UDC 535.42+535.44:535.211:535.341

Express method for measuring the refractive index of transparent fibres

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Abstract

An express method for measuring the refractive index, which is one of the main optical parameters of transparent fibres, is suggested. The method uses focusing properties of a cylindrical lens, which such a fibre is. The possibility to accurately measure such characteristics of optical fibres as the shell and core diameters, numerical aperture, refractive index profile, loss, and dispersion is equally important for fibre manufacturers and designers of optical communication systems who should choose the fibre that meets their requirements best. Almost all measurement methods use the refraction of light rays at the interface between the media. To do this, one should make samples of given shape and size, which are individual for each measuring instrument. The suggested method takes into account the fact that when light strikes upon a refractive cylinder (glass rod, fibreglass), the focusing occurs perpendicular to its axis with a focal region where light rays converge. Behind this region, the rays diverge again. The position of the focal region is determined by the refractive index of the cylinder. It can be inside the cylinder, outside it, or on the surface of the cylinder. During the observation of the fibre using a microscope, one can see that the light, which has passed through the fibre, forms a bright band on its backside against a dark background. The bandwidth depends on the refractive index of the fibre. The calculations using the methods of geometric optics were carried out. These methods may be applied over a wide range of fibre diameters. Using strict formulas of diffraction theory, the distribution of radiation energy in the fibre and its vicinity was calculated. A digital analysis of the resulting pattern was carried out. The results of the analysis coincided with the results obtained using the methods of geometric optics. An algorithm for determining the refractive index was worked out. The measurements of the refractive indices of artificial and natural fibres like fibreglass, webs and human hair (blonde-haired person, brown-haired person, grey hair) were provided.

Keywords: refractive index; transparent fibre; measurement; focusing; cylindrical lens.

Received: 27.07.2022 Edited: 15.09.2022 Approved for publication: 21.09.2022

Introduction

The refractive index is one of the main optical parameters of a substance. The possibility to accurately measure such characteristics of optical fibres as the shell and core diameters, numerical aperture, refractive index profile, loss, and dispersion is equally important for fibre manufacturers and designers of optical communication systems who should choose the fibre that meets their requirements best. Many methods have been suggested, and a large number of complex devices have been developed for their implementation. Some of these devices were designed to measure the characteristics of fibres in the process of their manufacture; others were designed to use fibres in communication systems.

For visible light, the most transparent media have refractive indices between 1 and 2. Gases, due to their low density at atmospheric pressure, have a refractive index close to 1. Almost all solids and liquids have a refractive index greater than 1.2, except aerogel, which is a solid with a very low density, and its refractive index is in the range from 1.002 to 1.3. Moissanite is at another end of the range. Its refractive index reaches 2.65. Most plastics have refractive indices in the range from 1.3 to 1.7, but their values of some polymers reach up to 1.76.

In the infrared range, the refractive indices can be much higher. Germanium is transparent in the wavelength range from 2 to 14 microns and has a refractive index of about 4. In the second half of the 2000s, a new type of material was discovered, called topological insulators, which has a very high refractive index up to 6 in the near and middle infrared ranges. These properties are potentially important for applications in infrared optics.

Almost all measurement methods use the refraction of light rays at the interface between the media. To do this, one should make samples of given shape and size, which are individual for each measuring instrument [1–3]. The measurement process is determined by standards and methods, for example [4, 5]. The measurement error of such methods is small. This is usually the fourth to sixth decimal place.

The disadvantage of the methods mentioned above is that measurements are carried out on purpose-made samples of raw material, but not on finished products. They may not always be used if the object has a specific shape, such as cylindrical or spherical.

There are methods for rapid control of the properties of optical products. In [6], a simple method for determining the refractive index of liquids in thin-walled cylindrical vessels or solid cylindrical bodies, including fibreglass, is described. The method is based on the phenomenon of light refraction in cylindrical lenses. If some straight line forms an angle α with the axis of the cylinder, then the pattern of the line is rotated by this angle when the straight line is observed through a transparent cylinder (Fig. 1).

