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Using the average air temperature along the trace for atmospheric correction in laser ranging: accuracy analysis

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Abstract

One of the factors affecting the results of laser ranging measurements on ground-level traces is the difference between the speed of light propagation in the Earth's atmosphere and the speed of light in vacuum. This difference is accounted for by the introduction of a correction for the mean integral refractive index of air along the measured trace. Research to improve the accuracy of such a correction is one of the most important scientific fields in geodesy and metrology. At the same time, the empirical approach to accounting for the atmospheric influence, which does not have a strict justification, is used in geodetic practice and involves determining the mean integral refractive index by the average air temperature along the measured trace. The paper is dedicated to the analysis of the accuracy of this empirical approach and to the estimation of the possibility of its application for providing traceability in the practice of linear measurements.

Keywords: measurement; accuracy; laser rangefinder; accounting for atmospheric influence; air temperature.

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1. Introduction

To provide metrological traceability of linear measurements from the 1 m length standard to distance measurements in geodesy, geodynamics, navigation, space research and others, the influence of the Earth's inhomogeneous atmosphere on the results of laser range-finding measurements carried out on the ground-level traces should be accounted for correctly. So, it is first necessary to know the average integral refractive index of air along the measured trace, as it determines the main part of the atmospheric additional delay of the optical signal, related to the difference of the signal propagation speed in the atmosphere and in vacuum. This part of the delay should always be excluded from the results of high-precision measurements. A much smaller contribution is given by another component of the atmospheric delay due to the refractive bending of the signal trajectory. It is necessary to consider this component only for sufficiently long traces, while for traces of less than 5 km it is usually negligible.

Recently, the requirements for the accuracy of determining the mean integral refractive index of the air have increased significantly. It has led to the rapid development of both methods for direct determination of the atmospheric delay (hardware methods) [1] and indirect determination based on the representation of the integral of the refractive index along the trace of the optical signal using quadrature or interpolation formulas (point approximation methods) [2, 3]. The importance of the point approximation methods is that they allow representing the average integral refractive index as a function of the local values of meteorological parameters (pressure P, temperature T and air humidity e) in a few discrete points located along the measured trace. The values of meteorological parameters at these points can be easily measured by well-known methods. At the same time, in some cases of geodetic practice, despite continuously increasing requirements to the accuracy of measurements, to account for the influence of atmosphere, the empirical approach based on evaluating the average air temperature along the measured trace is used without strict substantiation [4].

The purpose of this paper is to perform metrological analysis of the empirical approach on simple examples, which are considered below for traces up to 5 km long, and to establish conditions under which its application is justified.

2. Research results

In general, the ratio for the desired length of the straight line L connecting the end points of the measured trace can be represented as

$$L = \mathcal{L} - \delta\, \mathcal{L}, \quad (1)$$

where $\mathcal{L}=\frac{c\cdot\tau}{\overline{n}}$, $\delta\mathcal{L}=\int_{\sigma}d\sigma-L$, $\overline{n}=\mathcal{L}^{-1}\cdot\int_{\sigma}n(\sigma)d\sigma$.

Here, c is the speed of light in vacuum; τ is the signal travel time between the end points of the measured trace (directly measured value); $\mathcal{L}$ is the length of the laser radiation trajectory curved due to the refraction in a non-homogeneous atmosphere; $\delta\mathcal{L}$ is the refractive lengthening of the trajectory; $\overline{n}$ is the mean integral value of the air refractive index $n(\sigma)$ along the trajectory.

In general, the trajectory, along which the integrals in ratio (1) for $\delta\mathcal{L}$ and $\overline{n}$ are taken, satisfies the ray equation of geometric optics (σ is the ray coordinate counted along the trajectory).

Since the paper considers traces up to 5 km long, the value $\delta\mathcal{L}$ is negligibly small and is not accounted for hereafter. That is, it is assumed that $L=\mathcal{L}$, and the integrals in formula (1) are taken along a straight line connecting the end points of the measured baseline. In this case, to account for the influence of the atmosphere, it is necessary to determine only the average integral value of the refractive index along L

$$\overline{n}=\frac{1}{L}\int_{0}^{L}n(x)dx, \tag{2}$$

where x is the coordinate counted along L (straight line).

The dependence of the refractive index on x is due to spatial inhomogeneity of meteorological parameters of the atmosphere. Various variants of the functional ratio $n = f(T, P, e)$ accounting for this dependence are known. The most accurate version is the Siddor formula [5], which should be corrected as justified in [6]. In this paper, estimates are made to the magnitude order of the values of the analysed quantities. For such estimations, it is advisable to use less cumbersome ratios. In particular, the simplified formula for the visible range of optical radiation is recommended in [4], which provides the accuracy required for the estimates

$$n=1+83.11\cdot10^{-6}\cdot\frac{P}{T}-11.4\cdot10^{-6}\cdot\frac{e}{T}, \tag{3}$$

where air temperature T is expressed in Kelvin degrees K, and air pressure P and humidity e are expressed in hectopascals hPa.

