UDC 681.518.3 DOI:

Informational uncertainty of measuring instruments

G. Manko, E. Titova

Ukrainian State University of Chemical Technology, Haharina Ave., 8, 49005, Dnipro, Ukraine
elenatittova@gmail.com

Abstract

Recently, there has been a revival of interest in the use of the information approach in the theory of measurements. Unlike the traditional approach, information theory does not evaluate error or uncertainty, but entropy and the amount of information. This article analyzes a number of recent publications that develop ideas for the information approach. The limitations and disadvantages of both the entropy approach and the concept of uncertainty are indicated. As a compromise solution, it is proposed to use a criterion based on the Bongard's uncertainty and useful information. The concept of information uncertainty is proposed, which is estimated by the amount of negative useful information, that is, misinformation introduced by the measuring instrument. Some methods for calculating information uncertainty are described.

The problems of using the uncertainty approach are noted. This approach does not imply the use of such a generalized characteristic of measuring instruments as an accuracy class. The article proposes an analogue of the accuracy class in the form of relative informational uncertainty, expressed as a percentage. This will make it possible to evaluate the quality of the measuring instrument by a single parameter, the calculation of which requires a minimum of computational operations.

Keywords: uncertainty; entropy; information criterion; distribution law; measuring instrument; accuracy class.

Received: 23.07.2021 Edited: 04.10.2021 Approved for publication: 11.10.2021

1. Introduction

As it is known, measurement is an information process. Therefore, it seems obvious that it is possible to use information characteristics to assess the quality of measuring instruments (MIs).

Attempts to use the informational approach in measurement theory have been made repeatedly. A number of works [1, 2, etc.] presented the basics of information theory of measurement (ITM). It was proposed to estimate the amount of information in the measurement by the difference between the unconditional and conditional Shannon entropy. In this case, the accuracy of the measurement result is determined not by the root-mean-square or limiting error, but depends on the shape of the error probability distribution law.

It is known that a complete description of a random variable is provided by the probability distribution of its values. The traditional measurement theory operates with only the first two statistical moments (mean, variance). ITM estimates the amount of information obtained in the measurement process. In this case, the distribution law of the measurand is fully taken into account. This is what ensures the worth of ITM.

The works above were felt to be of purely theoretical interest. ITM demanded to abandon habitual notions. The basic concepts of traditional measurement theory are "error" and "uncertainty". ITM estimates the amount of information received in the measurement process and the loss of information due to imperfection of the MI.

Nevertheless, after a while, interest in using the information approach resumed.

The paper [3] analyzes the process of measurement using the information theory, puts forward the essence of measurement process and builds an information model for the measurement process, which can explain why the interference and error are always existed in the result of measurement. The article [4] describes the fractal-entropy analysis of the results of measurements. To evaluate the results of measurements in dynamic systems, it is proposed to use an expression linking Shannon entropy with the fractal dimension of the time series of measurement results. Boris Menin [5] compares the features of the application of the theory of measurements and the measure of the similarity of the model to the phenomenon under study on the basis of calculating the amount of information contained in the physical phenomena and technological processes.

In a number of works, the information approach is used to estimate the measurement uncertainty.

Jae-Yoon Jung et al. [6] propose an entropy-based uncertainty measure which captures the dynamic behavior of processes. It enables experts to better understand the nature of processes at runtime. In the paper [7] the informational uncertainty is expressed as the difference of information before receiving a message and after receiving it. The relative change in the informational uncertainty of measurement systems can be used to calculate the required number of extra measurements. In [8], a new information approach is proposed, which makes possible to calculate the absolute minimum uncertainty in the measurement of the investigated quantity of the phenomenon. The entropy is used here as a function of the state of a thermodynamic system. So the validity of using the term "informational" is questionable.

Summarizing, we can conclude that all the works considered are based on the use of the entropy approach. A number of constraints and flaws of this approach should be noted.

Shannon's entropy can be determined for random processes only. To calculate the measurement information, it is necessary to know the conditional probability distributions. Calculations are quite time consuming. To date, no effective entropy estimates of measurement accuracy have been developed.

The purpose of this article is to describe the methods of using information criteria for assessing the quality of MIs and establishing their suitability for use.

