UDC 616-093

Simulation of temperature dependence measurements of resistance measures by SVD

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Abstract

One of the important characteristics of high-precision resistance measures is the temperature dependence of resistance. Since this dependence is generally non-linear, it is most often approximated by a second-degree polynomial.

Polynomial coefficients: resistance of the measure at a reference temperature ($T_0 = 20\,°C$ or $23\,°C$) R_0, a coefficient characterizing the linear dependence of resistance on temperature α, and a coefficient characterizing the quadratic dependence of resistance on temperature β are determined experimentally by measuring the resistance of the measure R_T at different temperatures T and by solving the resulting system of equations.

To increase accuracy, multiple measurements are performed, which results in a redefined system of equations allowing solutions to be found by various methods.

The paper considers the solution of a redefined system of linear equations using the SVD (*singular value decomposition*) method, if the inaccuracy of measurements of R_T and T is caused by random factors. To simulate random factors, random values distributed according to the normal law were used.

The SVD method was implemented using the MATLAB software package.

The paper presents some results from simulating the process of measuring the temperature dependence of resistance measures.

Keywords: simulation; measurement; resistance measures; temperature dependence; singular value decomposition (SVD).

Received: 07.12.2023 Edited: 21.12.2023 Approved for publication: 26.12.2023

1. Introduction

One of the important characteristics of reference resistors is the dependence of their resistances on temperature. The temperature dependence of the resistance of electrical measures is usually expressed as, e.g. [1–4]:

$$R_T = R_0 \cdot \left[1 + \alpha\left(T - T_0\right) + \beta\left(T - T_0\right)^2\right], \qquad (1)$$

where R_T, R_0 is the resistance of measure at temperature T and at reference temperature T_0, respectively; α and β are the temperature coefficients of resistance.

When experimentally finding R_0, α, β, measurements of resistance R_T are performed at three or more temperature values, and the resulting system from equations (1) is solved.

In papers [5, 6] some aspects of the measurement of temperature coefficients are considered, and the obtained estimates of the measurement uncertainty based on the GUM guidelines [7] are analysed.

Given the fact, in this paper, we will consider the influence of random variation in the results of measuring the resistance of the measure and the results of measuring the temperature in the thermostat where the measure under consideration is placed.

2. Presentation of the main material

To average the influence of random factors, multiple measurements are usually performed. In the task, this means that a series of measurements are performed at several temperatures.

We will analyse the influence of random factors on determining the values of R_0, α and β by simulating the solution to equation (1) for given values of R_T and a certain set of temperatures T.

In this case, we take the given values R_{0in}, α_{in}, β_{in} and a certain set of temperatures T_{in} as the initial values. Under this condition, unperturbed values of R_{Tin} can be obtained from (1).

To simulate a random variation of R_T and T we assume that the deviations from the initial values of R_{Tin} and T_{in} are random variables with a normal distribution law, a zero mean and a given standard deviation of StD(δT) and StD(δR_T), respectively.

When using multiple measurements, the system of equations (1) turns out to be redefined, and to solve it, the MATLAB program-numerical computing platform was used. Moreover, if the system of equations (1) is presented in matrix form

$$\boldsymbol{Ax}=\boldsymbol{B}, \qquad (2)$$

then the solution (2) can be found by the singular value decomposition (SVD) [8, 9]:

$$\boldsymbol{x} = \boldsymbol{V}^{*}\mathbf{pinv}(\boldsymbol{S})^{*}\boldsymbol{U}'^{*}\boldsymbol{B}, \qquad (3)$$

where $\boldsymbol{U}$, $\boldsymbol{S}$, $\boldsymbol{V}$ are obtained from [$\boldsymbol{U,S,V}$] = svd($\boldsymbol{A}$).

By the magnitude of the singular values (s = =svd($\boldsymbol{A}$)) obtained by the SVD, it is possible to assess the validity of the solutions obtained.

The simulation was realised as follows:

1) The initial values R_{0in}= 1; T_0= 23 °C; $\Delta T_{in} = T - T_0 = \{0,-1,-2,-3\}$°C were chosen.

