UDC 528.06 DOI:

The measurement uncertainty of air object spatial coordinates by rho-theta fixing

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Abstract

The features of measurement uncertainty evaluation of the coordinates of an air object by the rho-theta fixing are discussed. Measurement models are presented that link its coordinates in the local rectangular coordinate system with the spherical coordinates of air object, found using a rangefinder and a goniometer. The models include a correction for determining the location of the base station, a correction for determining the angle of elevation due to inaccuracies in the leveling of the station platform and azimuth, and a correction related to the inaccuracy of the station's reference to the north. The measurement uncertainty budgets of rectangular coordinates which can be a basis for creation of software for automation of calculation of measurement uncertainties are resulted. Estimates of expanded uncertainties are found by the method of kurtosis. Expressions for the relative standard uncertainties of coordinate measurements are written and an example of their estimation for real data is given.

Keywords: coordinate measurement; rho-theta fixing; measurement uncertainty, kurtosis method.

Received: 10.01.2022 Edited: 22.02.2022 Approved for publication: 28.03.2022

Introduction

The problem of an air object (AO) spatial coordinates determining is widely used in geodesy, radio navigation, radio-, optical and acoustic locations [1–4]. Depending on the number of reference stations used in this case (radar, optoelectronic or acoustic) and their capabilities, this problem is solved by various methods. The rho-theta fixing (positional method) [5], considered in this article, is limited to the use of only one reference station (RS) containing a rangefinder and a goniometer (direction finder). It belongs to the active methods of location, since it requires radiation from the RS to determine the distance to the object.

Fig. 1 shows the local coordinate system, in which the *OY* axis is directed along a plumb line to the earth's surface; the *OX* axis is located in a plane perpendicular to the plumb line at point *O* and makes an angle with the meridian plane of the origin of the coordinate system, defined as the geodetic azimuth, counted in the clockwise direction from the north direction; the *OZ* axis is drawn so that a right-handed rectangular coordinate system is formed. From the location of the station *O*, the rangefinder determines the slant range *OP* = ρ, and the direction finder sets the direction to the target, i.e. its azimuth α and elevation angle β (the angle between the direction to the target and its projection onto the *XOZ* plane).

Through these parameters of the spherical coordinate system, the local Cartesian coordinates (x, y, z) of the object are determined in accordance with the equations [6]:

$$x = \rho\cos(\beta + \delta_H)\cos(\alpha + \delta_N) + \delta_x, \quad (1)$$

$$y = \rho\sin(\beta + \delta_H) + \delta_y, \quad (2)$$

$$z = \rho\cos(\beta + \delta_H)\sin(\alpha + \delta_N) + \delta_z, \quad (3)$$

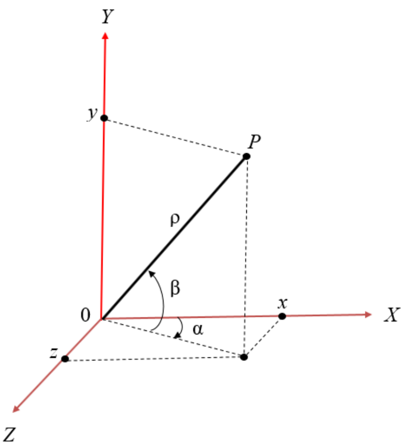

Fig. 1. Relationship between Cartesian and spherical coordinates of the object

in which δ_x, δ_y, δ_z are the corrections for determining the location of point O; δ_H is the correction for determining the elevation angle associated with the inaccuracy of leveling the RS platform; δ_N is the correction for determining the azimuth, associated with the inaccuracy of referencing the RS to the direction to the north.

