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Uncertainty in case of lack of information: extrapolating data over time, with examples of climate forecast models

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Abstract

The basic scientific tool for predicting is called a "forecast model", a mathematical model underpinned by observations. Generally, it is the evolution of some parameters of the present-day law(s) over time that are considered of fundamental importance in a specific case. The relevant available data are obviously limited to the past period of time, which is admittedly a limited period in most cases, when the law in question is considered valid and verified with sufficient precision – while no direct information is available about the future trend. A mathematical (set of) function(s) is extrapolated ahead over time to show present and next generations what they should be supposed to observe in the future. A problem arises from the fact that no (set of) mathematical function that could be used for a model is infinitely "flexible", i.e. apt to "correctly" interpolate any cluster of data, and the less a data set is, the less the parameters of the function(s) are. A data consistency is considered good when there is a balance between a mere "copying" the behaviour over time (e.g. when a function has to follow a given profile) and a satisfactory "averaging" the behaviour, especially over longer periods of time, without "masking" changing points. Furthermore, the *data uncertainty* is an embellishment, which the information often lacks, provided with extrapolations. Instead of it, the data uncertainty must be taken into account, and appropriate information must always be provided, since the quality of the adjustment of the available data is crucial for the quality of the subsequent extrapolation. Therefore, the forecast should better consist of an *area* (typically increasing its width over time) where future determinations are assumed to fall within a given probability range. Thus, it should be perfectly clear that the extrapolation of the past data into the future, i.e. a current evaluation that can be propagated to next generations, is affected by a high risk and that careful precautions and limitations should be taken.

Keywords: risk level; forecast model; data uncertainty; modelling accuracy; forecasting reliability.

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1. Introduction

The studies at high level in metrology allow the scientist to acquire a special competence in treating experimental data, understanding their features and the level of confidence that one can assign to them, especially those related to the uncertainty. Scientists in other disciplines, on the contrary, often do not have the necessary feeling for the need to use specific tools and refined procedures when the data analysis goes beyond simple examination.

This under-estimation is not infrequent and often makes a procedure very critical, like the extrapolation of data not founded on solid bases: excessive length of the extrapolation, though appealing for non-scientific purposes, or insufficient control over the constrained-level of the portion of the used function, is often found, especially when the uncertainty of the available data should be taken into account.

This paper, after recalling the foundation of science, addresses the urgent popular issue of the extrapolation of experimental data ahead over time for forecasting purposes (i.e. when *no information* is still available), and presents the difficulties and limitations that are intrinsic in that task, and the consequent risk of propagating false information, especially in the field of thermodynamics. As an example, the paper will explicitly address the specific discipline of climate forecasting, which is very popular today.

2. Science as the crossroad of disciplines

Science is a complex frame and the crossroad of different disciplines. The modern scientific method

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according to the Britannica definition is: “a mathematical and experimental technique employed in the sciences. More specifically, it is the technique used in the construction and testing of a scientific hypothesis. The process of observing, asking questions, and seeking answers through tests and experiments is not unique to any one field of science. In fact, the scientific method is applied broadly in science, across different fields. Many empirical sciences ... use mathematical tools borrowed from probability theory and statistics, together with outgrowths of these ...”

One crossroad is between experimental observations and theoretical studies, which are two disciplines that are interrelated, one providing experimental data, and the other one being responsible for the inference of the underlying “laws”: in general, it is difficult to state which one comes first in the scientific process.

Another crossroad concerns philosophy, in terms of which “...philosophers of science have addressed general methodological problems, such as the nature of scientific explanation and the justification of induction”, according to the same Britannica.

In both cases, one needs to intercommunicate the findings and hypotheses in a lexical form (oral or written) to the Community. Only the “language” is different: in case of theory, it is the mathematical one, which is considered a universal symbolic tool (carrying no ambiguity); in case of data, it is a universally defined and accepted symbolic language (typically algebra); in case of the philosophical foundations, it is a logical symbolic language or its idiom, i.e. a specific local language.

In all cases, it is assumed that next generations will also be able to correctly understand the past and present communications – directly or through historical education – and compare them with their novel findings.

