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Features of measurement uncertainty evaluation during calibration of digital ohmmeters

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Abstract

The scheme for transferring the size of the unit of resistance during the calibration of digital ohmmeters at direct current is considered. The procedure for the measurement uncertainty evaluation is described: recording the measurement model and its refinement, evaluation of input and measured values, evaluation of standard uncertainties of the input and measured values, evaluation of the expanded uncertainty. The refined model includes the dependence of the resistance of the reference resistor on temperature and a correction to the drift of the resistance value of the reference resistor since its last calibration. To evaluate the expanded uncertainty, the kurtosis method was used. An uncertainty budget has been made, including the kurtosis of input and measured values. The use of the Excel package makes it possible to implement, based on this budget, a program for automation of a digital ohmmeter of type 2318 at a point of 1 mOhm using an electrical resistance coil R310 with an accuracy class of 0.01 is considered. The influence of nonlinearity of the measurement model on the estimates of the numerical value of the measurand and its combined standard uncertainty is studied. To verify the results, the distribution law of the measurand was modelled by the Monte Carlo method. An algorithm for determining the expanded uncertainty Machine web application for the missing confidence level of 0.9545 is proposed. The comparison of the results of the measurement.

Keywords: digital ohmmeter; calibration; resolution; nonlinearity; measurement uncertainty; kurtosis method; uncertainty budget; Monte Carlo method.

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Introduction

Many digital ohmmeters (DOs) are used in Ukraine in various areas of the national economy and industry. To ensure the traceability of resistance measurement results with a reference for comparison, it is necessary to calibrate them. In this case, it is essential to evaluate the measurement uncertainty in accordance with the requirements of the international standard ISO/IEC 17025:20017 [1]. A reliable estimate of the expanded uncertainty cannot be obtained without accounting for the distribution laws of input quantities included in the measurement model, which is usually done by the Monte Carlo method [2]. For calibration goals, a reliable estimate of the expanded uncertainty can be obtained by the kurtosis method [3, 4]. The use of this method makes it possible to automate the calculation of the uncertainty, and the estimates of the expanded uncertainty will be close to the estimates obtained by the Monte Carlo method.

The purpose of the paper is to consider the features of the measurement uncertainty evaluation during the calibration of DOs.

1. Calibration model

During the calibration process, a direct measurement of the resistance of the reference resistor is carried out with the ohmmeter to be calibrated. The calibration scheme is shown in Fig. 1.



Fig. 1. Scheme of calibration of ohmmeter at direct current

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The simplest mathematical measurement model in this case has the following form [5]:

$$\Delta = \left(R_c + \Delta_c\right) - \left(R_s + \Delta_s\right),\tag{1}$$

where Δ is the systematic error of the ohmmeter at the calibration point; R_c is the resistance measured with the ohmmeter to be calibrated; Δ_c is the correction to the resolution of the ohmmeter to be calibrated; R_s is the resistance reproduced by the reference resistor; and Δ_s are the corrections to the drift of the value of the resistor since its last calibration.

The dependence of the resistance of the reference resistor on the calibration temperature of the ohmmeter t_c is determined by the following expression [6]:

$$R_{s} = R_{20} [1 + \alpha (t_{c} - 20) + \beta (t_{c} - 20)^{2}], \qquad (2)$$

where R_{20} is the resistance of the reference resistor at a temperature of 20 °C; α and β are its temperature coefficients of resistance.

Considering this dependence, the refined mathematical model (equation) of the measurement will have the following form:

$$\Delta = (R_c + \Delta_c) - [R_{20}[1 + \alpha(t_c - 20) + \beta(t_c - 20)^2] + \Delta_s].$$
(3)

2. Evaluation of input and measured quantities

An estimate of the resistance \hat{R}_c measured with the ohmmeter to be calibrated is found after a single measurement.

The mathematical expectation of the correction to the resolution of the ohmmeter to be calibrated is assumed to be zero: $\hat{\Delta}_c = 0$.

