taking into account the zero values of the second derivatives) by the formula:

$$\Delta[u^2(R_C)] = \frac{1}{4}[c(V_S)_2 u^2(V_S)]^2[\eta(V_S) + 2] + [c(R_S, V_S)u(R_S)u(V_S)]^2 + [c(R_S, V_C)u(R_S)u(V_C)]^2 + \\ + [c(\Delta_S, V_C)u(\Delta_S)u(V_C)]^2 + [c(\Delta_S, V_S)u(\Delta_S)u(V_C)]^2 + \\ + [c(\Delta_t, V_C)u(\Delta_t)u(V_C)]^2 + [c(\Delta_t, V_S)u(\Delta_t)u(V_S)]^2 + [c(V_C, V_S)u(V_C)u(V_S)]^2, \quad (9)$$

where in $c(x_i, x_j)$ – mixed partial derivatives of the second order of the measured quantity with respect to the corresponding input quantities; $\eta(x_i)$ – excess of input quantities.

The nonzero values of the second order mixed partial derivatives are:

$$c(R_S,V_C)=c(\Delta_S,V_C)=c(\Delta_t,V_C)=1/\hat{V}_S=1\ \mathrm{V}^{-1};$$

$$c(R_S,V_S)=c(\Delta_S,V_S)=c(\Delta_t,V_S)=V_C/\hat{V}_S^2=1.000005\mathrm{V}^{-1};$$

$$c(V_C,V_S)=-\hat{R}_S/\hat{V}_S=-1000.6\ \Omega/\mathrm{V}.$$

For the above estimates of input quantities and their standard uncertainties, as well as for excesses corresponding to the laws of distribution of input quantities adopted in [1] $\eta(R_S)=0$; $\eta(\Delta_s)=\eta(\Delta_t)=\eta(V_S)=\eta(V_C)=-1.2$, we have $\Delta[u^2(R_C)]=2.2\times10^{-11}\ \Omega^2$.

The unbiased estimate of the total standard uncertainty is found by the formula:

$$u_0(R_C)=\sqrt{u^2(R_C)+\Delta[u^2(R_C)]}. \tag{10}$$

For the above values $u^2(R_C)$ and $\Delta[u^2(R_C)]$, we have $u_0(R_C)=0.01892348$, which indicates that the bias of the standard uncertainty of the measured quantity is insignificant.

**4. Computing expanded uncertainty**

Since there is no displacement of the measured quantity, this indicates that the nonlinearity of the model does not introduce additional asymmetry into the distribution law of the measured quantity, therefore, to calculate the expanded uncertainty, the method of kurtosis proposed by the authors can be applied [4].

In this case, the kurtosis of the measured value is calculated by the formula:

$$\eta(R_C)=\frac{1}{u^4(R_C)}[\eta(R_S)c^4(R_S)u^4(R_S)+\eta(\Delta_S)c^4(\Delta_S)u^4(\Delta_S)+$$
$$+\eta(V_C)c^4(V_C)u^4(V_C)+(V_S)c^4(V_S)u^4(V_S)]. \tag{11}$$

For the above estimates of the input quantities, and standard uncertainties, and their excesses, we have $\eta(R_C)=-0.35$.

The coverage factor value for a probability of 0.95 is calculated by the formula [4]:

$$k_{0.95}=\begin{cases}0.1085\eta^3+0.1\eta+1.96, \text{ at } \eta<0;\\ 1.96, \text{ at } \eta\geq 0.\end{cases} \tag{12}$$

Since $\eta(R_C)=-0.35$, calculate $k_{0.95}=1.88$.

In this case, the expanded uncertainty will be

$$U(R_C)=k_{0.95}u(R_C)=0.0355\,\Omega. \tag{13}$$

**5. Uncertainty budget**

The budgets of the measurement uncertainty, estimated by the method of excess when calibrating the ERM using a potentiometer, are given in Table 1, 2.

Table 1

Measurement uncertainty budget for indirect resistance calibration

| Input value | Assessment input value | Standard uncertainty | Excess input value | Sensitivity factor | Uncertainty contribution |
|---|---|---|---|---|---|
| | $\hat{R}_S$ | $u(R_S)$ | $\eta(R_S)$ | $\frac{V_C}{V_S}$ | $u(R_S)\frac{V_C}{V_S}$ |
| $\Delta_S$ | 0 | $u(\Delta_S)$ | $\eta(\Delta_S)$ | $\frac{V_C}{V_S}$ | $u(\Delta_S)\frac{V_C}{V_S}$ |
| $\Delta_t$ | 0 | $u(\Delta_t)$ | $\eta(\Delta_t)$ | $\frac{\alpha R_{\mathrm{nom}}V_C}{V_S}$ | $u(\Delta_t)\frac{\alpha R_{\mathrm{nom}}V_C}{V_S}$ |
| $V_C$ | $\hat{V}_C$ | $u(V_C)$ | $\eta(V_C)$ | $\frac{R_S}{V_S}$ | $u(V_C)\frac{R_s}{V_s}$ |
| $V_S$ | $\hat{V}_S$ | $u(V_S)$ | $\eta(V_S)$ | $-\frac{R_S V_C}{V_S^2}$ | $-u(V_S)\frac{R_S V_C}{V_S^2}$ |
| Output value | Measurement result | Summary standard uncertainty | Kurtosis of measurand | Sensitivity factor | Expanded uncertainty |
| $R_C$ | Formula(3) | Formula(7) | Formula(11) | Formula(12) | Formula(13) |