This assumption means that the previously transmitted information may be invariant over time, but it does *not* mean that the whole previous knowledge remains invariant over time because the main science goal is to progress in knowledge, possibly also correcting, or even contrasting (Kuhn’s revolutions), the previous one.

3. Prediction in science

One of the most popular expectations of the non-scientific Community is that science allows *prediction*. However, quite often, that expectation does not realize that the prediction itself is always limited to a certain level of confidence and never provides any certainty in statistical meaning. Scientists, on the other hand, are expected to not ignore the latter fact as they are supposed to never ignore the uncertainty associated with any of their findings or thinking, and the possibility for errors in them, or of the evolution affecting them.

However, as amply discussed in philosophy of science, the problem is about how to take the doubt or the (conditional) certainty into account. This issue is particularly critical when concerning the top-level of the knowledge pyramid: the recognized “laws of nature”, even when expressed in their less ambiguous mathematical form.

A law is basically assessed to be valid by intersubjective consensus until a contrast is “demonstrated”, empirically (by observations), or formally (from mathematical contradictions). Actually, there is a third possible reason, which is connected to the human ways of communicating with each other: the contradiction as regards the foundations of human logic, e.g. concerning the cause-effect principle.

In conclusion, approaching truth is a “vast programme” (how can we understand whether we are approaching it if we do not know where it is, as observed again by Kuhn?), and the scientist should be humble in this respect because here another basic feature of science comes. It consists of the fact that science, basically, is not looking for “truth”, but simply for a consistent explanation – satisfactory to us – of facts by a sufficiently long roadmap made of observations showing a sufficient degree of repeatability and of theoretical inferences.

According to the above roadmap, a diversity of the positions almost invariably and intrinsically confronts with each other, requiring time and often adjustment to advance in knowledge until an issue can converge univocally and be considered acceptable by the whole Community. Nobody today still believes that the Earth is flat, but it took centuries of disputes and of experimental evidences before getting the certainty that the spherical model is the right one for us. In this case, the conclusion may have been made easier by the fact that the discipline of mechanics can be considered simpler to manage as regards the other ones, namely thermodynamics.

One of the tools, which is used by the followers of each position to support it, is the use of a well-known method – very much appreciated outside the science – to show how good a prediction can be obtained from the asserted position, i.e. how well the yet “unknown” looks like to follow the “known”.

The basic scientific tool for performing the *prediction* is called a “forecast model”, a mathematical model underpinned by observations [1]. Generally, it is the evolution of some parameters of the *present-day* law(s) over time that are considered of fundamental importance in a specific case.

The relevant available data are obviously limited to the past period of time, which is admittedly a limited period in most cases, when the law in question is considered valid and verified with sufficient precision. That is a risky task because in most cases the past precision has increased with the time, being that a goal of experimental science, but not always the data that are “weighted” for their precision, so the confidence in the precision of the function adjusted to these data could already be affected by the precision inhomogeneity. Then a mathematical (set of) function(s) is extrapolated ahead over time to show present and next generations what they should be supposed to observe in the future: this is commonly done for the weather forecast, normally for a subsequent period of a *few days* — why not more?

4. Modern use of prediction

The expected duration of the prediction validity depends, first, on the considered law: in case of mechanics, e.g. the orbit of the big sky bodies like the Earth, the forecast can confidently be done for extremely long periods of time. In case of thermodynamics (see later), most of us know very well the uncertainty about the weather forecast, which is a branch of it.

With the rapid spread of informatics, the use of computer models has rapidly grown up and became one of the preferred tools in the Internet socials for "informing" people. This has pushed an increasing number of scientists to exercise in this risky field — risky because the boundary between the science and politics is almost invisible and certainly quite uncertain.

Then, a problem arises from the fact that no (set of) mathematical function that could be used for a model is infinitely "flexible", i.e. apt to "correctly" interpolate any cluster of data, and the less a data set is, the less the parameters of the function(s) are. A data consistency is considered good when there is a balance between a mere "copying" the behaviour over time (e.g. when a function has to follow a given profile) and a satisfactory "averaging" the behaviour, especially over longer periods of time without "masking" changing points.