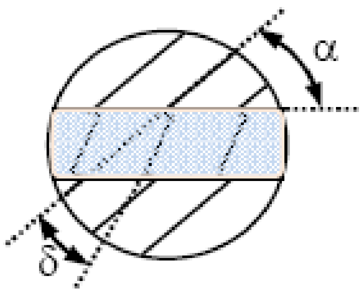


Fig. 1. Image distortion by a cylindrical lens

The angle δ depends on the angle α and the refractive index of the cylinder n and does not depend on the diameter of the cylinder:

$$\delta = \operatorname{arctg}\left(\frac{n}{2-n}\operatorname{tg}\alpha\right) - \alpha,$$

therefore

$$n = \frac{2\cos\alpha\sin(\alpha+\delta)}{\sin(2\alpha+\delta)}.$$

In [7], to measure the refractive index of a fibre, it was suggested to use a feature of the backscattering indicatrix [8]. There is a region of the increased intensity of scattered light bounded on both sides by its maxima, which is called the "rainbow region" (Fig. 2).

The angular size of the "rainbow region" is determined by the formula

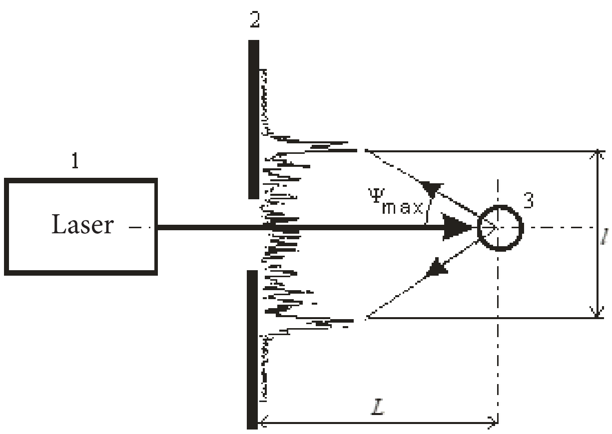


Fig. 2. Scattering indicatrix of a cylinder in the "rainbow region": 1 – source of light, 2 – screen, 3 – cylinder

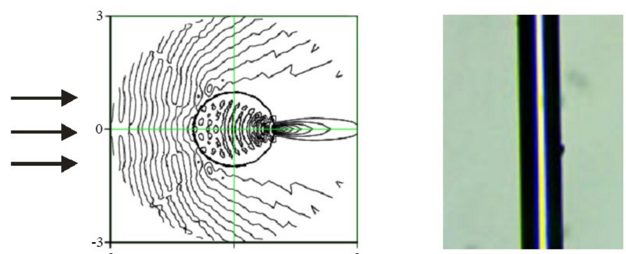
$$\psi_{\max} = 4\arcsin\left(\frac{4-n^2}{3n^2}\right)^{1/2} - 2\arcsin\left(\frac{4-n^2}{3}\right)^{1/2}.$$

It depends only on the refractive index and does not depend on the diameter of thick cylinders ($D \gg \lambda$).

The accuracy of the described express methods is not high, and it is approximately two decimal places. Their advantages are simplicity and the possibility to work with finished products made of fibreglass.

The method suggested here takes into account the fact that when light strikes upon a refractive cylinder (glass rod, fibreglass) perpendicular to its axis, the focusing occurs with a focal region where light rays converge. Behind this region, the rays diverge again. The position of the focal region is determined by the refractive index of the cylinder.

Fig. 3 shows the distribution of light intensity inside a cylinder with a refractive index n = 1.5 and in its vicinity calculated using strict formulas of diffraction theory. One can see a converging beam of the light inside the cylinder, a focal region near the rear surface of the cylinder, and a region of high intensity on the surface of the cylinder. Fig. 3*b* shows the microphotograph of a fibreglass obtained in transmitted light. In the middle of the cylinder there is a light band, which is a region of high intensity of the light focused by the cylinder.



a *b*

Fig. 3. Focusing of the light by a cylinder: *a* – light intensity distribution, *b* – microphotograph in transmitted light

This effect can be used to measure the refractive index of the fibre material. The measurement process consists of the fibre pattern processing, which is obtained using a microscope. One cannot expect the high accuracy of this method. However, its advantages are the simplicity of measurements and processing of results. In addition, one can measure the parameters of the finished product, taking into account possible changes in the material that occurred during the manufacturing process.