Basic part

If the mathematical expectations of all corrections are taken equal to zero, then the values of the measurands $\hat{x}, \hat{y}, \hat{z}$ can be determined from expressions (1) – (3) by substituting in them the values of the input quantities $\hat{\rho}$, $\hat{\alpha}$, $\hat{\beta}$:

$$\hat{x} = \hat{\rho}\cos(\hat{\beta})\cos(\hat{\alpha}); \quad (4)$$

$$\hat{y} = \hat{\rho}\sin(\hat{\beta}); \quad (5)$$

$$\hat{z} = \hat{\rho}\cos(\hat{\beta})\sin(\hat{\alpha}). \quad (6)$$

The expression for the standard measurement uncertainty of the coordinate x can be written in accordance with the rule of variances summation [7] as:

$$u(x) = \sqrt{\begin{array}{l}[c_\rho(x)u(\rho)]^2 + c_\beta^2(x)\left[u^2(\beta) + u^2(\delta_H)\right] + \\ + c_\alpha^2(x)[u^2(\alpha) + u^2(\delta_N)] + u^2(\delta_x)\end{array}}, \quad (7)$$

where the corresponding sensitivity coefficients $c_\rho(x)$, $c_\beta(x)$ and $c_\alpha(x)$, are determined by the expressions:

$$c_\rho(x) = \frac{\partial x}{\partial \rho} = \cos\beta\cos\alpha; \quad (8)$$

$$c_\beta(x) = \frac{\partial x}{\partial \beta} = -\rho\sin\beta\cos\alpha; \quad (9)$$

$$c_\alpha(x) = \frac{\partial x}{\partial \alpha} = -\rho\cos\beta\sin\alpha. \quad (10)$$

Similarly, we can write an expression for the standard measurement uncertainty of the coordinate y:

$$u(y) = \sqrt{[c_\rho(y)u(\rho)]^2 + c_\beta^2(y)\left[u^2(\beta) + u^2(\delta_H)\right] + \tilde{u}^2(\delta_y)}, \quad (11)$$

in which the corresponding sensitivity coefficients $c_\rho(y)$, $c_\beta(y)$ are found as:

$$c_\rho(y) = \frac{\partial y}{\partial \rho} = \sin\beta; \quad (12)$$

$$c_\beta(y) = \frac{\partial y}{\partial \beta} = \rho\cos\beta, \quad (13)$$

as well as the expression for the standard measurement uncertainty of the coordinate z:

$$u(z) = \sqrt{\begin{array}{l}[c_\rho(z)u(\rho)]^2 + c_\beta^2(z)\left[u^2(\beta) + u^2(\delta_H)\right] + \\ + c_\alpha^2(z)[u^2(\alpha) + u^2(\delta_N)] + \tilde{u}^2(\delta_z)\end{array}}, \quad (14)$$

where the corresponding sensitivity coefficients $c_\rho(z)$, $c_\beta(z)$ and $c_\alpha(z)$ have the form:

$$c_\rho(z) = \frac{\partial z}{\partial \rho} = \cos\beta\sin\alpha; \quad (15)$$

$$c_\beta(z) = \frac{\partial z}{\partial \beta} = -\rho\sin\beta\sin\alpha; \quad (16)$$

$$c_\alpha(z) = \frac{\partial z}{\partial \alpha} = \rho\cos\beta\cos\alpha. \quad (17)$$

The standard measurement uncertainty of the target slant range ρ is determined from the corresponding boundaries $\pm\theta_\rho$ of the maximum permissible error (MPE), assuming a uniform distribution of the measurement error ρ within these boundaries:

$$u(\rho) = \theta_\rho / \sqrt{3}. \quad (18)$$

The standard uncertainties of the corrections $u(\delta_x)$; $u(\delta_y)$; $u(\delta_z)$ for determining the location of the RS standing point are determined from the corresponding boundaries of the MPE $\pm\theta(\delta_x)$; $\pm\theta(\delta_y)$; $\pm\theta(\delta_z)$, assuming a uniform distribution of corrections δ_x, δ_y, δ_z within these boundaries:

$$u(\delta_x) = \theta(\delta_x) / \sqrt{3}; \quad (19)$$

$$u(\delta_y) = \theta(\delta_y) / \sqrt{3}; \quad (20)$$

$$u(\delta_z) = \theta(\delta_z) / \sqrt{3}. \quad (21)$$

In expressions (7), (11), (14), $u(\alpha)$ and $u(\beta)$ are the instrumental standard uncertainties of measuring the angular coordinates α and β with a goniometer, which can be found through the boundaries of the MPE instrumental errors in measuring the azimuth $\pm\theta_\alpha$ and elevation angle $\pm\theta_\beta$ and, respectively, assuming a uniform distribution of instrumental errors inside these boundaries as:

$$u(\alpha) = \theta_\alpha / \sqrt{3}; \quad (22)$$

$$u(\beta) = \theta_\beta / \sqrt{3}. \quad (23)$$

If the boundaries of the MPE of referencing to the north direction are taken equal to $\pm\theta_N$, then, assuming a uniform distribution of the referencing error within these boundaries, we can write:

$$u_N(\alpha) = \theta_N / \sqrt{3}. \quad (24)$$

If the boundaries of the leveling error MPE of along each of the axes are taken equal to $\pm\theta_H$, then, assuming a uniform distribution of the leveling error within these boundaries, we can write (Fig. 2):

$$u_H(\beta) = \theta_H / \sqrt{3}. \quad (25)$$

Since the standard uncertainties of all input quantities were determined according to type B and a uniform distribution law was assigned to them, the expanded uncertainties of measurement coordinate (x, y, z) are best searched for by the kurtosis method [8]:

$$U(x) = k(\eta_x)u(x); \tag{26}$$

$$U(y) = k(\eta_y)u(y); \tag{27}$$

$$U(z) = k(\eta_z)u(z), \tag{28}$$

where the coverage factors for a confidence level of 0.95 are found by the formula:

$$k_{0.95} = 0.1085\eta^3 + 0.1\eta + 1.96 \tag{29}$$

and the kurtosis of the distribution for (x, y, z) will be equal to:

$$\eta(x) = \frac{-1.2\left\{[c_\rho(x)u(\rho)]^4 + c_\beta^4(x)\left[u^4(\beta) + u^4(\delta_H)\right] + c_\alpha^4(x)[u^4(\alpha) + u^4(\delta_N)] + u^4(\delta_x)\right\}}{u_c^4(x)}; \tag{30}$$

$$\eta(y) = \frac{-1.2\left\{[c_\rho(y)u(\rho)]^4 + c_\beta^4(y)\left[u^4(\beta) + u^4(\delta_H)\right] + u^4(\delta_y)\right\}}{u_c^4(y)}; \tag{31}$$

$$\eta(z) = \frac{-1.2\left\{[c_\rho(z)u(\rho)]^4 + c_\beta^4(z)\left[u^4(\beta) + u^4(\delta_H)\right] + c_\alpha^4(z)[u^4(\alpha) + u^4(\delta_N)] + u^4(\delta_z)\right\}}{u_c^4(z)}. \tag{32}$$

Formulas (30) – (32) take into account that the kurtosis of all input quantities having a uniform distribution is equal to -1.2.

The uncertainty budgets of coordinates (x, y, z) measurements of the object will have the form given in Tables 1–3.

Table 1

Uncertainty budget for the measurement of the x-coordinate

Input quantities	Values of input quantities	Standard uncertainties of input quantities	Kurtosis of input quantities	Sensitivity coefficients	Uncertainty contributions
ρ	$\hat{\rho}$	$u(\rho)$ (18)	-1.2	$c_\rho(x)$ (8)	$c_\rho(x)u(\rho)$
α	$\hat{\alpha}$	$u(\alpha)$ (22)	-1.2	$c_\alpha(x)$ (9)	$c_\alpha(x)u(\alpha)$
β	$\hat{\beta}$	$u(\beta)$ (23)	-1.2	$c_\beta(x)$ (10)	$c_\beta(x)u(\beta)$
δ_H	0	$u(\delta_H)$ (25)	-1.2	$c_\beta(x)$ (10)	$c_\beta(x)u(\delta_H)$
δ_N	0	$u(\delta_N)$ (24)	-1.2	$c_\alpha(x)$ (9)	$c_\alpha(x)u(\delta_N)$
δ_x	0	$u(\delta_x)$ (19)	-1.2	1	$u(\delta_x)$
Measurand	Measurand Value	Combined standard uncertainty	Measurand kurtosis	Coverage factor	Expanded uncertainty
x	$\hat{x}$ (4)	$u_c(x)$ (7)	$\eta(x)$ (30)	$k(\eta)$ (29)	U (26)