2. Informational uncertainty of measurement

Soviet cyberneticist M.M. Bongard is known for his works on the pattern recognition problems. In his main book [9], he introduced the concept of "useful information", which turned out to be little-noticed. Nevertheless, it can be effectively used in measurement theory. Useful information can be either positive or negative. That is, the misinformation introduced by the imperfection of the MI can be measured.

According to [9], when the answer probability distribution is p, but an observer uses the hypothesis that the distribution is q, the problem uncertainty is:

$$N(p/q) = -\sum_{i=1}^{n} p_i \log q_i . \tag{1}$$

If a problem had an uncertainty N_0 prior to receiving message, and uncertainty N_1 after receipt of the message, then the message carries the following amount of useful information

$$I = N_0 - N_1 . \tag{2}$$

Shannon's entropy $H(p)$ is a particular case of uncertainty. Entropy is the uncertainty for an algorithm that knows and uses to best advantage the probability distribution of the problem answers. If this is not known, uncertainty increases.

Let's apply Bongard's developments to the measurement process. We will assume that p is the true distribution of the measured value, and q is the distribution of the measurement results. Then entropy is the uncertainty for MI, which gives the true value of the measurand. The amount of useful information that a measuring instrument provides is

$$I = H(p) - N(p/q). \tag{3}$$

As shown in [9], if the p_i are fixed, problem uncertainty is minimal when $q_i = p_i$. In this case $N(p/q) = N(p/p) = H(p)$, i.e., the problem uncertainty is equal to the entropy of the answer probabilities. As a result, the amount of useful information (3) is always negative, and we should talk about the misinformation introduced by the MI:

$$D = N(p/q) - H(p) = \sum_i p_i \log \frac{p_i}{q_i} . \tag{4}$$

Expression (4) defines the informational uncertainty (IU) of the measurement. IU takes zero value for ideal measurements (there is no uncertainty) and the maximum value in case of $H(p) = 0$. Dividing the IU by this maximum value, we obtain the relative informational uncertainty (RIU)

$$\nu = \frac{D}{N(p/q)} = 1 - \frac{\sum_i p_i \log p_i}{\sum_i p_i \log q_i} . \tag{5}$$

RIU (5) is a generalized criterion for the quality of MI which takes into account the shape of the curve of the measurand distribution. Thus, it is possible to assess how a particular device is suitable for measuring a given specific measurand.

It is recommended to obtain the Q-distribution by statistical processing of a sample of measurement results. To obtain the P-distribution, one should use the conventional true values obtained using control instrument. In an extreme case, one can accept the hypothesis about the standard distribution law of the true values of the measurand. As a standard distribution, one can take the normal, as the most common, or uniform, as suggesting the greatest a priori uncertainty. This method complies with the calculation of Type A uncertainty according to the GUM.

Calculation by Type B is possible using the following methods. If there is information about the form of the measurand probability distribution, then an estimate of the uncertainty can be obtained analytically. For this, expression (4) should be presented in integral form using the differential entropy.

If only interval estimates of the measurement results are known, then two situations can be considered: the value of the measured quantity is either covered by the confidence interval, or lies outside the confidence interval. The probability of the first situation is equal to the confidence level β, the probability of the second is equal to (1 −β,). If α is the a priori (before the interval estimate) confidence level, then IU can be calculated by the formula:

$$D = \beta \log \frac{\beta}{\alpha} + (1-\beta) \log \frac{1-\beta}{1-\alpha} . \tag{6}$$

3. Uncertainty and accuracy class

The concept of "accuracy class" (AC) was widely used to compare the MI accuracy in the "pre-reform era". The same basic accuracy of two measuring instruments does not mean that they have the same quality if their measurement ranges differ significantly. Uncertainty approach (UA) provides even fewer opportunities for comparative assessment of the accuracy of dissimilar instruments, since the uncertainty is estimated in units of measurement for measurand.

Today, many specifications of the MI quality and measuring methods are used. In addition to the concepts of "error" and "uncertainty", the "accuracy", "repeatability", "reproducibility", "trueness", and "precision" have been introduced into metrological practice. At the same time, AC served as a unified indirect assessment of the MI quality.

Ukrainian standards formulate the concept of AC as generalized characteristic of measuring instruments, determined by the limits of permissible basic and additional errors, as well as a number of other properties that affect the accuracy of measurements. With UA, this characteristic cannot be used. Therefore, attempts are being made to find a compromise in using the concepts of error and uncertainty.