Theoretical ratio

Let us consider the propagation of a thin beam through a cylinder (Fig. 4).

A beam parallel to the x-axis strikes upon a cylinder with radius r to point B with coordinates

$$x_0 = -r\cos\varphi, \quad y_0 = r\sin\varphi,$$

where φ is an incident angle.

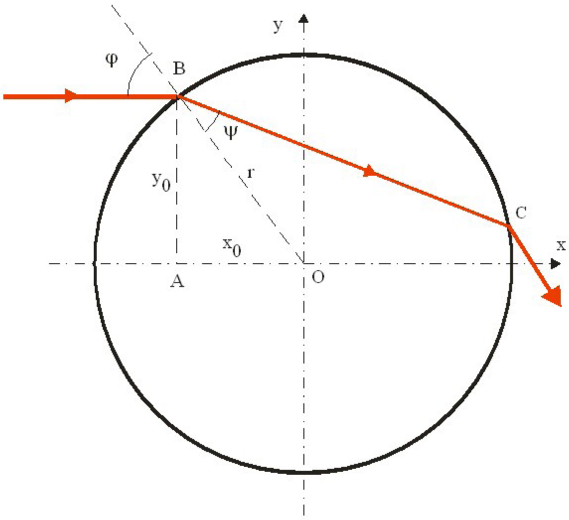


Fig. 4. Beam propagation through a cylinder

Upon entering the cylinder, the beam is refracted according to the law

$$\frac{\sin\varphi}{\sin\psi}=n,$$

where ψ is the angle of refraction, n is the refractive index.

Let us get the equation of the refracted beam $y = kx + b$. From geometric constructions, it follows that the angular coefficient is a straight line

$$k=-\mathrm{tg}\left(\varphi-\psi\right), \tag{1}$$

and the parameter that defines the intersection point of the line with the y-axis is

$$b=r\left[\sin\varphi-\mathrm{tg}\left(\varphi-\psi\right)\cos\varphi\right]. \tag{2}$$

Let us get the intersection points of the straight line $y = kx + b$ with the circle $x^2 + y^2 = r^2$, i.e. the coordinates of the points where the ray enters and leaves the cylinder. The solution of the resulting system of equations is

$$\begin{aligned} x_1&=\frac{-kb-\sqrt{k^2b^2-\left(k^2+1\right)\left(b^2-r^2\right)}}{k^2+1}, \qquad y_1=kx_1+b,\\ x_2&=\frac{-kb+\sqrt{k^2b^2-\left(k^2+1\right)\left(b^2-r^2\right)}}{k^2+1}, \qquad y_2=kx_2+b. \end{aligned} \tag{3}$$

Parameters k and b are determined by (1) and (2).

The first pair of coordinates corresponds to the entrant point of the beam into the cylinder,

$$x_1=x_0,\quad y_1=y_0.$$

The second pair corresponds to the egress point of the beam from the cylinder. The distance between point C and the x-axis is half of the bright band on the surface of the fibreglass observed during measurements with a microscope (Fig. 3*b*). Its width is determined by the refractive index of the fibre.

The dependence of the width of the bright band h on the refractive index n is shown in Fig. 5 by line 1 (geometric optics) and line 2 (calculated using strict formulas of diffraction theory). For low refractive indices, the focal region is located outside the cylinder, and the width of the light band h on its surface is large. If the refractive index increases, the focusing of light increases, the focal region gets closer to the surface of the cylinder, and the width of the bright band on its surface decreases. If the refractive index further increases, the focal region shifts inside the cylinder, a diverging light beam strikes upon its surface from the inside, and the width of the bright band increases again.