2) The initial α_{in} and β_{in} values were selected from the following ranges: $\alpha_{in} \in$ [0, 1E-05]; $\beta_{in} \in$ [-3E-06, 1E-07]. From equation (1), the initial values of $R_{Tin} = f(R_{0in}, \alpha_{in}, \beta_{in}, \Delta T_{in})$ were calculated.

3) Inaccuracies of measurements of $(T-T_0)$ and R_T were simulated by set of random variables δT and δR_T with normal distribution, zero means and given standard deviations of StD(δT) and StD(δR_T), respectively.

4) In the simulation, it was assumed that each m-th multiple measurement consists of 25 repeated measurements at four temperature values ΔT_{in}. The values R_{0m}, α_m and β_m were calculated using the SVD method.

For convenience, equation (1) is transformed into the form:

$$x_1 + (\Delta T_{in} + \delta T)\cdot x_2 + (\Delta T_{in} + \delta T)^2 \cdot x_3 = (R_{Tin} + \delta R_T), \quad (4)$$

where $x_1 = R_{0m}$, $x_2 = \alpha_m \cdot R_{0m}$, $x_3 = \beta_m \cdot R_{0m}$, and R_{0m}, α_m, β_m, which is the result of m-th multiple measurement.

Since one multiple measurement consists of 25 repeated measurements at each of the four temperature set points, then to obtain the measurement result, it is necessary to solve a system of one hundred equations:

$$\begin{pmatrix} 1 & (0+\delta T_1) & (0+\delta T_1)^2 \\ 1 & (-1+\delta T_2) & (-1+\delta T_2)^2 \\ 1 & (-2+\delta T_3) & (-2+\delta T_3)^2 \\ 1 & (-3+\delta T_4) & (-3+\delta T_4)^2 \\ 1 & (0+\delta T_5) & (0+\delta T_5)^2 \\ 1 & (-1+\delta T_6) & (-1+\delta T_6)^2 \\ 1 & (-2+\delta T_7) & (-2+\delta T_7)^2 \\ 1 & (-3+\delta T_8) & (-3+\delta T_8)^2 \\ \ldots & \ldots & \ldots \\ 1 & (0+\delta T_{97}) & (0+\delta T_{97})^2 \\ 1 & (-1+\delta T_{98}) & (-1+\delta T_{98})^2 \\ 1 & (-2+\delta T_{99}) & (-2+\delta T_{99})^2 \\ 1 & (-3+\delta T_{100}) & (-3+\delta T_{100})^2 \end{pmatrix} \times \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} (R_{Tin1}+\delta R_{T1}) \\ (R_{Tin2}+\delta R_{T2}) \\ (R_{Tin3}+\delta R_{T3}) \\ (R_{Tin4}+\delta R_{T4}) \\ (R_{Tin1}+\delta R_{T5}) \\ (R_{Tin2}+\delta R_{T6}) \\ (R_{Tin3}+\delta R_{T7}) \\ (R_{Tin4}+\delta R_{T8}) \\ \ldots \\ (R_{Tin1}+\delta R_{T97}) \\ (R_{Tin2}+\delta R_{T98}) \\ (R_{Tin3}+\delta R_{T99}) \\ (R_{Tin4}+\delta R_{T100}) \end{pmatrix}, \qquad (5)$$

in which sets of one hundred random numbers $\{\delta T_1, \ldots, \delta T_{100}\}$ and the same amount of random numbers $\{\delta R_{T1}, \ldots, \delta R_{T100}\}$ are introduced.

Based on the solution of the system of equations (5), the m-th multiple measurement result is expressed as $R_{0m} = x_1$; $\alpha_m = x_2 / R_{0m}$; $\beta_m = x_3 / R_{0m}$.

Random numbers δT and δR_T with a normal distribution law were generated by a generator built into the MATLAB platform. The standard deviation varied within the following limits: StD(δT) $\in$ [0, 5E-02] and StD(δR_T) $\in$ [0, 1E-07].

5) To obtain the statistical characteristics of the measurement results, the solution of the system (5) was repeated N_r = 1200 times (m = 1, 2, ..., 1200) for each set of initial values R_{0in}, α_{in}, β_{in}, ΔT_{in}, and given values of StD(δT), StD(δR_T).

The results of simulation

Some simulation results are presented in the following tables and graphs, where R_{0in} = 1. Average values were calculated as the arithmetic mean with N_r repetitions.