The value of the resistance \hat{R}_{20} reproduced by the reference resistor at 20 °C, as well as its temperature coefficients of resistance α and β , are taken from its calibration certificate.

The mathematical expectation of the correction associated with the drift of the resistance value of the reference resistor since its last calibration is taken equal to zero: $\hat{\Delta}_s = 0$.

Therefore, the estimate of the systematic error of the ohmmeter at the calibration point (bias) will be equal to:

$$\hat{\Delta} = \hat{R}_c - \hat{R}_{20} [1 + \hat{\alpha} (\hat{t}_c - 20) + \hat{\beta} (\hat{t}_c - 20)^2].$$
(4)

3. Evaluation of standard uncertainties of input and measured quantities

The standard uncertainty of the correction to the resolution d of the ohmmeter to be calibrated is estimated assuming its uniform distribution within the limits $\pm d/2$, as:

$$u_B(\Delta_c) = \frac{d}{2\sqrt{3}}.$$
 (5)

The standard uncertainty of the resistance value reproduced by the reference resistor $u_B(R_{20})$ at 20 °C is found from the value of the expanded uncertainty U_{20} and coverage factor k_{20} specified in its calibration certificate:

$$u_B(\hat{R}_{20}) = \frac{U_{20}}{k_{20}}.$$
 (6)

The standard uncertainty associated with the drift Δd of the value of the reference resistor since its last calibration is evaluated assuming its uniform distribution within the limits $\pm \delta_s$, as:

$$u(\hat{\Delta}_s) = \frac{\delta_s \cdot \hat{R}_{20}}{\sqrt{3} \cdot 100\%},\tag{7}$$

where δ_s is permissible limits for the relative deviation of the resistance of the reference resistor from the value obtained during the previous calibration.

The standard uncertainty associated with the inaccuracy of providing the specified temperature of the reference resistor \hat{t}_c is evaluated assuming its uniform distribution within the limits $\pm \theta_t$ as follows:

$$u_B(\hat{t}_c) = \frac{\theta_t}{\sqrt{3}}.$$
(8)

Since the quantities in equation (3) are not correlated, the standard measurement uncertainty during the ohmmeter calibration will be calculated as follows:

$$u_{c}(\hat{\Delta}) = \sqrt{\frac{u_{B}^{2}(\hat{\Delta}_{c}) + c_{20}^{2}u_{B}^{2}(\hat{R}_{20}) + u_{B}^{2}(\hat{\Delta}_{s}) + c_{t}^{2}u_{B}^{2}(\hat{t}_{c})},$$
(9)

where c_{20} is the sensitivity coefficient of the measured value to the change in resistance R_{20} :

$$c_{20} = \frac{\partial \Delta}{\partial R_{20}} = 1 + \alpha (t_c - 20) + \beta (t_c - 20)^2; \quad (10)$$

 c_t is the sensitivity coefficient of the measurand value to the change in the calibration temperature t_c :

$$c_t = \frac{\partial \Delta}{\partial t_c} = -R_{20}[\alpha + 2\beta(t_c - 20)]. \tag{11}$$

Since dependence (3) is nonlinear relatively to t_c , the values $\hat{\Delta}$ and $u_c(\hat{\Delta})$ obtained using expressions (4) and (9) must be calculated with corrections [4, 7, 8]:

$$\delta_t = -\frac{1}{2} \frac{\partial^2 \Delta}{\partial t_c^2} u^2(\hat{t}_c) = \beta R_{20} \frac{\theta_t^2}{3}; \qquad (12)$$

$$\delta[u_c^2(\Delta)] = \frac{1}{4} \left(\frac{\partial^2 \Delta}{\partial t_c^2}\right)^2 u^4(\hat{t}_c)(\eta+2) = \delta_t^2(\eta+2), \quad (13)$$

where $\eta = -1.2$ is the kurtosis of the temperature distribution t_c within the limits $\pm \theta_t$.