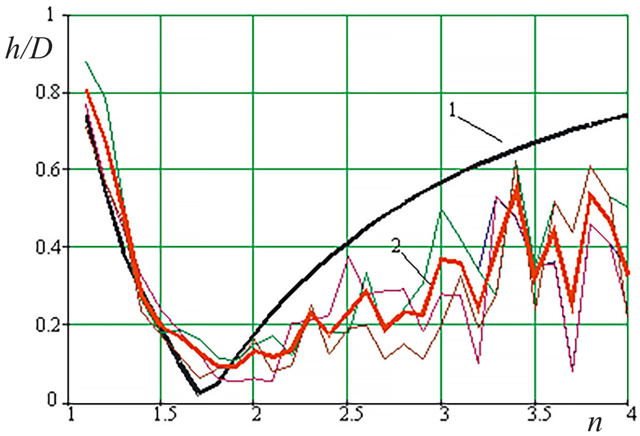
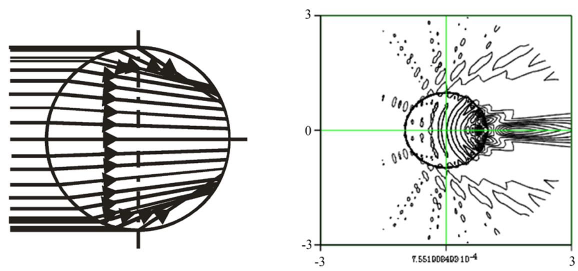


Fig. 5. Dependence of the width of the bright band on the cylinder on the refractive index, 1 – geometric optics, 2 – strict formulas

This is also clearly seen in Fig. 6*a*, 7*a*, 8*a*, which show the paths of rays in the cylinders with the refractive indices of 1.2, 1.5 and 2.

Fig. 6*b*, 7*b* and 8*b* show the distribution of light intensity inside the cylinder and around it calculated by strict formulas describing light diffraction on the cylinder [1]. One can see a good coincidence of the pictures built on the laws of geometric optics with the ones built on the formulas of light diffraction.

The thin lines in Fig. 5 show the dependencies $h(n)$ obtained by digital processing of the field distribution patterns similar to the ones in Fig. 6*b*, 7*b*, 8*b*. They are built for the values of the parameter $\rho=\pi D/\lambda$. from 10 to 100. This parameter characterizes the ratio between the cylinder diameter and the radiation wavelength. For the indicated values of ρ, the cylin-



a *b*

Fig. 6. The path of rays in the cylinder (n = 1.2)

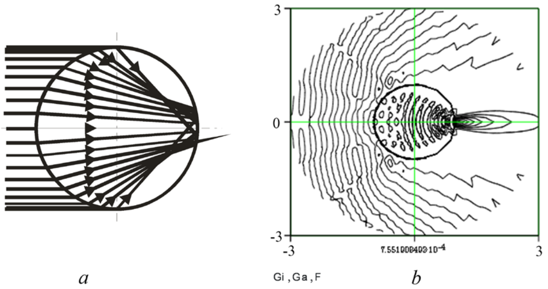


Fig. 7. The path of rays in the cylinder (n = 1.5)

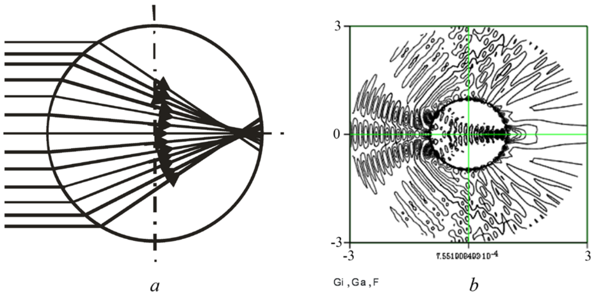


Fig. 8. The path of rays in the cylinder (n = 2)

der is thick, and the laws of geometric optics can be applied to such case.

For 1.1 < n < 1.8, the trend of all curves is almost the same. After the minimum, the trend of curve 1 for the case of geometric optics and for the curves for strict calculations are also qualitatively the same, i.e. they increase. However, strict curves grow much more slowly. The thicker curve 2 is the average value of the thin curves. Jumps on it are not a consequence of measurement errors during digitization. Fig. 9 clearly shows regular alternating maxima and minima with a period of 0.26 on smoothed curve 2. The focusing of light is either greater or weaker in these resonances.