Table 1

Average values and Standard Deviations of R_{0m}, α_m, and β_m with N_r repetitions

Row	The initial data					Average values with N_r repetitions			Standard Deviations with N_r repetitions		
	α_{in}, 1/°C	β_{in}, 1/(°C)^2	δR_T	δT, °C	N_r	$(R_{0Nr}$ / R_{0in}-1), 10^{-8}	$(\alpha_{Nr}$ / α_{in} -1), %	$(\beta_{Nr}$ / β_{in} -1), %	StD(R_{0m}), 10^{-8}	StD(α_m), %	StD(β_m), %
1	6E-06	-6.7E-07	0	0	120	-1E-07	-2E-08	-1E-08	0	0	0
2	0	-1E-07	1E-08	0	1200	0.003	0.001× ×10^{-7}*	0.02	0.187	0.032× ×10^{-7}*	1.02
3	1E-07	0E+00	1E-08	0	1200	0.002	0.08	0.0003× ×10^{-7}*	0.193	3.15	0.010× ×10^{-7}*
4	1E-07	-1E-07	0	0.05	1200	0.032	2.7	-1.13	0.139	5.66	2.21
5	1E-07	-1E-07	1E-08	0	1200	0.003	0.05	-0.015	0.194	3.15	1.005
6	1E-07	-1E-07	1E-08	0.02	1200	0.013	0.58	-0.21	0.203	3.87	1.33
7	1E-07	-1E-07	1E-08	0.05	1200	0.032	2.92	-1.23	0.228	6.61	2.57
8	1E-06	-1E-06	2E-08	0.02	1200	0.058	0.5	-0.2	0.645	2.417	0.97
9	1E-06	-1E-06	1E-08	0.05	1200	0.31	2.84	-1.22	1.42	5.86	2.318
10	6E-06	-3E-06	4E-08	0.02	1200	0.04	0.238	-0.21	2.76	1.44	1.14
11	6E-06	-3E-06	1E-07	0.05	1200	0.33	1.35	-1.15	1.17	3.55	2.67
12	1E-05	-1E-06	2E-08	0.02	1200	-0.38	0.018	-0.18	4.15	0.78	2.65
* – the value is given in absolute units											

According to Table 1, the nonlinearity of equation (1) and the presence of random components when measuring R_T and T can lead to quite significant bias in the result of measurements of α and β.

In addition, the results of the study show that the correlation of the obtained values R_{0m} with the values of α_m and β_m is less than the correlation between α_m and β_m. Thus, within the framework of the conducted research, the following correlation coefficients were obtained: $r(R_{0m}, \alpha_m) = 0.5–0.7$; $r(R_{0m}, \beta_m) = 0.4–0.5$; $r(\alpha_m, \beta_m) = 0.95–0.96$.

Fig. 1–6 show the convergence of average values and standard deviations when N_r changes from 10 to 1200 with a step of 10 for 12 implementations of the simulating process, i.e. N_r = 10, 20, ..., 1200. In this case, the initial data correspond to row 12 of Table 1.

From the above graphs, it follows that the convergence of measurements is quite slow: a "steady-state" average value is achieved only after 100–300 repetitions of multiple measurements from one hundred observations.

