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Improving metrological characteristics of measuring instruments by discrete wavelet noise filtering using the recursion method

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Abstract

The method of recursive discrete wavelet noise filtering for improving metrological characteristics of measuring instruments was investigated for the first time. Methods with a common threshold for all decomposition levels, methods without threshold with a simple zeroing of detail coefficients until the minimum mean square (RMS) error is reached, and methods with universal threshold for detail coefficients at each decomposition level were studied. Twenty different types of measurement signals from the popular PyWavelets library were analyzed. The functions of filtering methods with a common threshold were determined, for which the use of recursion reduces the filtering error from 10 to 50%. For methods without threshold and with universal threshold, the recursion does not reduces the error by multiple filtering of measurement signals. To apply the recursion to the method with a common threshold for all decomposition levels, a mathematical model based on the fundamental equations of wavelet filtering was constructed. The character of distribution of the filtering RMS error depending on the number of reversible cycles is investigated. It was summarized that for the measurement signal models under consideration, the maximum error reduction occurs between the zero cycle, in which the initial measurement signal is filtered, and the first level of recursion. Further reduction of the filtering error with increasing number of recursion cycles occurs according to the law close to hyperbolic.

Keywords: recursive algorithm; RMS error; numerical optimization; model signal; discrete wavelet transform; noise filtering; decomposition level.

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Introduction

Today, wavelet filtering is one of the most promising techniques of information processing, which is proved by an extensive scope of application: cleaning of measurement signals from noise, identification of signal anomalies, spectral analysis, and data compression. The discrete wavelet transform is used for signal analysis and synthesis. A convincing advantage of the discrete wavelet transform is its comparative economy in the number of operations and memory requirements with respect to the continuous wavelet transform [1].

The essence of wavelet signal analysis is to decompose a signal into a number of approximating and detailing coefficients, and since the latter ones characterize the noise component, the processing method of detailing components determines filtering efficiency in general [2]. One of the most well-known approaches for detailing coefficients processing is thresholding (zeroing) the coefficients that do not exceed a given value (threshold) [2, 3]. This method of noise removal is called thresholding [4], and the quality of noise reduction depends on the type of a thresholding function and the way it is applied [5]. The use of a local method with an adaptive universal threshold does not allow its capabilities to be fully realized [6].

Literature analysis and problem statement

The recursive filtering algorithm is considered in [7], and the use of recursion by the discrete wavelet filtering method is considered in [8]. According to [8], the suggested batch method of wavelet filtering has a low efficiency of 0.0035 dB per iteration and requires significant computing resources [9] due to the imposition of restrictions on both detail coefficients and approximation coefficients.

The authors of [10] also describe replacing the hierarchical computational design by a horizontal-recursive one and solving the problem of constructing basis wavelets that satisfy the recursion requirements. However, the paper is more of an overview giving examples of new and known wavelets for which there are effective (recursive) algorithms to compute the local discrete wavelet transform.

In [11], the experience of applying the recursive approach to Butterworth filters is described. According to the authors, the main difference is that the signal transformation is carried out through recursive filtering using IIR filters. Such filters use one or more of their outputs as input.

The authors of [12] suggest separating the signal from the noise using nonlinear thresholding, avoiding computationally resource-intensive block thresholding algorithms on the scale-time wavelet plane. Efficiency is achieved by estimating the pre-event noise characteristic statistics using empirical cumulative distribution functions and then applying these characteristics to the threshold of the entire time series using hard or soft nonlinear thresholding.

The most urgent task is to evaluate specific functions of radars and sonars [13–16].

The purpose of this paper is to increase the efficiency of discrete wavelet filtering by the recursion method using examples of noisy complex functions: "MishMash" (simulating radar and sonar signals) and "HeaviSine" (simulating anomalies in the form of sharp changes in the signal).

1. Recursive mathematical model of discrete wavelet filtering with a common threshold

The initial data for constructing a mathematical model of recursive filtering is a noisy signal:

$$f_\eta(t_i) = f(t_i) + \eta, \tag{1}$$

where $f(t_i)$ is the function of a pure measuring signal; $f_\eta(t_i)$ is the function of a noisy signal; η is white noise of normal distribution.