Table 2

Uncertainty budget for the measurement of the y-coordinate

Input quantities	Values of input quantities	Standard uncertainties of input quantities	Kurtosis of input quantities	Sensitivity coefficients	Uncertainty contributions
ρ	$\hat{\rho}$	$u(\rho)$ (18)	-1.2	$c_\rho(y)$ (12)	$c_\rho(y)u(\rho)$
β	$\hat{\beta}$	$u(\beta)$ (23)	-1.2	$c_\beta(y)$ (13)	$c_\beta(y)u(\beta)$
δ_H	0	$u(\delta_H)$ (25)	-1.2	$c_\beta(y)$ (13)	$c_\beta(y)u(\delta_H)$
δ_y	0	$u(\delta_y)$ (20)	-1.2	1	$u(\delta_y)$
Measurand	Measurand Value	Combined standard uncertainty	Measurand kurtosis	Coverage factor	Expanded uncertainty
y	$\hat{y}$ (5)	$u_c(y)$ (11)	$\eta(y)$ (31)	$k(\eta)$ (29)	U (27)

Table 3

Uncertainty budget for the measurement of the z-coordinate

Input quantities	Values of input quantities	Standard uncertainties of input quantities	Kurtosis of input quantities	Sensitivity coefficients	Uncertainty contributions
ρ	$\hat{\rho}$	$u(\rho)$ (18)	-1.2	$c_\rho(z)$ (15)	$c_\rho(z)u(\rho)$
α	$\hat{\alpha}$	$u(\alpha)$ (22)	-1.2	$c_\alpha(z)$ (17)	$c_\alpha(z)u(\alpha)$
β	$\hat{\beta}$	$u(\beta)$ (23)	-1.2	$c_\beta(z)$ (16)	$c_\beta(z)u(\beta)$
δ_H	0	$u(\delta_H)$ (25)	-1.2	$c_\beta(z)$ (16)	$c_\beta(z)u(\delta_H)$
δ_N	0	$u(\delta_N)$ (24)	-1.2	$c_\alpha(z)$ (17)	$c_\alpha(z)u(\delta_N)$
δ_z	0	$u(\delta_z)$ (21)	-1.2	1	$u(\delta_z)$
Measurand	Measurand Value	Combined standard uncertainty	Measurand kurtosis	Coverage factor	Expanded uncertainty
z	$\hat{z}$ (6)	$u_c(z)$ (14)	$\eta(z)$ (32)	$k(\eta)$ (29)	U (28)

From expressions (7), (11), (14), it is possible to write expressions for relative standard uncertainties of coordinates (x, y, z) measurement:

$$\tilde{u}(x)=\frac{u(x)}{\hat{\rho}}=\sqrt{[\tilde{u}(\rho)\cos\beta\cos\alpha]^2+[u(\beta)\sin\beta\cos\alpha]^2+[u(\alpha)\cos\beta\sin\alpha]^2+\tilde{u}^2(\delta_x)}, \quad (33)$$

$$\tilde{u}(y)=\frac{u(y)}{\hat{\rho}}=\sqrt{[\tilde{u}(\rho)\sin\beta]^2+[u(\beta)\cos\beta]^2+\tilde{u}^2(\delta_y)}, \quad (34)$$