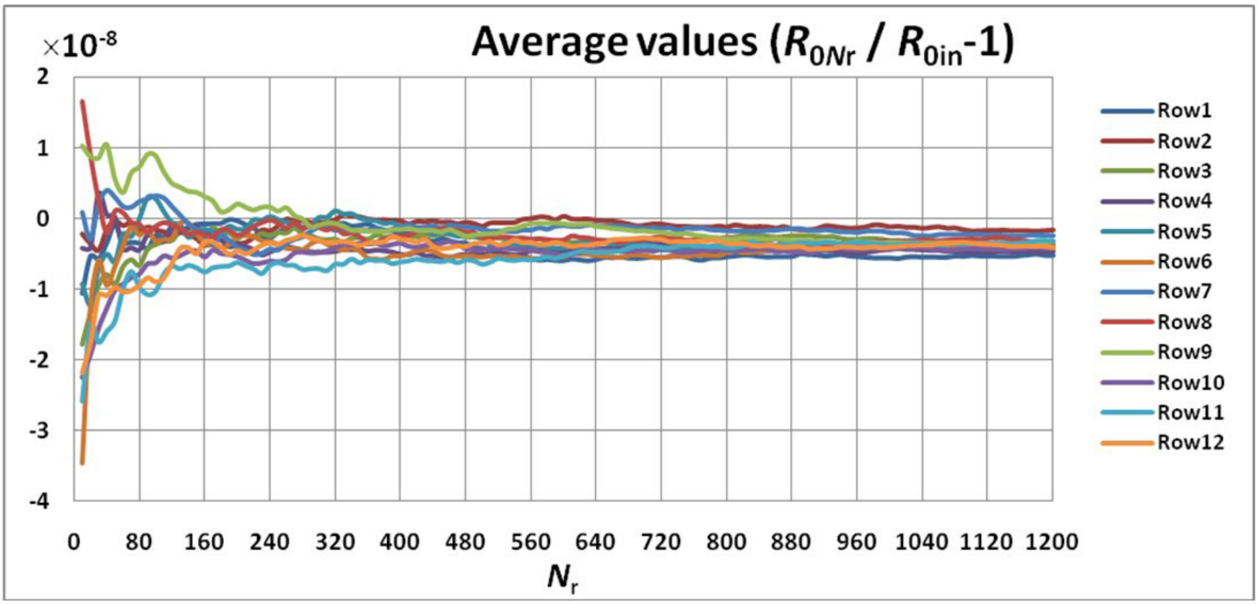

Fig. 1. Dependence of the average values R_{0Nr} on the number of repetitions N_r

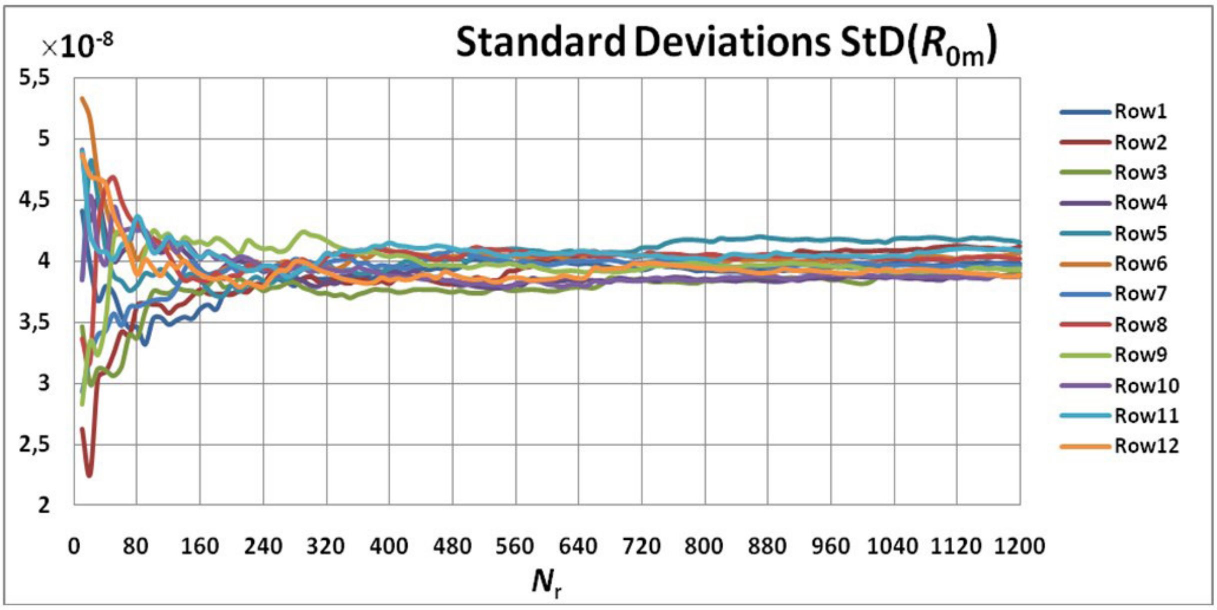

Fig. 2. Dependence of the Standard Deviations $\mathrm{StD}(R_{0m})$ on the number of repetitions N_r