In the process of multiple filtering, signal (1) is cleared of noise with a gradual decrease in the additive component η.

For the transition from time domain $f_\eta(t_i)$ to frequency domain, we use the expansion of signal (1) over all decomposition levels [17] taking into account that $a_{j+j_0,k}$ and $d_{j+j_0,k}$ are not individual coefficients, but series and integrals taken repeatedly for each set of values j, k:

$$\left.\begin{aligned} a_{j+j_0,k} &= \int_R f_\eta(t_i)\cdot\varphi_{j+j_0,k}(t)dt \\ d_{j+j_0,k} &= \int_R f_\eta(t_i)\cdot\psi_{j+j_0,k}(t)dt \end{aligned}\right\}, \tag{2}$$

where R is the function definition interval $f(t)$; $a_{j+j_0,k}$, $d_{j+j_0,k}$ are the approximation and detail coefficients of a noisy signal respectively; $\varphi_{J+J_0,k}(t), \psi_{J+J_0,k}(t)$ are the maternal and paternal wavelets respectively; j_0, j, k are the initial and current levels of the wavelet decomposition and the serial number of a wavelet coefficient.

To return from frequency domain (2) to time domain on the n-th recursion cycle, we obtain relation (3):

$$\begin{aligned} f_\eta^n(t_i) &= \sum_k a_{j+j_0,k}^n \cdot \varphi_{j+j_0,k}^n(t) + \\ &+ \sum_{j=1}^{J}\sum_k F^n(\lambda_j^n)\cdot d_{j+j_0,k}^n \cdot \psi_{j+j_0,k}^n(t), \end{aligned} \tag{3}$$

where $f_\eta^n(t_i)$ is a filtered signal for the n-th recursion cycle; n is the number of a current recursion cycle; $a_{j+j_0,k}^n, d_{j+j_0,k}^n$ are the approximation and detail coefficients for the n-th recursion cycle. $\varphi_{j+j_0,k}^n(t), \psi_{j+j_0,k}^n(t), F^n(\lambda_j^n)$ are optimal parameters of discrete wavelet filtering for the n-th recursion cycle obtained from the conditions of the proportion minimum error (4).

$$E^n = \frac{1}{N}\cdot\sum_{i=0}^{N}(f(t_i) - f_\eta^n(t_i))^2, \tag{4}$$

where E^n is the minimum RMS filtering error for the n-th recursion cycle.

The suggested algorithm works as follows: we apply relation (2) to signal (1), return to the time domain according to (3), filter the signal, then calculate the error according to (4) and decompose the resulting signal according to (2). Repeating this sequence many times for different sets $\varphi_{j+j_0,k}^n(t), \psi_{j+j_0,k}^n(t), F^n(\lambda_j^n)$ [18] we obtain a set of errors E^0... E^n arranged in descending order.

It is thus obvious that the recursion should be stopped when the error does not change over two cycles:

$$E^{n-2} = E^{n-1} = E^n. \tag{5}$$

2. Numerical analysis of a recursive mathematical model of discrete filtering with a common threshold

Let us consider a complex "MishMash" [18] function simulating a measurement signal consisting of three sinusoids with different phases and frequencies:

$$\begin{cases} f_1(t) = \sin(\frac{\pi}{3}\cdot t\cdot m\cdot t^2) \\ f_2(t) = \sin(\pi\cdot m\cdot 0.6902\cdot t) \\ f_3(t) = \sin(\pi\cdot m\cdot 0.125\cdot t^2) \\ f(t) = f_1(t) + f_2(t) + f_3(t) \end{cases}, \tag{6}$$

where m=1024 is the number of samples of the function $f(t)$.

According to (1), let us add white noise of normal distribution with zero mathematical expectation and $\sigma = 0.4$ to the function $f(t)$. Let us perform recursion using (1)÷(5) according to the suggested algorithm.

