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On the accuracy of the gradient method for determining the mean integral refractive index of air for large-scale dimensional measurements

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Abstract

The results of the analysis of the accuracy capabilities of the gradient method for determining the mean integral value of the group refractive index of air along the path of the laser radiation propagating on the surface atmospheric layer are presented. The mean integral refractive index of air is used in precision laser ranging as a correction to the results of large-scale dimensional measurements, taking into account the difference between the speed of the laser radiation propagation in the atmosphere and the speed of light in vacuum. The performed analysis includes a critical review of publications underpinning this method, and the prospects for its use in high-precision laser measurements of horizontal baselines of up to 5 km long with an expanded uncertainty of less than or equal to 1 mm are considered. This method has been studied in the framework of the Project 18 SIB01 GeoMetre "Large-Scale Dimensional Measurements for Geodesy" executed in accordance with the European Metrology Programme for Innovation and Research (EMPIR).

Keywords: refractive index; large-scale dimensional measurements; gradient method.

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1. Introduction

Today, more and more services rely on geodetic reference frames data, which form the backbone of georeferencing. A lot of the Earth science measurements are referenced to the International Terrestrial Reference Frame (ITRF), which is the realization of the International Terrestrial Reference System (ITRS). At its 24th meeting, The General Conference on Weights and Measures (CGPM) recommended that the ITRS be adopted as the unique international reference system for terrestrial reference frames for all metrological applications [1].

Global observations of natural phenomena, such as ice sheets decrease, sea level increase, monitoring of deformation of large civil engineering projects and critical infrastructure assets, require most accurate linear measurements with uncertainties of less than 1 mm. Applications in ground surveying, natural disaster detection, and monitoring of the stability of large structures require length references that demonstrate a long-term stability and high accuracy.

The ITRF coordinates are the result of multiple sources of the input data on measurements performed all over the world, thus the traceability chain to the SI definition of the metre is therefore highly complex. One of the key elements of the traceability of the coordinate observations to the SI definition of the metre is the ground survey and local ties.

Traceability of macroscale measurements to the SI with a high accuracy is among the trends of the development of dimensional measurement technologies [2]. But the accuracy of large-scale dimensional measurements is currently limited by the accuracy of the determination of the refractive index of air (due to the effect on laser wavelength-based length scales and refraction causing laser beam-bending effects).

Correct consideration of the influence of the Earth's inhomogeneous atmosphere on the results of laser ranging measurements performed on near-Earth tracks is important for providing the metrological traceability of distance measurements to the SI base unit, the metre, in geodesy, geodynamics, and navigation [3]. Such consideration is necessary, in particular, to exclude additional delay of optical signal caused by the presence of the atmosphere and related to the difference of velocities of the signal propagation in the atmosphere and in vacuum. This delay is proportional to the mean integral refractive index of air along the measured trace, which depends on meteorological parameters of the atmosphere. The paper discusses a scientific rationale of a new gradient method studied within the GeoMetre Project [4] to determine the feasibility of its application for determining the mean integral refractive index of air used as a correction in high-precision laser ranging measurements of baselines of up to 5 km long with an expanded uncertainty of less than 1 mm.

The main issues that are considered within this method are formulated in publications [5–7]: a) what models justify the possibility of using the sensors of meteorological parameters of the atmosphere to determine the mean integral index of the air refraction, b) what are the requirements for the number of discrete points of the trace on which the sensors should be placed, c) what are the requirements for the accuracy characteristics of the sensors.

The approach developed in [5–7] is rigorous from a metrological point of view. This approach can be considered an addition to the examples set out in the Guide [8] in terms of measurement models for integral (averaged over the spatial coordinate) quantities, for example, temperature, refractive index of air, etc.

The methodology (procedure) that characterizes this approach suggests two variants of determining the mean integral refractive index of air, which are based on the application of the quadrature and (or) interpolation relations, allowing representing the output value given by the integral:

$$\bar{n} = \frac{1}{L}\int_0^L n(\sigma)\,d\sigma \qquad (1)$$

through the functions of the local values of the refractive index n (depending on the corresponding local values of the atmospheric meteorological parameters) on the integration interval L and the refractive index gradients at the end points of the trace.

Taking into account the prospects of using the gradient method in practice, its general definition can be formulated as follows: the gradient method for determining the mean integral refractive index of air is a general name for methods based on the use of quadrature formulas containing summands which depend on the values of the air refractive index gradient at the discrete points on the integration interval along the distance measured by a laser rangefinder.