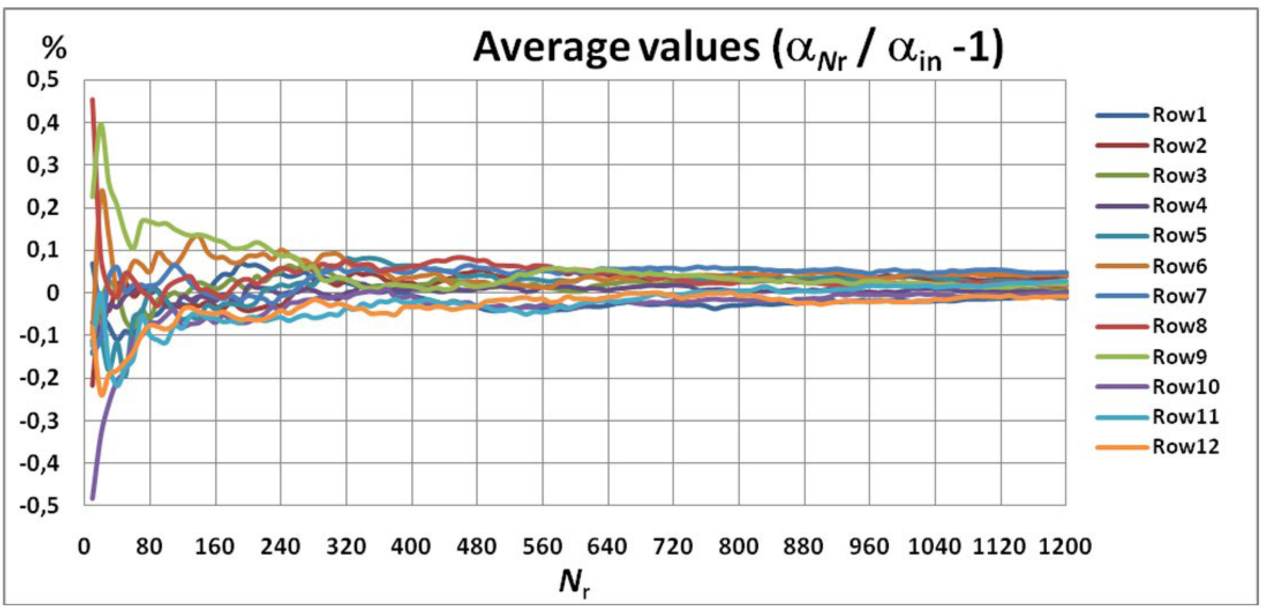

Fig. 3. Dependence of the average values α_{Nr} on the number of repetitions N_r

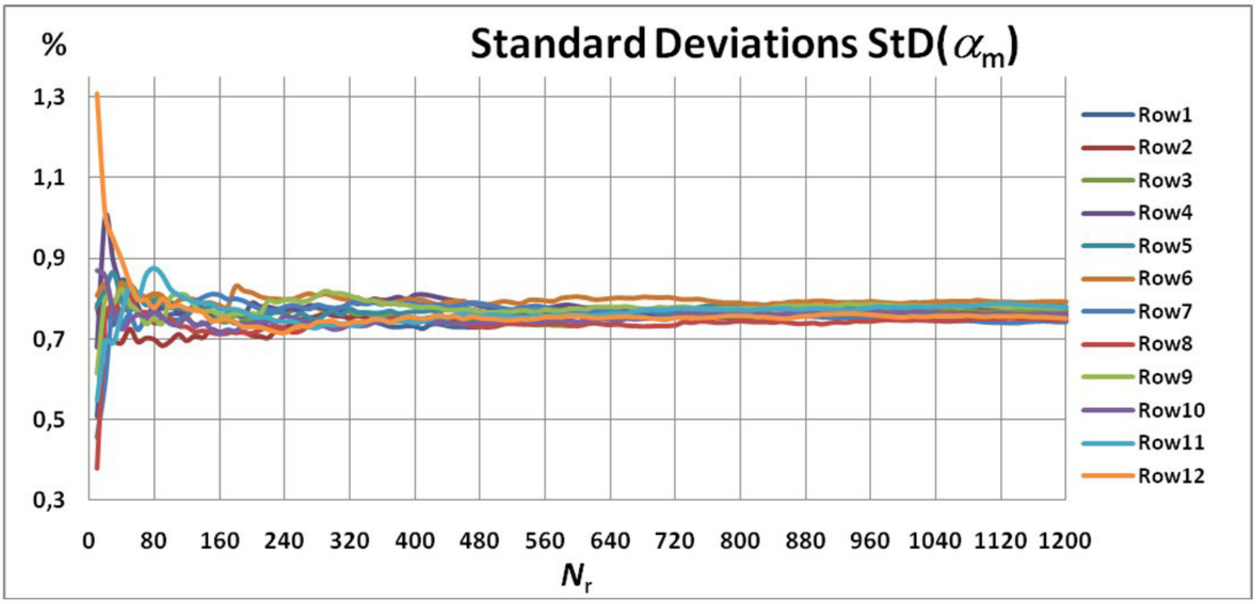

Fig. 4. Dependence of the Standard Deviations StD(α_m) on the number of repetitions N_r