The correction δ_i can be neglected if the following inequality is observed [7]:

$$\delta_{t} < \frac{1}{3} \sqrt{u_{c}^{2}(\hat{\Delta}) + \delta[u_{c}^{2}(\hat{\Delta})]}.$$
(14)

Features of measurement uncertainty evaluation during calibration of digital ohmmeters

X _i	X_{i}	$u(x_i)$	η	\mathcal{C}_{i}	$u_i(y)$
R _c	\hat{R}_c	-	-	1	-
Δ_c	0	(5)	-1.2	1	$u_{B}(\Delta_{c})$
R ₂₀	$\hat{R_s}$	(6)	0	(10)	$c_{20}u~(\hat{R}_{20})$
Δ_{s}	0	(7)	-1.2	-1	$-u_{\scriptscriptstyle B}(\hat{\Delta}_{\scriptscriptstyle S})$
t _c	\hat{t}_c	(8)	-1.2	(11)	$-c_t u_B(\hat{t}_c)$
Y	У	$u_c(y)$	η	k	U
Δ	(4)	(9)	(16)	(17)	(18)

Measurement uncertainty budget during ohmmeter calibration

The correction $\delta[u_c^2(\Delta)]$ can be neglected if the following inequality is observed [8]:

$$\delta[u_c^2(\hat{\Delta})] \Big| < \frac{1}{9} u_c^2(\hat{\Delta}). \tag{15}$$

4. Evaluation of expanded measurement uncertainty

Since all the contributions of the uncertainty included in expression (9) are evaluated according to type B, it is advisable to calculate the expanded uncertainty by the kurtosis method [3, 4], which makes it possible to obtain an estimate of the expanded uncertainty accounting for the distribution laws of input quantities.

Considering the fact that the kurtosis for the normal distribution law is equal to zero and for the uniform distribution is equal to -1.2, the kurtosis of the measurand will be equal to:

$$\eta = -1.2 \frac{u_B^4(\hat{\Delta}_c) + c_{20}^4 u_B^4(\hat{R}_{20}) + u_B^4(\hat{\Delta}_s) + c_i^4 u_B^4(\hat{t}_c)}{u_c^4(\hat{\Delta})}.$$
 (16)

Then the coverage factor for the confidence level of 0.9545 will be calculated by the following formula:

$$k(\eta) = \begin{cases} 0.12\eta^3 + 0.1\eta + 2, \text{ at } \eta < 0; \\ 2, \text{ at } \eta \ge 0, \end{cases}$$
(17)

and the expanded uncertainty will be found as follows:

$$U = k(\eta) \cdot u_c(\Delta), \tag{18}$$

where $u_c(y)$ is the combined standard uncertainty calculated by formula (9).

The uncertainty budget is presented in Table 1.

5. An example of measurement uncertainty evaluation during digital ohmmeter calibration

At a temperature of 20 °C, a digital ohmmeter of type 2318 is calibrated at a point of 1 mOhm using an electric resistance coil P310 with an accuracy class of 0.01,

from the calibration certificate of which the following data are taken: the real value $\hat{R}_s = 0.9998$ mOhm; the expanded uncertainty of the resistance reproduction by the coil P310 $U_s = 10^{-4}$ mOhm, the coverage factor $k_s = 2$ for the confidence level of 0.9545; and temperature coefficients of resistance $\alpha = 4.6 \times 10^{-6}$ 1/°C and $\beta = -0.39 \times 10^{-6}$ 1/°C². The coil resistance drift since the last calibration δ_s is not more than 0.002%; the limits of the ambient temperature change during the calibration are $\theta_t = \pm 2$ °C. The readings of DOs during the calibration are $R_c = 0.999$ mOhm.

Table 1

For these initial data, the standard uncertainty of the resolution correction d=0.0001 mOhm of the ohmmeter being calibrated will be determined from expression (5):

$$u_B(\Delta_c) = \frac{d}{2\sqrt{3}} = \frac{0.001}{2\sqrt{3}} = 2.89 \cdot 10^{-4} \text{ mOhm.}$$

The standard uncertainty of the resistance value reproduced by the reference resistor $u_B(R_{20})$ is found by formula (5):

$$u_B(\hat{R}_{20}) = \frac{U_{20}}{k_{20}} = \frac{10^{-5}}{2} = 5 \cdot 10^{-6}$$
 mOhm.