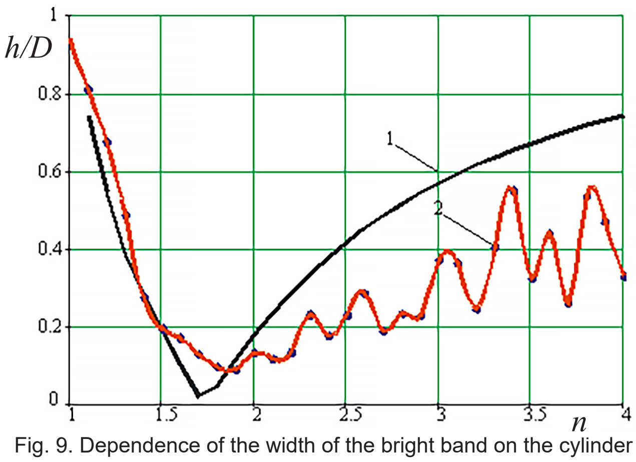


Fig. 9. Dependence of the width of the bright band on the cylinder on the refractive index, 1 – geometric optics, 2 – diffraction theory

The monotonically decreasing branch of the dependence $h(n)$ can be used to measure the refractive index of fibreglass by performing required measurements with a microscope. The accuracy of such measurements is low. However, for a quick assessment of the refractive index, this method is convenient. There is no need to make samples of a certain shape, as in the case with other methods, and no need to use special liquids with a known refractive index.

Fig. 10 shows the section of a theoretical curve for the refractive indices from 1.1 to 1.8 with a thick dotted line on a larger scale. By measuring the value of h/D using a microscope, this curve can be used to determine the refractive index of fibreglass. The curve is well described by the dependence

$$n = \frac{1}{0.466\frac{h}{D} + 0.578}. \quad (4)$$

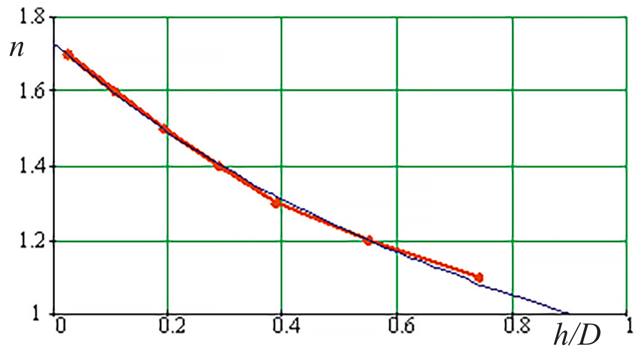


Fig. 10. Graduation curve

It is shown in Fig. 10 with a thin line.

Let us estimate the error of this method by considering its two components:

1. The error of determining the value of h/D during digitization of the field patterns in the cylinder.

2. The uncertainty caused by the variation of values when using expression (4).

The patterns of the field in the cylinder have been processed for several values of the parameter $\rho = \pi D/\lambda$. The obtained values for the same refractive index were averaged, and the standard deviation S was calculated. Its values were in the range from 0.03 to 0.07. During estimating the error of the method, their average value S = 0.05 was chosen.

A difference between the values of the refractive index at the nodal points in Fig. 6 and the values determined by the approximating curve does not exceed 0.02.

Thus, the absolute error of determining the refractive index using this method is several digits of the second decimal place. It is comparable with the errors of the express methods described in [6, 7].

This is a rather large error, but, given the simplicity of the method and the convenience of its use, it can be useful for online control of the properties of fibreglass and other transparent fibres.

Experiment

The refractive indices of several fibres was measured. The measurement results are presented in Table 1.

Such measurement method is suitable for a wide range of diameters, from 6 to 100 µm.

The result of measurements of the refractive index of fibreglass correlates well with the values of the refractive indices of the glasses used in fibre optics.

Table 1

Measurement results

№	Fibre type	Diameter, μm	n	Δn
1	fibreglass	30	1.54	0.020
2	blonde hair 1	75	1.12	0.016
3	blonde hair 2	104	1.23	0.018
4	brown hair	46	1.09	0.010
5	grey hair	50	1.28	0.020
6	spider's web	6	1.41	0.017

There is a lack of information about the refractive indices of the human hair in the literature. In [9–12], its values were taken in the range from 1.47 to 1.55 as approximate. The values of this parameter obtained here are therefore of interest since they differ from those available in the literature.