In equation (1), σ is a ray coordinate counted along the signal trajectory of length L. In general, this trajectory is curved due to the refraction effect. However, in the case of lengths up to 5 km long, the refractive curvature of the trajectory is negligibly small (the trajectory virtually coincides with a straight line passing through the end points of the trace), so the coordinate counted along a straight line of length L connecting the end points ($\sigma_0 = 0$, $\sigma_L = L$) of this trace can be considered a ray coordinate σ [6].

According to [6], the expanded uncertainty of length measurements less than 1 mm for traces of up to 5 km long will be provided if the value is determined with an uncertainty not exceeding ~10^{-7}. Further, we will discuss possible variants to realise this requirement by the considered method.

2. Accuracy capabilities of the method

2.1. Theoretical analysis, modelling, and numerical experiment

In the first variant of the procedure under consideration, when processing measurement results (obtaining output quantities from the measurement data on input quantities), a measurement model by the gradient method is based on calculating the integral (1) using the Euler-Maclaurin quadrature [5]:

$$\bar{n}_{EM} = \frac{(n_0 + n_L)}{2N} + \frac{1}{N}\sum_{i=1}^{N-1} n(\sigma_i) - \frac{L\left(n_L^{(1)} - n_0^{(1)}\right)}{12N^2} + R_{EM}, \qquad (2)$$

where $n_0 = n(\sigma_0)$; $n_L = n(\sigma_L)$; $n_0^{(1)} = \dfrac{dn_0}{d\sigma}$; $n_L^{(1)} = \dfrac{dn_L}{d\sigma}$ are the values of the refractive index and its first derivative at the end points of the trajectory; $R_{EM} = \dfrac{L^4}{720N^4}\dfrac{d^4 n}{d\sigma^4}\Big|_{\sigma=\sigma_{xx}}$ is a remainder term; σ_{xx} is a point of the trajectory on the integration interval.

The second variant of the procedure uses the ratio of the gradient method, for which the integral (1) is calculated using the interpolation Hermite polynomials [5]:

$$\bar{n}_H = \frac{1}{L}\sum_{i=1}^{N-1} A_i n(\sigma_i) + \frac{1}{L}\sum_{i=0,N}\left(A_{i0} n(\sigma_i) + A_{i1}\frac{dn}{d\sigma}\Big|_{\sigma=\sigma_i}\right) + R_H, \qquad (3)$$

where $R_H = \dfrac{1}{L} \times \dfrac{d^{N+3} n}{d\sigma^{N+3}}\Big|_{\sigma=\sigma_{xxx}} \displaystyle\int_0^L \frac{\Omega(\sigma)}{(N+3)!}d\sigma$ is a remainder term;

σ_{xxx} is a point of the trajectory on the integration interval.

Equation (2) is valid only if there is a uniform distribution of intermediate measurement points σ_i of the local values of refractive index $n(\sigma_i)$ along the trace, and equation (3) is valid for both uniform and non-uniform locations of these points. In these relations, the following designations are taken: N is the number of breakdowns of the trace by the points at which the local values of the refractive index are determined (N + 1 is the number of the above points on the trace, including the end points); L is the length of the trace (baseline). The refractive index derivatives along the coordinate at the end points are determined by the refractive index gradients along the line connecting end points. The weight coefficients

A_i, A_{i0}, and A_{i1} and the value $\Omega(\sigma)$ in equation (3) are calculated by the equations given in [5].

The Siddor formula [9] is used as a relation for the air refractive index $n=n(T, P, h, \lambda)$ included in equations (1)–(3) (taking into account Pollinger's corrections [10]). The explicit form of this formula is not given due to its unwieldiness. In this formula, λ is the laser radiation wavelength, and the inhomogeneity of the atmosphere is accounted for by the spatial inhomogeneity of meteorological parameters (T – temperature, P – pressure, h – relative humidity of air) that vary along the beam trajectory.

The values of the gradient g of the air refractive index along the trajectory of the laser radiation propagation at the end points are determined by the derivatives of the refractive index functions at these points, entering equations (2) and (3). According to [5], they can be determined by the finite difference method with a linear smoothing:

$$g = \frac{dn}{d\sigma} = \frac{n_{01} - n_{-01}}{\Delta\sigma} \tag{4}$$

using the data on the local values of the refractive index n_{01}, n_{-01} at the points, equidistant at the distance $\pm\frac{\Delta\sigma}{2}$ from the end points. The values n_{01}, n_{-01} are determined according to the relation $n=n(T, P, h, \lambda)$ through the values of temperature T, pressure P and relative humidity h at these points.

The number of intermediate points on the trace $N-1$, at which the local values of the air refractive index are determined, is chosen so that to ensure a correct estimation of the remainder terms in equations (2) and (3) and related components of Type B uncertainty, arising due to the fact that the remainder terms are not considered in practically used measurement models.