The standard uncertainty associated with the drift of the resistance value of the reference resistor since its last calibration $u(\Delta_s)$ is estimated by formula (6):

$$u(\Delta_d) = \frac{\delta_d \cdot \hat{R}_s}{\sqrt{3} \cdot 100\%} = \frac{0.002 \cdot 0.9998}{\sqrt{3} \cdot 100} = 1.154 \cdot 10^{-5} \text{ mOhm.}$$

The standard uncertainty associated with the deviation of the ambient temperature during the calibration of the ohmmeter $u_B(t_c)$ from the declared one is estimated by formula (8):

$$u_B(t_c) = \frac{\theta_t}{\sqrt{3}} = \frac{2}{\sqrt{3}} = 1.154$$
 °C.

Sensitivity coefficients c_{20} and c_t calculated by formulas (10) and (11) will be equal to:

$$c_{20} = \frac{\partial \Delta}{\partial R_{20}} = -[1 + \alpha(t_c - 20) + \beta(t_c - 20)^2] = -[1 + 4.6 \cdot 10^{-6} (20 - 20) - 0.39 \cdot 10^{-6} (20 - 20)^2] = -1;$$

$$c_t = \frac{\partial \Delta}{\partial t_c} = -R_{20}[\alpha + 2\beta(t_c - 20)] = -0.9998[4.6 \cdot 10^{-6} - 2 \cdot 0.39 \cdot 10^{-6} (20 - 20)] = -4.6 \cdot 10^{-6} \text{ mOhm /°C.}$$

Measurement uncertainty budget when calibrating DOs of type 2318 at 1 mOhm point

X _i	x _i	$u(x_i)$	η	<i>C</i> _{<i>i</i>}	$u_i(y)$, mOhm
R _c	0.999 mOhm	-	-	1	-
Δ_c	0 mOhm	2.89×10 ⁻⁴ mOhm	-1.2	1	2.89×10-4
R ₂₀	0.9998 mOhm	5×10 ⁻⁵ mOhm	0	-1	-5×10 ⁻⁵
Δ_s	0 mOhm	1.154×10 ⁻⁵ mOhm	-1.2	-1	-1.15×10-5
t _c	20 °C	1.154 °C	-1.2	-4.6×10 ⁻⁶ mOhm /°C	-5.31×10-6
Y	y, mOhm	$u_c(y)$, mOhm	η	k	U, mOhm
Δ	-0.0008	2.933×10-4	-1.127	1.716	0.000503

The standard measurement uncertainty during the ohmmeter calibration by formula (9) will be:

$$u_{c}(\hat{\Delta}) = \sqrt{u_{B}^{2}(\hat{\Delta}_{c}) + c_{20}^{2}u_{B}^{2}(\hat{R}_{20}) + u_{B}^{2}(\hat{\Delta}_{s}) + c_{t}^{2}u_{B}^{2}(\hat{t}_{c})} =$$
$$= \sqrt{(2.89 \cdot 10^{-4})^{2} + (5 \cdot 10^{-5})^{2} + (1.154 \cdot 10^{-5})^{2} + (1.154 \cdot 4.6 \cdot 10^{-6})^{2}} =$$

The kurtosis of the measurand calculated by formula (16) will be equal to:

$$\eta = -1.2 \frac{u_B^4(\hat{\Delta}_c) + c_{20}^4 u_B^4(\hat{R}_{20}) + u_B^4(\hat{\Delta}_s) + c_t^4 u_B^4(\hat{t}_c)}{u_c^4(\hat{\Delta})} =$$
$$= \frac{(2.89 \cdot 10^{-4})^4 + (5 \cdot 10^{-5})^4 + (1.15 \cdot 10^{-5})^4 + (5.31 \cdot 10^{-6})^4}{(2.94 \cdot 10^{-4})^4} = -1.127.$$

Then the coverage factor for the confidence level of 0.9545 calculated by formula (17) will be:

$$k(\eta) = 0.12\eta^{3} + 0.1\eta + 2 = 0.12 \cdot (-1.127)^{3} + 0.1 \cdot (-1.127) + 2 = 1.716,$$

and the expanded uncertainty calculated by formula (18) will be equal to:

$$U = k(\eta) \cdot u_c(y) = 1.716 \cdot 2.93 \cdot 10^{-4} = 5.03 \cdot 10^{-4} \text{ mOhm.}$$

The measurement uncertainty budget during the calibration of DOs at a point of 1 mOhm is given in Table 2.