Table 2

Uncertainty budget for calibrating ERM 1000 Ω using a voltage potentiometer

| Input value | Assessment input value | Standard uncertainty | Excess input value | Sensitivity factor | Uncertainty contribution, Ω |
|---|---|---|---|---|---|
| $R_S$ | 1000.006 Ω | 0.005 Ω | 0 | 1.000005 | 0.005 |
| $\Delta_S$ | 0 | 0.0116 Ω | -1.2 | 1.000005 | 0.011547 |
| $\Delta_t$ | 0 | 0.577 °C | -1.2 | 0.02 Ω/ °C | 0.01154 |
| $V_C$ | 1.000005 V | 5.77 μV | -1.2 | 1000.006 Ω/V | 0.00577 |
| $V_S$ | 1 V | 5.77 μV | -1.2 | −1000.001 Ω/V | -0.00577 |
| Output value | Measurement result | Summary standard uncertainty | Kurtosis of measurand | Sensitivity factor | Expanded uncertainty |
| $R_C$ | 1000.001 Ω | 0.0189 Ω | -0.35 | 1.92 | 0.036 Ω |

The expanded uncertainty was estimated by the Monte Carlo method [5], which showed good agreement with the estimate obtained by the kurtosis method: $R_C = 1000.001\,\Omega$, $u(R_c) = 0.019\,\Omega$, $U(R_C) = 0.038\,\Omega$, $k_{0.95} = 2$.

**Conclusions**

1. The paper presents model measurements of the ERM resistance using a voltage potentiometer, taking into account the instability of the reference resistance for the time elapsed since the previous calibration and the change in the ambient temperature.

2. The estimation of the displacement of the numerical value of the measured quantity and its standard uncertainty, taking into account the nonlinearity of the measurement model, has been carried out. The check showed that both biases are insignificant for the input uncertainties considered in the example.

3. The expanded uncertainty was estimated by the kurtosis method, which showed good agreement with the estimate obtained by the Monte Carlo method.

# Метод ексцесів при оцінюванні невизначеності вимірювань у процесі калібрування мір електричного опору за допомогою потенціометра

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**Анотація**

Проаналізовано існуючі методи вимірювань при калібруванні мір електричного опору: пряме вимірювання за допомогою цифрового омметра, вимірювання за допомогою моста постійного струму, нульовим методом за допомогою компаратора опорів, непрямим методом через вимірювання падіння напруги на еталонному резисторі та резисторі, який калібрується. Розглянуто калібрування мір електричного опору непрямим методом, реалізованим через вимірювання падінь напруги, за допомогою потенціометра, на послідовно з’єднаних у ланцюгу постійного струму, який повинен бути стабільним протягом проведення вимірювань, еталонному резисторі та резисторі, який калібрується. При складанні моделі вимірювань додатково враховувалися внески невизначеності, обумовлені нестабільністю опору еталонної міри опору за час, який минув з моменту попереднього калібрування, і зміною температури навколишнього середовища під час калібрування. Розраховувалися зміщення оцінок вимірюваної величини і сумарної стандартної невизначеності, обумовлені нелінійністю моделі вимірювань. Облік законів розподілу вхідних величин при обчисленні розширеної невизначеності здійснено методом ексцесів. Розглянуто приклад оцінювання невизначеності вимірювань під час калібрування однозначної міри електричного опору Р331 з номінальним опором

1000 Ом шляхом порівняння її значення за допомогою потенціометра Р345 зі значенням відкаліброваної еталонної міри. Виявлені домінуючі внески невизначеностей. Складено бюджет невизначеностей, який можна використовувати для автоматизації оцінювання невизначеностей вимірювань та здійснення їх менеджменту. Оцінки значення вимірюваної величини і її стандартної та розширеної невизначеностей, які були отримані за допомогою запропонованого методу, показали хороший збіг з оцінками, отриманими за допомогою методу Монте-Карло.

**Ключові слова:** міра електричного опору; калібрування; потенціометр; невизначеність вимірювань; метод ексцесів.

# Метод эксцессов при оценивании неопределенности измерений в процессе калибровки мер электрического сопротивления с помощью потенциометра

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**Аннотация**

Проанализированы существующие методы измерений при калибровке мер электрического сопротивления: прямое измерение с помощью цифрового омметра, измерение с помощью моста постоянного тока, нулевым методом с помощью компаратора сопротивлений, косвенным методом через измерения падения напряжения на эталонном и калибруемом резисторах. Рассмотрена калибровка мер электрического сопротивления косвенным методом, реализованным через измерения падений напряжения, с помощью потенциометра, на последовательно соединенных в цепи постоянного тока, который должен быть стабильным на протяжении проведения измерений, эталонном резисторе и резисторе, который калибруется. При составлении модели измерений дополнительно учитывались вклады неопределенности, которые обусловлены нестабильностью сопротивления эталонной меры сопротивления за время, прошедшее с момента предыдущей калибровки, и изменением температуры окружающей среды во время калибровки. Рассчитано смещение оценок измеряемой величины и суммарной стандартной неопределенности, обусловленное нелинейностью модели измерений. Учет законов распределения входных величин при вычислении расширенной неопределенности произведен методом эксцессов. Рассмотрен пример оценивания неопределенности измерений при калибровке однозначной меры электрического сопротивления Р331 с номинальным сопротивлением 1000 Ом путем сравнения ее значения с помощью потенциометра Р345 со значением откалиброванной эталонной меры. Обнаружены доминирующие вклады неопределенностей. Составлен бюджет неопределенности, который можно использовать для автоматизации оценивания неопределенности измерений и осуществления их менеджмента. Оценки значения измеряемой величины и ее стандартной и расширенной неопределенности, полученные с помощью предложенного метода, показали хорошее совпадение с оценками, которые были получены с помощью метода Монте-Карло.

**Ключевые слова:** мера электрического сопротивления; калибровка; потенциометр; неопределенность измерений; метод эксцессов.

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