$$\tilde{u}(z)=\frac{u(z)}{\hat{\rho}}=\sqrt{[\tilde{u}(\rho)\cos\beta\sin\alpha]^2+[u(\beta)\sin\beta\sin\alpha]^2+[u(\alpha)\cos\beta\cos\alpha]^2+\tilde{u}^2(\delta_z)}, \quad (35)$$

where $\tilde{u}(\delta_x)=u(\delta_x)/\hat{\rho}$; $\tilde{u}(\delta_y)=u(\delta_y)/\hat{\rho}$; $\tilde{u}(\delta_z)=u(\delta_z)/\hat{\rho}$ are the relative standard uncertainties of the corrections δ_x, δ_y, δ_z; $\tilde{u}(\rho)=u(\rho)/\rho$ is the relative standard uncertainty of measurement ρ.

For values $\hat{\rho}$ =1000 m; θ_ρ=1.5 m; $\theta(\delta_x)=\theta(\delta_y)=\theta(\delta_z)$ =1.8 m; θ_α =0.6 mrad; θ_β =0.4 mrad; θ_N = 0.1 mrad; θ_H =6 mrad given in [9], the dependences of $\tilde{u}(x)$, $\tilde{u}(y)$, $\tilde{u}(z)$ on α and β are obtained, shown in Fig. 2.

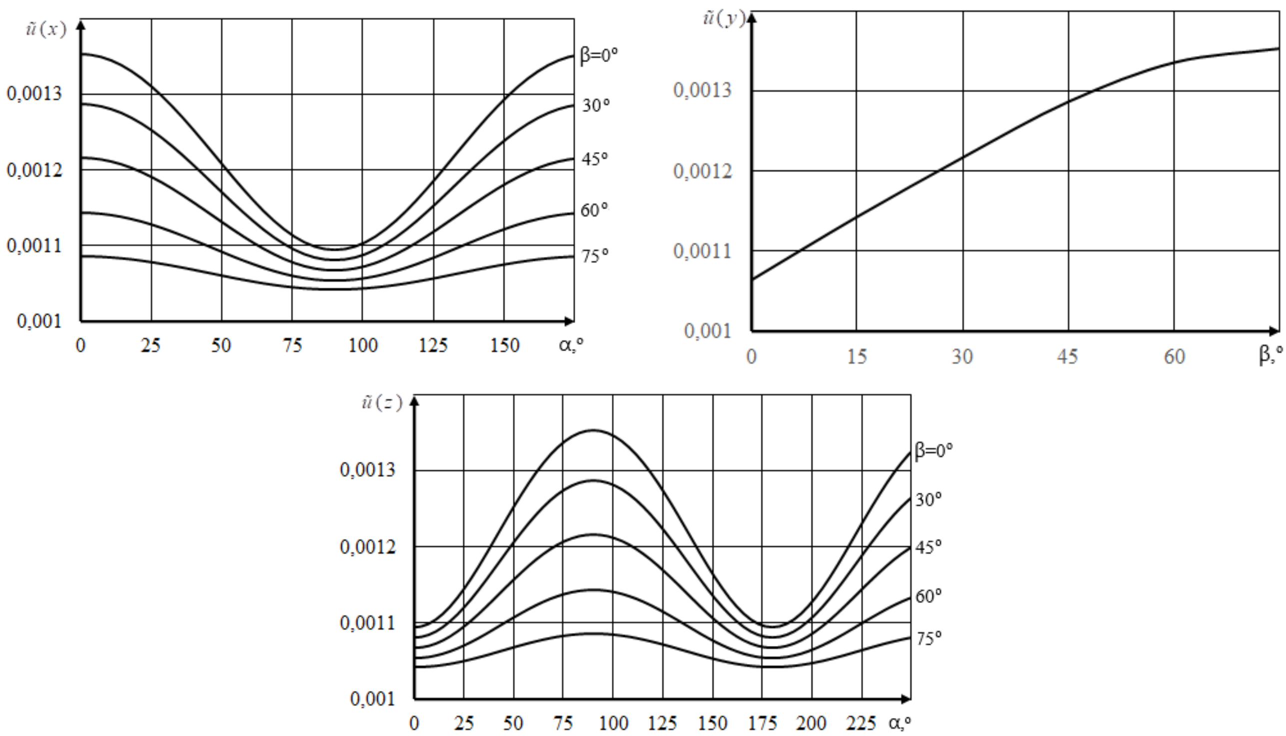

Fig. 2. The dependences of $\tilde{u}(x)$, $\tilde{u}(y)$ and $\tilde{u}(z)$ on α and β