To analyze the results of the recursion, we obtain a relation to determine the noise power reduction $\Delta U = U_2^n - U_1^0$, where U_1^0 is the noise power for filtering without recursion, U_2^n is the noise power for filtering after recursion with n cycles:

$$U_1 = 10\cdot\log_{10}\left(\frac{\sum f(t_i)^2}{\sum[f_{\eta 1}(t_i) - f(t_i)]^2}\right), \tag{7}$$

where $\sum f(t_i)^2$ is the sum of the squares of the samples of the function $f(t_i)$ of a "pure" signal; $\sum[f_{\eta 1}(t_i)-f(t_i)]^2$ is the sum of the squares of the sample differences of the function $f_{\eta 1}(t_i)$ of a filtered signal without recursion and a "pure" signal.

$$U_2 = 10 \cdot \log_{10}\left(\frac{\sum f(t_i)^2}{\sum[f_{\eta 2}(t_i)-f(t_i)]^2}\right), \qquad (8)$$

where $\sum[f_{\eta 2}(t_i)-f(t_i)]^2$ is the sum of the squares of the sample differences of the function $f_{\eta 2}(t_i)$ of a filtered signal after *n* recursion cycles and a "pure" signal.

$$\Delta U = 10 \cdot \log_{10}\left(\frac{\sum[f_{\eta 1}(t_i)-f(t_i)]^2}{\sum[f_{\eta 2}(t_i)-f(t_i)]^2}\right), \qquad (9)$$

where ΔU is the noise power reduction due the use of recursion at discrete wavelet filtering of signals.

For function (6), taking into account the signal to noise ratio of 9 dB before filtering, at $E^0 = 0.1454$, the filtering parameter are: wavelet – 27 dB; $F^0(\lambda_j^0)$–"*soft*"; $\lambda_j^0 = 0.2$ [18]. According to the suggested algorithm we obtain E^{15} = 0.1303 and the signal to noise ratio of 10 dB, and the filtering parameters are: wavelet – bior3.3; $F^{15}(\lambda_j^{15})$–"*hard*"; $\lambda_j^{15} = 0.1$. The error reduction is 10.4% (Fig. 1).

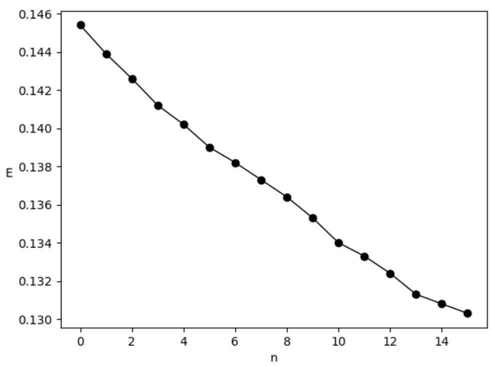

Fig. 1. The distribution of the root-mean-square filtering error *E* over *n* recursion cycles for noisy model function (7), where *E* is the error; *n* is the number of recursion cycles

On the same graph, let us plot the initial function (6) of a measuring signal and the function obtained after recursive filtering $f_\eta^{15}(t_i)$ (Fig. 2).

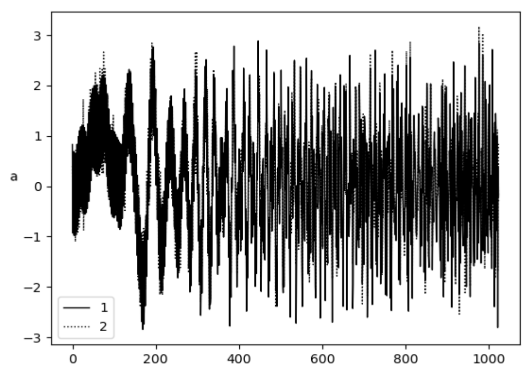

Fig. 2. Graphs of "clean" measuring signals – 1, and filtered measuring signals taking into account recursion − 2, according to relations (6), where: *a* is the relative signal amplitude, and *m* is the signal length