This also applies to the refractive index of the web, the values of which could not be found in the available literature.

Conclusions

1. An express method for measuring the refractive index of transparent fibres is suggested, which is based on measuring the light flux in a cylinder as in a focusing lens.
2. The method can be applied over a wide range of fibre diameters.
3. Experimental tests of the method using artificial and natural fibres were carried out.

Експрес-метод вимірювання показника заломлення прозорих волокон

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Анотація

Запропоновано експрес-метод вимірювання показника заломлення, який є одним з основних оптичних параметрів речовини, прозорих волокон. Метод використовує фокусуючі властивості циліндричної лінзи, якою є таке волокно. Можливість точно вимірювати характеристики оптичного волокна, такі як діаметри оболонки й серцевини, числову апертуру, профіль показника заломлення, втрати й дисперсію, однаково важлива як для виробників волокна, так і для розробників оптичних систем зв'язку, яким слід обирати волокно, що найбільше відповідає поставленим вимогам. Практично всі методи вимірювань використовують заломлення світлових променів на межі поділу середовищ. Для цього виготовляються зразки заданої форми та розмірів відповідно конкретного вимірювального приладу. У запропонованому авторами методі використовується той факт, що при падінні світла на заломлюючий циліндр (скляний стрижень, скловолокно) перпендикулярно його осі відбувається фокусування. Утворюється фокальна область, до якої сходяться світлові промені. За цією областю промені знову розходяться. Положення фокальної області визначається показником заломлення циліндра. Вона може знаходитися всередині циліндра, поза ним або на поверхні циліндра. Під час спостереження за волокном під мікроскопом світло, що проходить через волокно, утворює на задньому боці світлу смугу на темному тлі. Ширина лінії залежить від показника заломлення волокна.

У цій роботі проведено розрахунки за допомогою методів геометричної оптики. Метод може застосовуватись у великому діапазоні значень діаметрів волокон. За точними формулами теорії дифракції розраховано розподіл енергії випромінювання у волокні та його околиці. Проведено цифровий аналіз отриманої картини. Результати аналізу збіглися з результатами, отриманими на основі геометричної оптики. Розроблено алгоритм визначення показника заломлення. Проведено вимірювання показників заломлення штучних та природних волокон – скловолокна, павутини та волосся людини (блондина, шатена, брюнета, сивини).

Ключові слова: показник заломлення; прозоре волокно; вимірювання; фокусування; циліндрична лінза.

Экспресс-метод измерения показателя преломления прозрачных волокон

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Аннотация

Предложен экспресс-метод измерения показателя преломления прозрачных волокон. Возможность точно измерять такие характеристики оптического волокна, как диаметры оболочки и сердцевины, числовую апертуру, профиль показателя преломления, потери и дисперсию, одинаково важна как для изготовителей волокна, так и для разработчиков оптических систем связи, которым следует выбирать волокно, максимально отвечающее поставленным требованиям. Практически все методы измерений используют преломление световых лучей на границе раздела сред. В предложенном авторами методе используется тот факт, что при падении света на преломляющий цилиндр (стеклянный стержень, стекловолокно) перпендикулярно его оси происходит фокусировка. Во время наблюдения за волокном под микроскопом свет, проходящий через волокно, образует на его задней стороне светлую полосу на темном фоне. Ширина полосы зависит от показателя преломления волокна.

Проведены расчеты с помощью методов геометрической оптики. По строгим формулам теории дифракции рассчитано распределение энергии излучения в волокне и его окрестности. Проведен цифровой анализ получившейся картины. Результаты анализа совпали с результатами, полученными на основе геометрической оптики. Разработан алгоритм определения показателя преломления. Проведены измерения показателей преломления стекловолокна, паутины и волос человека (блондина, шатена, брюнета, седины).

Ключевые слова: показатель преломления; прозрачное волокно; измерение; фокусировка; цилиндрическая линза.

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