The paper [10] assumes the establishment of an acceptable risk for wrong measurements. It is recommended that decision-making in conformity assessment when accounting for uncertainty in testing follows the shared risk principle under the condition that the uncertainty of the complete measurement system – MPU – is less than 1/3 of the maximum permissible error – MPE – for the actual conformity assessment.

It is noted in [11] that the notion of accuracy class has the value of universality and can also be for the assessment of measuring transducers at the instrument design stage. A method of using the instrument accuracy class for the estimation of the Type B uncertainty is proposed.

In recommendation [12] concepts "uncertainty" and "error" are recommended to be used harmoniously. It is offered the general rule: in the most metrological situations, measurement results are characterized by uncertainty and accuracy of measuring instruments are characterized by error limits. Concept "error" is used at comparison with reference quantity value, and evaluation errors are obtained at calibration or verification of measuring instruments.

Still, it would be more convenient to be able to evaluate the instrument's accuracy of the device within the UA by a parameter like an AC. We propose to use the relative informational uncertainty (5), expressed as a percentage, as such a parameter. Its maximum admissible value ν_{adm} must be indicated in the technical documentation for MI. When the MI's conformity assessment procedure is carried out, the value of the RIU should be experimentally determined and compared with ν_{adm}.

4. Discussion and conclusions

As rightly noted in [5], the practical application of the information approach can cause an uneven response in the scientific community. At the same time, the general principles of the information-oriented approach do not contradict the firmly established conclusions of modern physics, but, on the contrary, allow to consider them as an opportunity for purposefully increasing the accuracy of calculations and experiments.

The authors are aware that the introduction of information estimates of uncertainty in metrological practice will face a number of difficulties of the objective and subjective nature. In particular, changes will be required to many standards. However, it should be borne in mind that the overwhelming majority of regulatory documents do not take into account UA, and some even contradict it. Since, in any case, there is a lot of work to be done to update such documents, it would be advisable to use an information approach.

Uncertainty assessment using information criteria (4) and (5) has the following advantages.

1. Full consideration of the distribution laws of the measurands and measurement results increases the accuracy of uncertainty estimation. Traditional methods underestimate the accuracy of measurements.
2. Taking into account the shape of the curve of the distribution law of the measurand provides an opportunity to assess how a particular MI is suitable for measuring this particular value.
3. The proposed method does not require correction factors such as coverage factor, which significantly reduces the arbitrariness in the assessment of uncertainty.
4. Estimation of measurement uncertainty in accordance with the above method requires a minimum of computational operations.
5. Assessing the quality of measuring instruments by a single parameter is convenient from a practical point of view.

Інформаційна невизначеність засобів вимірювання

Г.І. Манко, О.В. Тітова

Український державний хіміко-технологічний університет, просп. Гагаріна, 8, 49005, Дніпро, Україна
elenatittova@gmail.com

Анотація

Останнім часом відроджується інтерес до використання інформаційного підходу в теорії вимірювань. На відміну від традиційного підходу, інформаційна теорія оцінює не похибку або невизначеність, а ентропію і кількість інформації. Перевагою інформаційного підходу є врахування форми кривої розподілу ймовірностей вимірюваної величини, в той час як традиційна теорія вимірювань оперує тільки двома першими статистичними моментами, що достатньо тільки у випадку нормального розподілу. На практиці закон розподілу вимірювальної величини часто суттєво відрізняється від нормального.

Аналіз наукових публікацій показав, що на сьогодні усі розробки стосовно інформаційної теорії вимірювань базуються винятково на використанні поняття ентропії як міри невизначеності вимірювальної величини. У статті аналізуються обмеження і недоліки як ентропійного підходу, так і загальноприйнятої концепції невизначеності. Пропонується компромісне рішення – використання інформаційного критерію на основі поняття невизначеності Бонгарда. Як міра інформаційної невизначеності використовується кількість дезінформації, яка вноситься неточністю вимірювань. Для оцінки якості засобів вимірювань слід використовувати відносну інформаційну невизначеність, виражену у відсотках. Описані методи розрахунку інформаційної невизначеності.