Let us consider a "HeaviSine" [18] measurement signal:

$$f(t) = 4 \cdot \sin(4\pi \cdot t) - sign(t-0.3) - sign(0.72-t). \qquad (10)$$

For function (10), taking into account the signal to noise ratio of 18 dB before filtering, at $E^0 = 0.0117$, the filtering parameters are: wavelet – bior4.4; $F^0(\lambda_j^0)$–"*garotte*"; $\lambda_j^0 = 1.0$. According to the suggested algorithm we obtain $E^{10} = 0.0071$ and the signal to noise ratio of 31 dB, and the filtering parameters are: wavelet – bior3.3; $F^{10}(\lambda_j^{10})$–"*hard*"; $\lambda_j^{15} = 0.1$. The error reduction is 39.3% (Fig. 3).

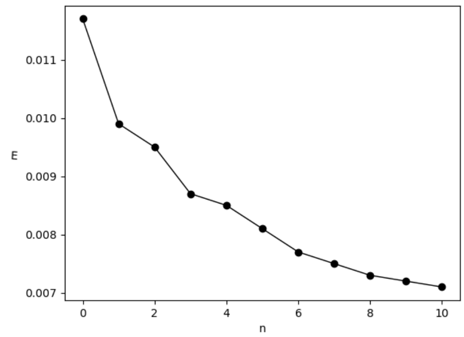

Fig. 3. Recursion efficiency for ratio function (10), where *E* is the error, and *n* is the number of recursion cycles

On the same graph, let us plot the initial function (10) of a measuring signal and the function obtained after recursive filtering $f_\eta^{15}(t_i)$ (Fig. 4).

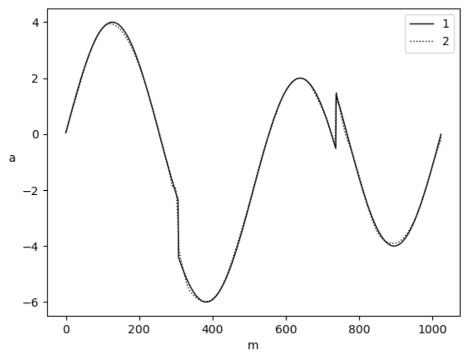

Fig. 4. Graphs of "pure" measuring signals – 1, and filtered measuring signals taking into account recursion – 2, according to relation (10), where: *a* is the relative amplitude of the signal, and *m* is the signal length

Having analyzed the plots for "pure" and filtered measuring signals (Fig. 4) it was summarized that the recursion allows achieving a high level of coincidence of a filtered signal with (10).

To obtain a recursive mathematical model of discrete wavelet filtering without threshold, it is sufficient to change the second summand of relation (3) to $\sum_{j=j_{opt}}^{J}\sum_k d_{j+j_0,k}^n \cdot \psi_{j+j_0,k}^n(t)$ by starting the first summation from the $j=j_{opt}$ decomposition level because all wavelet detail coefficients are replaced by zeroes from the first decomposition level to the $J=J_{opt}$ decomposition level. Data obtained from model function (6) with 10 dB of noise from [18] demonstrate that the filtering error does not change in each recursion cycle and remains $E^0 = 0.1581$ with a bior1.1 wavelet. Numerical verification on model function (10) with 18 dB of noise [18] before filtering demonstrates that $E^0 = 0.0176$ and a filtering parameter is a bior1.3 wavelet. According to our suggested algorithm, we obtained $E^6 = 0.017$, the signal to noise ratio of 27 dB and a bior1.1 wavelet as a filtering parameter. The error reduction is 3%.