Conclusions

1. In measurement models linking AO coordinates in the local Cartesian system with their spherical coordinates found using a rangefinder and a goniometer, it is necessary to include a correction for determining the location of point O, a correction for determining the elevation angle associated with the inaccuracy of leveling the RS platform, as well as a correction for determining azimuth associated with the inaccuracy of referencing the RS to the direction to the north.

2. The expressions for the standard uncertainties of measuring the rectangular coordinates of the AO in absolute and relative forms are obtained.

3. Since the standard uncertainties of all input quantities were determined according to type B and a uniform distribution law was assigned to them, then the expanded measurement uncertainties of the coordinates are best searched for by the kurtosis method.

4. Uncertainty budgets for measuring the AO rectangular coordinates are given, which can serve as a basis for automating the calculation of measurement uncertainty.

Невизначеність вимірювання просторових координат повітряного об’єкта далекомірно-кутомірним методом

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Анотація

Визначення просторових координат повітряного об’єкта широко використовується у геодезії, радіонавігації, радіо-, оптичній та акустичній локаціях. Залежно від кількості застосовуваних при цьому опорних станцій (радіолокаційних, оптико-електронних або акустичних) та їх можливостей, це завдання вирішується різними методами. У статті розглядаються особливості оцінювання невизначеності вимірювань координат повітряного об’єкта далекомірно-кутомірним способом. Наводяться моделі вимірювань, що зв’язують його координати у місцевій декартовій системі координат зі сферичними координатами повітряного об’єкта, знайденими за допомогою далекоміра та кутоміра. У моделях включено поправку на визначення розташування опорної станції, поправку на визначення кута місця, пов’язану з неточністю горизонтування платформи станції та визначення азимуту, а також поправку, пов’язану з неточністю прив’язки станції до напрямку на північ. Відповідно до правила підсумовування дисперсій записані вирази для стандартних невизначеностей вимірювання прямокутних координат об’єкта. Наводяться бюджети невизначеності вимірювань прямокутних координат, які можуть бути основою для створення програмних засобів для автоматизації розрахунку невизначеностей вимірювань. Методом ексцесів знаходяться оцінки розширених невизначеностей. Записані вирази для відносних стандартних невизначеностей вимірювань координат. Наводиться приклад оцінювання відносних стандартних невизначеностей вимірювань прямокутних координат повітряного об’єкта для реальних даних. Побудовані графічні залежності відносних стандартних невизначеностей вимірювань прямокутних координат повітряного об’єкта від азимуту та кута місця.

Ключові слова: координатні вимірювання; далекомірно-кутомірний метод; невизначеність вимірювань; метод ексцесів.

Неопределенность измерения пространственных координат воздушного объекта дальномерно-угломерным методом

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Аннотация

В статье рассматриваются особенности оценивания неопределенности измерения координат воздушного объекта (ВО) дальномерно-угломерным способом. Приводятся модели измерений, связывающие его координаты в местной декартовой системе координат со сферическими координатами цели, найденными с помощью дальномера и угломера. Приводятся бюджеты неопределенности измерения прямоугольных координат ВО. Методом эксцессов находятся оценки расширенных неопределенностей. Записаны выражения для относительных стандартных неопределенностей измерения координат и приводится пример их оценивания для реальных данных.

Ключевые слова: измерение координат; дальномерно-угломерный метод; неопределенность измерений; метод эксцессов.

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