Consequently, there is normally a balance between the number of data available — and the period length during which they were taken — and a future period where the function so obtained can "safely" be extrapolated providing a safe forecast — i.e. remaining accurate while behaving without any constraints except those (purely mathematical) set by the function itself.

For example, if the "safe" observation time is considered 50 years, it is hardly possible to imagine any sensible extrapolation to a further period of the same length, and the shorter the period length is, the more the variation in the extrapolated period can be problematic, especially for rapidly increasing (or decreasing) function derivatives or for non-simple shapes of it.

Furthermore, the data uncertainty is an embellishment, which the information often lacks, provided with extrapolations. Instead of it, the data uncertainty must be taken into account and appropriate information must always be provided (which does not happen in many instances) since the quality of the adjustment of the available data is crucial for the quality of the subsequent extrapolation. There are instances when the data uncertainty is so large that their consistency is already sufficient to consider them unreliable and the extrapolation meaningless. The adjustment of the weighted data is always advised to limit this deficiency.

More frequently, providing the results from more than one model is preferred as a multiplicity that may allow an indirect evaluation of the forecast possible variability. This comparison of models can certainly mitigate the risk of false extrapolations if made with different adjusting (set of) equations — and of different complexity — on the same data.

Therefore, the forecast should better consist of an area (typically increasing its width over time) where future determinations are assumed to fall within a given probability range. Generally, the trend is monotonic because changing points cannot usually be foreseen. In a few cases, the latter are also foreseen: in that case, the extrapolation can also show a change in the sign of the first or/and of the second derivative (e.g., a future decline in the local/world human population, or exhaust/born of causes for the past/present trend).

5. A few examples of predicting the Earth thermodynamic parameters

A particularly risky field of prediction is that for thermodynamic phenomena, e.g. dominating on our planet.

The field of mechanics is generally simpler to handle because it is basically deterministic and little time dependent even when having a dynamics, and being most often limited to studies on a few bodies.

The extension of studies to "many bodies" is totally a different affair — a bit less in the astronomic field — when considering the "colligative" behaviour of a "discrete ensemble of bodies", i.e. depending on the body numerosity only, and not on their chemical-physical nature. This is the case of thermodynamics. The difficulties are usually somewhat mitigated by considering its dynamics as a sequence of equilibrium states, in which case granularity is usually ignored, and continuous mathematical functions are used to describe the time behaviour (stationary systems).

However, that is a simplification that cannot hold in too large systems when the validity of models extrapolated ahead over time becomes more and more questionable, especially in non-homogeneous systems and in case of complex (physical-chemical) interactions between bodies, or in the case of discrete systems. The development of science for the case of discreteness in physics and chemistry is accelerating, but at present, it is still quite unsatisfactory. This topic has already been addressed in a previous paper [2].

Thus, in the current situation, the increasing importance of reliable forecast for a much longer span over time than presently available for weather forecast cannot be considered sufficient to match with the present development of sound mathematical

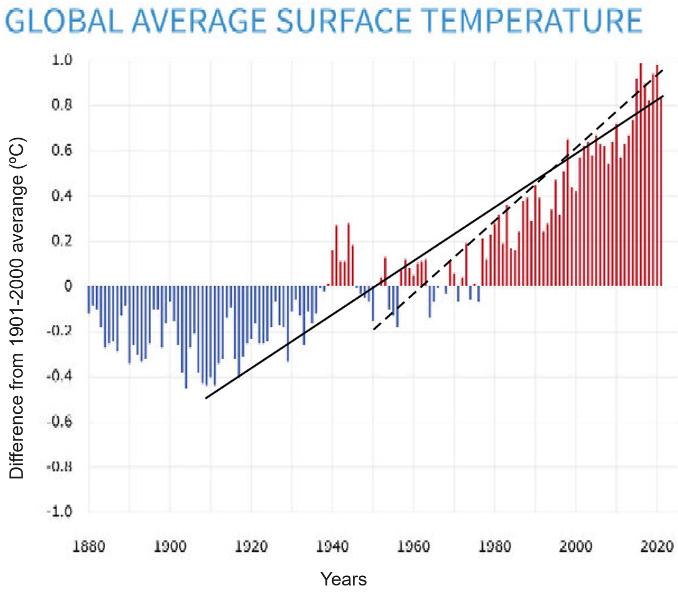

Fig. 1. Mean Global Earth Surface Temperature, 1880-2021 [7]. The long-term adjustment has an annual increase of (+0.10 ± 0.14) °C in the range of 1910−2020. The shorter-term adjustment of the IPCC shows a quite larger annual increase of (+0.17 ± 0.21) °C in the range of 1950−2020

and statistical tools, requiring considerable progress to cover the expectations of the next generation.