To simplify our calculations, we assume that air pressure and humidity do not change along a horizontal trace L, and only temperature $T(x)$ depends on the coordinate x along the trace. Note that from the formula for refractive index (3) it follows that, with a strict approach, the mean integral refractive index (2) in this case should be determined in terms of the mean integral value of the quantity, which is inverse to temperature

$$\left\langle\frac{1}{T(x)}\right\rangle=\frac{1}{L}\cdot\int_{0}^{L}\frac{dx}{T(x)}, \tag{4}$$

and not in terms of the mean integral value of the temperature

$$\langle T(x)\rangle=\frac{1}{L}\cdot\int_{0}^{L}T(x)dx, \tag{5}$$

as provided by the empirical approach under consideration.

The methodological component of the error of the empirical approach is determined by the difference $\overline{n}_{acc}-\overline{n}_{emp}$ of the two results of calculating the value $\overline{n}$ according to formula (2) (from this difference the corresponding uncertainty component $|\overline{n}_{acc}-\overline{n}_{emp}|/3^{1/2}$ is estimated). For the first result ($\overline{n}_{acc}$ is a rigorous approach), the integral (2) accounting for formula (3) with P and e unchanged along the trace is expressed through integral (4) using the ratio

$$\overline{n}_{acc}=1+(83.11\cdot10^{-6}\cdot P-11.4\cdot10^{-6}\cdot e)\cdot\left\langle\frac{1}{T(x)}\right\rangle \tag{6}$$

and for the second result ($\overline{n}_{emp}$ is the empirical approach), integral (2) accounting for formula (3) under similar conditions is expressed through integral (5) to obtain a value

$$\overline{n}_{emp}=1+(83.11\cdot10^{-6}\cdot P-11.4\cdot10^{-6}\cdot e)\cdot\frac{1}{\langle T(x)\rangle}. \tag{7}$$

As a result, the ratio for estimating methodical error takes the form

$$\overline{n}_{acc}-\overline{n}_{emp}=(83.11\cdot10^{-6}\cdot P-11.4\cdot10^{-6}e)\left[\left\langle\frac{1}{T(x)}\right\rangle-\frac{1}{\langle T(x)\rangle}\right]. \tag{8}$$

Next, consider two options for $\overline{n}_{acc}-\overline{n}_{emp}$. The first option assumes a linear change in the air temperature along the trace, which is usually observed on a homogeneous underlying surface

$$T(x)=T_0+g_T\cdot x, \tag{9}$$

where g_T is the horizontal air temperature gradient (along the trace). By using (9) and formulas (4)–(8), accounting for equality $T_L=T_0+g_T\cdot L$, we obtain

$$\overline{n}_{acc}-\overline{n}_{emp}=(83.11\cdot10^{-6}\cdot P-11.4\cdot10^{-6}e)\cdot\frac{1}{T_0}\left[\frac{\ln\frac{T_L}{T_0}}{\frac{T_L}{T_0}-1}-\frac{2}{1+\frac{T_L}{T_0}}\right]. \tag{10}$$

The calculations according to formula (10) were performed for the following input parameters: T_L = 293.15 K, T_0 = 291.15 K, P = 1013.25 hPa, e = 13.33 hPa (the assumed temperature values for a 1 km long trace correspond to the horizontal temperature gradient g_T = 0.002 K/m). It is shown that $\overline{n}_{acc}-\overline{n}_{emp}$ = $1.12\cdot10^{-9}$, which corresponds to a methodical component of the distance error of 0.00146 mm on a 1 km trace. Therefore, to such traces (with temperature profile close to a linear one) we can apply the empirical approach, when the average temperature value along the trace is determined and used to calculate the average integral index of the air refraction. For a 5 km long trace, the above error of 0.00146 mm is realized in case of a linear change of the air temperature along the trace with a smaller gradient of g_T = 0.0004 K/m.

In the second case, the estimation was performed for a non-uniform temperature profile $T(x)$ represented by a second degree symmetric polynomial

$$T(x) = T_0 - 4\frac{(T_0 - T_{L/2})\cdot x}{L} + 4\frac{(T_0 - T_{L/2})x^2}{L^2}, \qquad (11)$$

with extremum $T_{L/2} = T(x{=}L/2)$ at the middle point of the trace $x{=}L/2$ and the same air temperature values at the end points $T(x{=}0) = T(x{=}L) = T_0$. The conditions, under which a similar non-monotonic profile is realized in practice, are typical for measurements on an inhomogeneous underlying surface with land-water-land areas, but in some cases, it is also possible for traces passing only over land [4, 7, 8].