To study the influence of nonlinearity of the measurement model on the estimates of the numerical value of the measurand and its combined standard uncertainty, corrections to these quantities were calculated using formulas (12) and (13) respectively:

$$\delta_t = \beta R_{20} \frac{\theta_t^2}{3} = -0.39 \cdot 10^{-6} \cdot 0.9998 \frac{4}{3} = -0.52 \cdot 10^{-6} \text{ mOhm};$$

$$\delta[u_c^2(\hat{\Delta})] = \delta_t^2 \cdot (\eta + 2) = (0.52 \cdot 10^{-6})^2 (-1.2 + 2) = 0.416 \cdot 10^{-12} \text{mOhm}^2.$$

The correction to the obtained numerical value of the measurand leads to the following result:

 $\hat{\Delta}_0 = -0.0008 + 0.52 \cdot 10^{-6} = -0.00079948$ mOhm.

The correction to the resulting value of the combined standard uncertainty leads to the following result:

$$u_{c0}(\hat{\Delta}) = \sqrt{u_c^2(\hat{\Delta}) + \delta[u_c^2(\hat{\Delta})]} = \sqrt{0.0002933^2 + 0.41610^{-12}} = 0.0002933 \text{ mOhm}^2.$$

Using the criteria for neglecting the correction given in (14) and (15) the following is obtained:

$$\begin{aligned} \left| \delta_t \right| &= 0.52 \cdot 10^{-6} < \frac{1}{3} \sqrt{u_c^2(\hat{\Delta}) + \delta[u_c^2(\hat{\Delta})]} = 0.98 \cdot 10^{-5}, \\ \left| \delta[u_c^2(\hat{\Delta})] \right| &= 0.416 \cdot 10^{-12} < \frac{1}{9} u_c^2(\hat{\Delta}) = 0.953 \cdot 10^{-11}. \end{aligned}$$

That is, corrections to the numerical value of the measurand and its combined standard uncertainty can be neglected.

To verify the obtained results, the distribution law of the measured quantity by the Monte Carlo method was modelled [2, 9]. The distribution law of the measurand is shown in Fig. 2. The following parameters of this law were obtained: mathematical expectation -0.0007996mOhm and standard deviation 0.0002932mOhm. The values of the expanded uncertainties for different levels of confidence are presented in Table 3.

Table 3

Expanded measurement uncertainties during calibration of DOs of type 2318 for different levels of confidence p

Trust level p	Expanded uncertainty, mOhm
0.99	0.0005475
0.95	0.0004915
0.9	0.0004555
0.68	0.0003415

Український метрологічний журнал, 2023, № 2, 22-27 25



Fig. 2. The law distribution of the measurand during calibration of digital ohmmeter of type 2318 at 1 mOhm obtained by the Monte Carlo method

To implement the Monte Carlo method, the NIST Uncertainty Machine [5] web application was used, which, unfortunately, does not consider the expanded uncertainty for the confidence level p=0.9545. Therefore, based on the obtained values of the expanded uncertainty $U_{0.99}$, $U_{0.95}$ and $U_{0.9}$ (Table 3), a quadratic approximating dependence of the form $U_p=ap^2+bp+c$ was calculated.