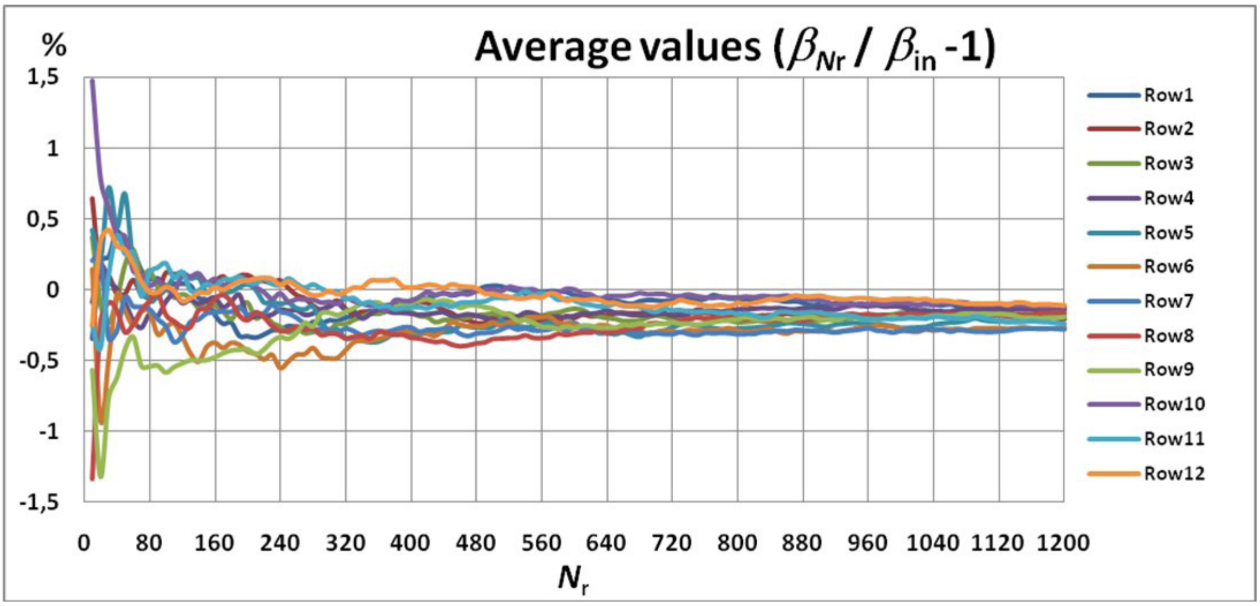

Fig. 5. Dependence of the average values β_{Nr} on the number of repetitions N_r

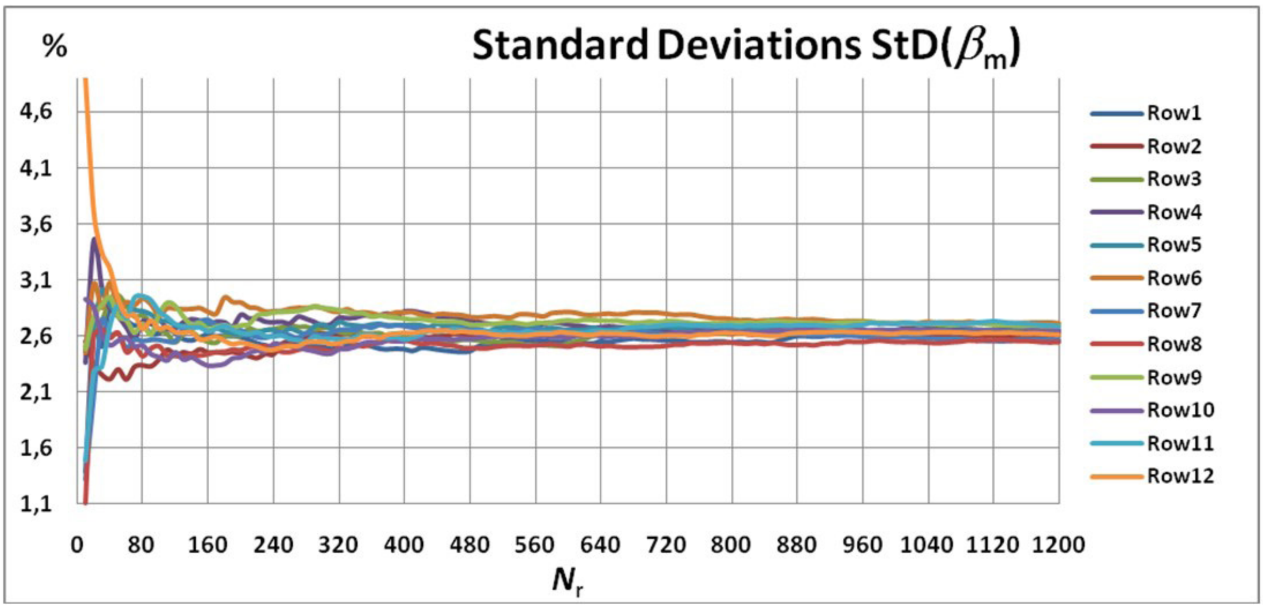

Fig. 6. Dependence of the Standard Deviations StD(β_m) on the number of repetitions N_r

Conclusions

The obtained simulation results can be used for:

• planning and performing measurements of the temperature dependence resistance of resistors;

• the measurement uncertainty evaluation, both in determining the temperature coefficient of resistance and the uncertainty of measurements performed using resistance measures.

Моделювання вимірювань температурної залежності мір опору за допомогою SVD

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Анотація

Однією з важливих характеристик високоточних мір опору є температурна залежність опору. Оскільки ця залежність, як правило, нелінійна, її найчастіше апроксимують поліномом другого ступеня.

Поліноміальні коефіцієнти: опір міри при опорній температурі (20 °С або 23 °С) R_0; коефіцієнт, що характеризує лінійну залежність опору від температури, α, та коефіцієнт, що характеризує квадратичну залежність опору від температури, β, – знаходять експериментально шляхом вимірювання опору міри R_T при різних температурах T і вирішення отриманої системи рівнянь.