Formula (8), accounting for dependence $T(x)$ assumed according to (11), takes the form

$$\overline{n}_{acc} - \overline{n}_{emp} = (83.11\cdot 10^{-6}\cdot P - 11.4\cdot 10^{-6} e)\times$$
$$\times\frac{1}{T_{L/2}}\left[\frac{arctg\sqrt{\dfrac{T_0 - T_{L/2}}{T_{L/2}}}}{\sqrt{\dfrac{T_0 - T_{L/2}}{T_{L/2}}}} - \frac{1}{1+\dfrac{1}{3}\cdot\dfrac{T_0 - T_{L/2}}{T_{L/2}}}\right]. \qquad (12)$$

The estimation by formula (12) was performed for T_0 = 293.15 K, P = 1013.25 hPa, e = 13.33 hPa, $T_{L/2}$ = 291.15 K, i.e. for the conditions that are created for a 1 km long trace if the temperature at the end points decreases inversely in the magnitude by 0.004 K/m, which provides a 2 K temperature decrease at the middle point of the trace as compared to the temperature at the end points. For a 5 km long trace, such conditions are created by a non-monotonic temperature change with a much smaller modulus of its horizontal gradient of 0.0008 K/m. The resulting methodological error does not exceed $\overline{n}_{acc} - \overline{n}_{emp} = 1.2\cdot 10^{-9}$, which is close to the result obtained for a linear temperature profile.

Thus, accounting for the influence of the atmosphere, based on determining the average air temperature along the trace (the empirical approach), can be sufficiently accurate for both monotonic and non-monotonic nature of the refractive index dependence on the coordinate, counted along the trace under consideration. However, to decide on the possibility of its practical application, it should be considered that the analysis performed in this paper is based on model air temperature profiles. If real profiles differ from the model ones, then the results of estimations in the best case can only provide understanding of the magnitude order of the estimated methodological errors and should be specified for each individual case.

To obtain more accurate data, all calculations according to the methodology suggested in this paper should be done with real profiles. At the same time, the requirements for the number of intermediate temperature measuring points for specific traces can be established, providing the required accuracy of accounting for the influence of the atmosphere when using the empirical approach based on determining the average air temperature value along the trace.

3. Conclusions

A quantitative analysis of the accuracy of the empirical approach to account for the influence of the Earth's atmosphere on the results of distance measurements carried out using electromagnetic waves of the optical range on ground-level traces has been carried out. The approach under consideration assumes the determination of the mean integral refractive index of air from the average air temperature along the trace. The analysis is performed for both monotonic and non-monotonic air temperature dependences on the coordinate counted along the measured trace.

On the example of a monotone profile described by the linear law of temperature changes along the trace, it has been shown that for a 1 km long trace, the methodological component of the distance measurement error does not exceed 0.00146 mm for horizontal temperature gradients of the order of 0.002 K/km. For a 5 km trace, the above error of 0.00146 mm is realized in case of a linear air temperature change along the trace with a gradient of 0.0004 K/m.

Similar results were obtained for a non-monotonic profile described by a symmetric parabola with opposite temperature gradients at the end points equal in modulo to 0.004°K/km for a 1 km trace and 0.0008°K/km for a 5 km trace. Such profiles are close to those, which are realized for traces with a sharply no uniform underlying surface, in particular when there are land-water-land areas on this surface.

Thus, the approach under consideration for the abovementioned variants of monotonic and non-monotonic dependences of the air temperature on the coordinate along the trace provides a negligibly small component of the methodological error of distance measurements on traces up to 5 km long, which allows using it in cases when real temperature profiles are close to model ones.

If real profiles under experimental conditions differ significantly from those used in the analysis in this paper, it is advisable to perform a preliminary evaluation of the accuracy of the discussed empirical approach just for these real profiles before deciding on its use. The methods outlined in this paper can be applied for such an evaluation.

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Використання середньої температури повітря вздовж траси для атмосферної корекції в лазерній далекометрії: аналіз точності

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Анотація

Серед чинників, що суттєво впливають на результати лазерних далекомірних вимірювань на навколоземних трасах, особливо важливе значення має відмінність швидкості поширення світла в атмосфері Землі від швидкості світла у вакуумі. У рамках геометричної оптики неоднорідних середовищ ця відмінність враховується шляхом введення в результати вимірювань коригувальної поправки, яка виражається через середньоінтегральний показник заломлення повітря уздовж вимірюваної траси.

Останнім часом помітно зросла активність досліджень із метою підвищення точності визначення такої поправки. Ці дослідження увійшли до числа найактуальніших і найперспективніших наукових напрямів для геодезії та метрології. Водночас для врахування впливу атмосфери у геодезичній практиці часто використовують емпіричний підхід, який не має суворого обґрунтування і спирається на визначення середньоінтегрального показника заломлення з використанням середньої температури повітря вздовж вимірюваної траси.

У статті викладено результати аналізу точності цього емпіричного підходу. Аналіз виконано з урахуванням того, що згідно із загальноприйнятими формулами показник заломлення повітря нелінійно залежить від температури. Оцінено можливості застосування проаналізованого підходу з урахуванням необхідності забезпечення простежуваності в практиці лінійних вимірювань.

Ключові слова: вимірювання; точність; лазерний далекомір; врахування впливу атмосфери; температура повітря.

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