Current unresolved problems in Climate analysis have already been noted by authors in Philosophy of Science and on Environment, e.g.: "Non-epistemic values pervade climate modelling, as is now well documented and widely discussed in the philosophy of climate science", [3], and "Internal variability in the climate system confounds assessment of human-induced climate change and imposes irreducible limits on the accuracy of climate change projections, especially at regional and decadal scales" [4].

A few examples are given to bring evidence of how critical can be the forecast over a long period of the future natural behaviour concerning popular parameters in climate forecast, such as the Earth global temperature and global sea-water level.

Fig. 1 shows a prediction of the Mean Global Earth Surface Temperature for 1880−2021 from NOAA [5] compared with the prediction from IPCC by making an author's adjustment of the original data, where the adjustment over a longer period indicates a quite lower annual increase, and with a better adjustment s.d. (Note: the s.d. of the adjustment should not be confused with the accuracy of the temperature data, which, arising from a collation of the data from meteorological stations, cannot be better than ± 0.5 °C [6−7]).

Fig. 2 shows the IPCC prediction of the Mean Sea Level increase up to 2300 (!) according to different models [8]. It is difficult to believe that a mathematical model can be so accurate for such a long future period without a high risk, being based on a much shorter period of observations, during which the level increase

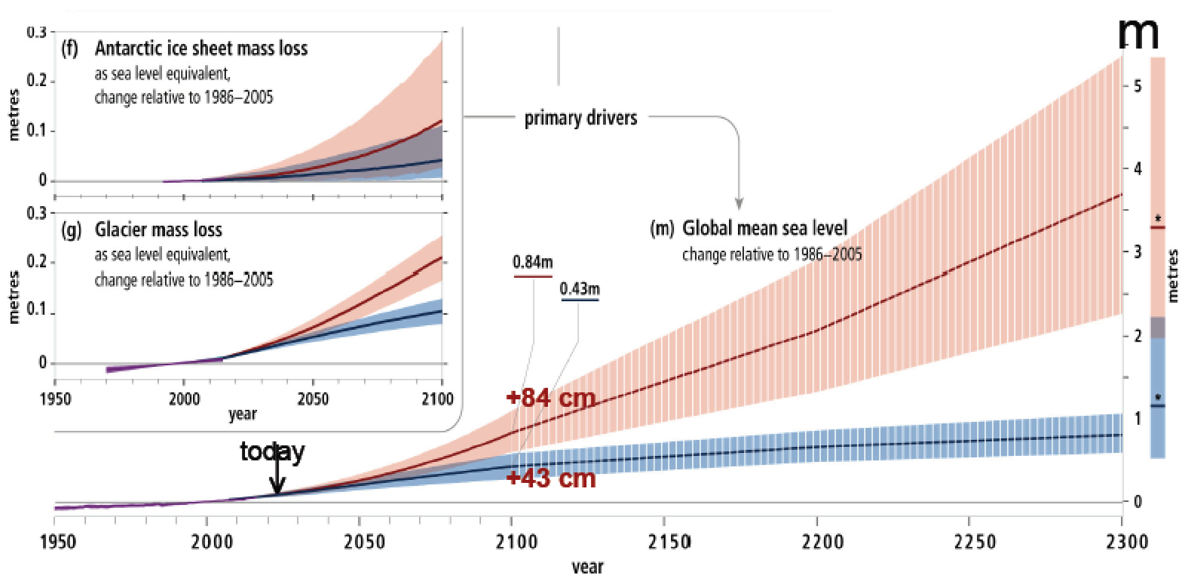

Fig. 2. Forecast of the Mean Sea Level Increase up to 2300 [8]. The PNAS 2016 (not shown) [9] prediction is from (+0.4 ± 0.1) m to (+0.85 ± 0.25) m up to 2100 depending on the model

has been limited to less than +0.1 m (uncertainty unreported), the same risk affecting the extrapolation up to 2100.