To obtain a recursive mathematical model of discrete wavelet filtering with universal threshold, we should take into account that the threshold λ_i in relation (3) is not given, but it is calculated by the following relations [15]:

$$\left.\begin{array}{l} \sigma = \dfrac{median(|d_{1,k}|)}{0.6742} \\ \lambda_j^{univ} = \sigma\sqrt{2\ln N_j} \end{array}\right\}, \tag{11}$$

where λ_j^{univ} is the universal threshold; N_j is the number of the detail coefficients $d_{j,k}$ on the j level of decomposition; $median(|d_{1,k}|)$ is the median of the array $|d_{1,k}|$ of the detail coefficients on the first level of decomposition. However, numerical verification (11) on model functions (6) and (10) of measurement signals with regard to the noise has shown that the filtering error in each recursion cycle does not change and remains E^0.

Conclusion

The use of recursion in discrete wavelet filtering is effective only for a method with a common threshold for wavelet detail coefficients. For the “MishMash” function, which is a complex signal of sonars and radars, the filtering error using the recursion method is reduced by 10.4%, and the noise is reduced by 1 dB. For the “HeaviSine” function, which simulates anomalies in signals, the filtering error using the recursion method is reduced by 39.3%, and the noise is reduced by 13 dB after 15 cycles of recursion.

Підвищення метрологічних характеристик засобів вимірювання шляхом дискретної вейвлет-фільтрації шумів методом рекурсії

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Анотація

Вперше досліджено метод рекурсивної дискретної вейвлет-фільтрації шумів для підвищення метрологічних характеристик засобів вимірювання. Досліджувалися методи: із загальним порогом для всіх рівнів декомпозиції; без порога із простим обнуленням коефіцієнтів деталізації до досягнення мінімальної середньоквадратичної похибки; з універсальним порогом коефіцієнтів деталізації для кожного рівня декомпозиції. Було досліджено двадцять різних типів вимірювальних сигналів із популярної бібліотеки PyWavelets. Визначено функції методів фільтрації із загальним порогом, для яких застосування рекурсії знижує похибку фільтрації від 10 до 50%. Рекурсія не забезпечує суттєвого зниження похибки для методів без порога та з універсальним порогом. Для застосування рекурсії до методу із загальним порогом для всіх рівнів декомпозиції побудовано математичну модель, основою якої є фундаментальні рівняння вейвлет-фільтрації. Вивчено характер розподілу середньоквадратичної похибки фільтрації від кількості реверсивних циклів. Показано, що для досліджуваних моделей вимірювальних сигналів максимальне зниження похибки відбувається між нульовим циклом, у якому фільтрується вихідний вимірювальний сигнал, і першим рівнем рекурсії. Подальше зниження похибки фільтрації зі зростанням числа циклів рекурсії відбувається за законом, близьким до гіперболічного.

Ключові слова: рекурсивний алгоритм; середньоквадратична похибка; чисельна оптимізація; модельний сигнал; дискретне вейвлет-перетворення; фільтрація шумів; рівень декомпозиції.

Повышение метрологических характеристик средств измерений путем дискретной вейвлет-фильтрации шумов методом рекурсии

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Аннотация

Впервые исследован метод рекурсивной дискретной вейвлет-фильтрации шумов для повышения метрологических характеристик средств измерений. Исследовались методы: с общим порогом для всех уровней декомпозиции; без порога с простым обнулением коэффициентов детализации до момента достижения минимальной среднеквадратической погрешности; с универсальным порогом для коэффициентов детализации на каждом уровне декомпозиции. Были исследованы двадцать различных типов измерительных сигналов из популярной библиотеки PyWavelets. Определены функции методов фильтрации с общим порогом, для которых применение рекурсии снижает погрешность фильтрации от 10 до 50%. Рекурсия не обеспечивает существенного снижения погрешности для методов без порога и с универсальным порогом. Для применения рекурсии к методу с общим порогом для всех уровней декомпозиции построена математическая модель, в основу которой положены фундаментальные уравнения вейвлет-фильтрации. Изучен характер распределения среднеквадратической погрешности фильтрации от количества реверсивных циклов.

Ключевые слова: рекурсивный алгоритм; среднеквадратическая погрешность; численная оптимизация; модельный сигнал; дискретное вейвлет-преобразование; фильтрация шумов; уровень декомпозиции.

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