Для підвищення точності проводять багаторазові вимірювання, що призводить до перевизначеної системи рівнянь, рішення якої можуть знаходитись різними методами. Одним з варіантів вирішення перевизначеної системи є застосування сингулярного розкладу матриці – так званого методу SVD (*singular value decomposition*).

У статті досліджено застосування SVD-методу при вимірюванні температурної залежності мір опору за умови, що неточність вимірювань R_T і T зумовлена випадковими факторами. Проведено машинний експеримент, у якому для моделювання випадкових факторів використовувалися випадкові величини, розподілені за нормальним законом. Реалізація SVD-методу здійснювалась за допомогою пакету прикладних програм MATLAB.

Моделювання проводилось за умови, що температурні коефіцієнти знаходяться у таких діапазонах: $\alpha \in$ [0, 1E-05]; $\beta \in$ [-3E-06, 1E-07], а цикл вимірювання температурної залежності проводиться за чотирьох значень температури від 23 °С до 20 °С із багаторазовим вимірюванням, що складається з 25 спостережень, R_T та T при кожному значенні T. Кожен цикл повторювався 1200 разів.

Наведено низку результатів моделювання процесу вимірювання температурної залежності мір опору та проведено статистичний аналіз отриманих результатів.

Ключові слова: моделювання; вимірювання; міри опору; температурна залежність; розкладання за сингулярними числами (SVD).

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