Відзначаються проблеми використання концепції невизначеності, яка не передбачає використання такої узагальненої характеристики засобів вимірювання, як клас точності. У статті пропонується аналог класу точності у вигляді критерію відносної інформаційної невизначеності, вираженої у відсотках. Це дозволить оцінювати якість вимірювального приладу і придатність його до використання єдиним параметром. Наведені переваги такого критерію, головними з яких є врахування форми закону розподілу і мінімальний обсяг обчислювальних операцій.

Ключові слова: невизначеність; ентропія; інформаційний критерій; закон розподілу; засіб вимірювання; клас точності.

Информационная неопределенность средств измерений

Г.И. Манко, Е.В. Титова

Украинский государственный химико-технологический университет, просп. Гагарина, 8, 49005, Днепр, Украина
elenatittova@gmail.com

Аннотация

В последнее время возрождается интерес к использованию информационного подхода в теории измерений. В отличие от традиционного подхода, информационная теория оценивает не погрешность или неопределенность, а энтропию и количество информации. В статье анализируются ограничения и недостатки как энтропийного подхода, так и концепции неопределенности. В качестве компромиссного решения предлагается использование информационного критерия на основе понятия неопределенности Бонгарда.

Отмечаются проблемы использования концепции неопределенности, которая не предполагает использования такой обобщенной характеристики средств измерения, как класс точности. В статье предлагается аналог класса точности в виде относительной информационной неопределенности, выраженной в процентах. Это позволит оценивать качество измерительного прибора единственным параметром, расчет которого требует минимума вычислительных операций.

Ключевые слова: неопределенность; энтропия; информационный критерий; закон распределения; средство измерения; класс точности.

References

1. Kavalerov G.I. and Mandelstam S.M. Vvedenye v ynformatsyonnuiu teoryiu yzmerenyi [Introduction to the information theory of measurements]. Moscow, Energy, 1974. 375 p. (in Russian).
2. Ornatsky P.P. Teoretycheskye osnovy ynformatsyonno-yzmerytelnoi tekhnyky [Theoretical basis of information and measurement technology]. Kyiv, High school, 1983. 455 p. (in Russian).
3. Shuangbao Ma and Yong Xue. The philosophical thoughts of measurement information theory and its applications. EDT, Shenzhen, China, 2010, pp. 473–475.
4. Machekhin Y.P., Kurskoy Y.S. Fractal-Entropy Analysis of the Results of Measurements in Nonlinear Dynamical Systems. *Measurement Techniques*, 2014, vol. 57, issue 6, pp. 609–614.
5. Menin B. Applying Measurement Theory and Information-based Measure in Modeling Physical Phenomena and Technological Processes. *European Journal of Engineering Research and Science*, 2018, vol. 3, no. 1, pp. 28–34. doi:https://doi.org/10.24018/ejers.2018.3.1.594
6. Jae-Yoon Jung, Chang-Ho Chin, Jorge Cardoso. An entropy-based uncertainty measure of process models. *Information Processing Letters*, 2011, vol. 111, issue 3, pp. 135–141.
7. Rybokas M. et al. The determination of measurement accuracy using information entropy. *Journal of Measurements in Engineering*, 2013, vol. 1, issue 1, pp. 59–62.
8. Menin B. h, k, N_A: Evaluating the Relative Uncertainty of Measurement. *American Journal of Computational and Applied Mathematics*, 2018, vol. 8(5), pp. 93–102. doi:10.5923/j.ajcam.20180805.02
9. Bongard M.M. Pattern Recognition. Rochelle Park, N.J.: Hayden Book Co., Spartan Books, 1970. 253 p.
10. WELMEC 4.2. Issue 1. Elements for deciding the appropriate level of confidence in regulated measurements. (Accuracy classes, MPE in-service, nonconformity, principles of uncertainty). WELMEC Secretariat, Vienna, 2006. 12 p.
11. Tokarska M., Gniotek K. The measuring instrument accuracy class as a measure of the Type B uncertainty. *Measurement Automation Monitoring*, Feb. 2015, vol. 61, no. 02, pp. 28–31.
12. COOMET R/GM/21:2011. Use of concepts “error of measurement” and “uncertainty of measurement”: General principles. 2011. 10 p. Available at: http://www.coomet.org/EN/doc/r21_2011.pdf