6. Final remarks

It should be perfectly clear that the extrapolation of the past data into the future, i.e. a current evaluation that can be propagated to next generations, is affected by a high risk. Risk level is rarely used as a tool for predicting to measure the reliability that one can assign to them. In many fields of prediction, this important parameter is not available, though it can extremely vary from case to case.

One basic reason is that a correct and full uncertainty analysis is not performed, in particular because an *uncertainty budget* [10] is not compiled, like in the current case of the climate field, or is not made available.

Невизначеність у разі відсутності інформації: екстраполяція даних у часі, з прикладами моделей прогнозування клімату

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Анотація

Основний науковий інструмент, що використовується для прогнозування, має назву “модель прогнозу” – математична модель, підкріплена спостереженнями. Зазвичай це еволюція в часі деяких параметрів сучасного закону (законів), що вважаються принципово важливими в конкретному випадку. Очевидно, що відповідні доступні дані обмежені минулим періодом часу – загальновизнано, що в більшості випадків цей період є обмеженим, коли закон, що розглядається, вважається дійсним і перевіреним із достатньою точністю – у той час, як пряма інформація про майбутню тенденцію не може бути доступною. Зазвичай математична функція екстраполюється у часі, щоб показати нинішньому та наступним поколінням те, що вони, як прогнозується, зможуть спостерігати в майбутньому. Проблема виникає через те, що жодна математична функція, яку можна було б використовувати для цієї моделі, не є нескінченно “гнучкою”, тобто здатною “правильно” інтерполювати будь-який набір даних, і чим менший цей набір, тим менші параметри функції. Відповідність даних вважається хорошою, коли існує баланс між простим “копіюванням” поведінки в часі (наприклад, коли функція має дотримуватися заданого профілю) та задовільненим “усередненням” поведінки, особливо за більш тривалі періоди часу, без “маскування” точок, що змінюються. Крім того, невизначеність даних є прикрасою, яка часто відсутня в інформації, що надається з екстраполяцією. Замість цього необхідно враховувати невизначеність даних і завжди надавати відповідну інформацію, адже якість налагодження наявних даних має вкрай важливе значення для якості наступної екстраполяції. Відповідно краще, якщо прогноз складається з області (що зазвичай розширюється з часом), у межах якої допускається, що майбутні визначення потрапляють в діапазон заданої ймовірності.

Таким чином, має бути цілком зрозуміло, що екстраполяція минулих даних у майбутнє, тобто поточна оцінка, яку може бути передано наступним поколінням, пов’язана з високим ризиком і що слід вживати ретельних запобіжних заходів та обмежень.

Ключові слова: рівень ризику; модель прогнозу; невизначеність даних; точність моделювання; надійність прогнозування.

Неопределенность в случае отсутствия информации: экстраполяция данных во времени, с примерами моделей прогнозирования климата

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Аннотация

В статье рассматривается актуальный вопрос оценивания неопределенности при экстраполяции данных во времени. При этом используется модель прогноза – математическая модель, подкрепленная наблюдениями как основной научный инструмент, используемый для прогнозирования. Поскольку доступные данные ограничены прошлым периодом времени, а прямая информация о будущей тенденции не может быть доступной, то при прогнозировании возникает проблема из-за того, что ни одна математическая функция, которую можно было бы использовать для этой модели, не способна достоверно интерполировать любой набор данных. Кроме того, неопределенность данных часто отсутствует в информации, предоставляемой с экстраполяцией.

Таким образом, должно быть вполне понятно, что экстраполяция прошлых данных в будущее, то есть текущая оценка, которая может быть передана следующим поколениям, связана с высоким риском и что следует принимать тщательные меры предосторожности и ограничения.

Рассмотрены примеры экстраполяции данных, связанных с прогнозированием климата.

Ключевые слова: уровень риска; модель прогноза; неопределенность данных; точность моделирования; надежность прогнозирования.

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