For this, a system of equations was written:

$$\begin{cases} U_{0.99} = a \cdot 0.99^2 + b \cdot 0.99 + c; \\ U_{0.95} = a \cdot 0.95^2 + b \cdot 0.95 + c; \\ U_{0.9} = a \cdot 0.9^2 + b \cdot 0.9 + c. \end{cases}$$

The solution to this system is:

$$a = (2.5 \cdot U_{0.99} - 4.5 \cdot U_{0.95} + 2 \cdot U_{0.9})/0.009;$$

$$b = (U_{0.99} - U_{0.95} - a \cdot 0.0776)/0.04;$$

$$c = U_{0.95} - a \cdot 0.95^2 - b \cdot 0.95.$$

For the values of the expanded uncertainty given in Table 2, $\hat{a} = 0.00755556$; $\hat{b} = -0.0132578$; $\hat{c} = 0.0062675$. were obtained.

Therefore, for the confidence level p=0.9545 the following is obtained:

 $U_{0.9545} = \hat{a} \cdot 0.9545^2 + \hat{b} \cdot 0.9545 + \hat{c} = 0.000497$ mOhm.

The coverage factor for this case is:

$$k = \frac{U_{0.9545}}{u_c(\hat{\Delta})} = \frac{0.000497}{0.0002932} = 1.695.$$

The obtained values of the expanded uncertainty and the coverage factor coincide with the values obtained by the kurtosis method.

Conclusions

1. The refined measurement model during the calibration of the digital ohmmeter provides the opportunity to perform calibrations in the temperature range of (15-30) °C for coils of class 0.01 and in the range (10-35) °C for coils of class 0.02 since it includes the resistance dependence of the reference resistor on temperature.

2. Since the refined measurement model is nonlinear, the bias of the estimate of the numerical value of the measurand and its combined standard uncertainty should be estimated.

2. Since all the uncertainty contributions included in the measurement model are evaluated according to type B, it is advisable to calculate the expanded uncertainty by the kurtosis method, which allows obtaining an estimate of the expanded uncertainty accounting for the distribution laws of input quantities and automation of the process of its calculation.

3. To verify the obtained results, the distribution law of the measured value by the Monte Carlo method was modelled, which showed good agreement between the results obtained by the kurtosis method.

4. To determine the expanded uncertainty by the NIST Uncertainty Machine web application for the missing confidence level of 0.9545, the algorithm obtained in the process of work should be used.

Особливості оцінювання невизначеності вимірювання при калібруванні цифрових омметрів

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Анотація

Розглянуто схему передавання розміру одиниці опору при калібруванні цифрових омметрів за постійного струму. Описано процедуру оцінювання невизначеності вимірювань: запис моделі вимірювань та її уточнення, оцінювання вхідних та вимірюваної величин, оцінювання стандартних невизначеностей вхідних та вимірюваної величин, оцінювання стандартних невизначеностей вхідних та вимірюваної величин, оцінювання розширеної невизначеності. Уточнена модель включає залежність опору еталонного резистора від температури й поправку на дрейф значення опору еталонного резистора з моменту його останнього калібрування. Для оцінювання розширеної невизначеності застосовано метод ексцесів. Складено бюджет невизначеності, що включає ексцеси вхідних та вимірюваної величин. Використання пакета Ехсеl дозволяє реалізувати на основі цього бюджету програму для автоматизації обчислень невизначеності вимірювань. Розглянуто приклад оцінювання невизначеності вимірювань при калібруванні цифрового омметра типу 2318 у точці 1 мОм за допомогою котушки електричного опору Р310 класом точності 0,01. Досліджено вплив нелінійності моделі вимірювання на оцінки числового значення вимірюваної величини та його сумарної стандартної невизначеності. Для верифікації отриманих результатів було проведено моделювання закону розподілу вимірюваної величини методом Монте-Карло. Запропоновано алгоритм визначення розширеної невизначеності вебдодатком NIST Uncertainty Machine для відсутнього в ній рівня довіри 0,9545. Порівняння результатів оцінювання невизначеності вимірювань методами ексцесів і Монте-Карло показало їхній хороший збіг.

Ключові слова: цифровий омметр; калібрування; роздільна здатність; нелінійність; невизначеність вимірювань; метод ексцесів; бюджет невизначеності; метод Монте-Карло.

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