For the first variant (equation (2)), in which the order of the derivative in the remainder term does not depend on N, the remainder term can be estimated by Runge's rule using measurement data on a sufficient number of intermediate points [11]. For the second variant (equation (3)), Runge's rule is inapplicable, since the order of the derivative in the remainder term depends on N.

A universal variant for estimating the remainder term and its associated Type B uncertainty is a numerical experiment, in which the value $\bar{n}_H$, for example, determined by equation (3), but without R_H, is compared with the exact value determined by equation (1). That is, to find the value of R_H, it is necessary to know the exact profile $n(\sigma)$ on the considered trace (or typical dependence $n(\sigma)$ to estimate R_H by the magnitude order). By performing a numerical experiment with different values of N, it is possible to find conditions, under which the remainder term is negligibly small, i.e. conditions, under which the Type B uncertainty component associated with the rejection of remainder terms in model equations (2) and (3) can be ignored.

This approach is chosen as the main one in the considered methodology, when all the information about the number of intermediate points $N-1$ and the accuracy requirements for sensors at end and intermediate points should be used only for those values N, at which the type B uncertainty component is negligibly small.

The number of intermediate points $(N-1)$ providing negligibly small value of Type B uncertainty due to the difference of the quadrature (interpolation) relations without remainder terms from the exact integral for $\bar{n}$, in the case of baselines of a reference geodetic polygon of the NSC "Institute of Metrology", in the absence of abrupt jumps in the refractive index, according to the results of numerical experiments (partially described in [6]), should not be less than 1 point for reference baselines of up to 1 km long and not less than 3–4 points for baselines of up to 5 km long.

For other baselines (other polygons), a reasonable choice of the number of intermediate points also requires a numerical experiment using the refractive index profiles that are typical for these baselines.

By neglecting the uncertainty component caused by discarding the remainder terms in equations (2) and (3), formulas for the combined uncertainty of determining the air refractive index can be derived, which allow us to formulate the requirements for the accuracy of sensors of meteorological parameters. These requirements must ensure the specified accuracy of determining the required value $\bar{n}\left(u_{\bar{n}} \sim 10^{-7}\right)$.

These requirements are set taking into account the following conditions: the uncertainty component of temperature measurements at two end points and $N-1$ intermediate points of the baseline, as well as the uncertainty components of pressure and humidity measurements only at two end points, account for 10% each (that is, 30% in total), and the component related to the determination of the gradient accounts for 70% of the combined uncertainty of the determination of the mean integral refractive index $u_{\bar{n}} \sim 10^{-7}$.

These conditions allow us to obtain the following ratios, assuming that the air pressure and humidity are measured only at the end points of the baseline (the values of these parameters for intermediate points are determined by recalculating from the data at the end points under the assumption of linear relationship).

In the case of equation (2):

$$\begin{aligned} u_T^2 &\le 0.1\times10^{-14}\,\frac{2N^2}{2N-1}\left(\frac{\partial n}{\partial T}\right)^{-2}, \\ u_P^2 &\le 0.1\times10^{-14}\times 2\left(\frac{\partial n}{\partial P}\right)^{-2}, \\ u_h^2 &\le 0.1\times10^{-14}\times 2\left(\frac{\partial n}{\partial h}\right)^{-2}, \\ u_g^2 &\le 0.7\times10^{-14}\times 72N^4L^{-2}, \end{aligned} \tag{5}$$

Table 1

The results for a baseline of 1 km long at $N = 1$

For (5). Euler-Maclaurin quadrature	For (6). Hermite polynomials
$14.252 \text{ Pa} < u_p < 16.476 \text{ Pa}$	$14.252 \text{ Pa} < u_p < 16.476 \text{ Pa}$
$0.0417 < u_h < 0.690$	$0.0417 < u_h < 0.690$
$0.0361\ °\text{C} < u_T < 0.0474\ °\text{C}$	$0.0361\ °\text{C} < u_T < 0.0474\ °\text{C}$

Table 2

The results for a 5 km baseline at $N = 4$

For (5). Euler-Maclaurin quadrature	For (6). Hermite polynomials
$14.252 \text{ Pa} < u_p < 16.476 \text{ Pa}$	$14.252 \text{ Pa} < u_p < 16.476 \text{ Pa}$
$0.0417 < u_h < 0.690$	$0.0417 < u_h < 0.690$
$0.0546\ °\text{C} < u_T < 0.0717\ °\text{C}$	$0.0538\ °\text{C} < u_T < 0.0706\ °\text{C}$

where $\frac{\partial n}{\partial T}, \frac{\partial n}{\partial P}, \frac{\partial n}{\partial h}$ are partial derivatives of the air refractive index by temperature, pressure, and relative humidity respectively calculated using the Siddor formula [9]; u_T, u_P, u_h are the uncertainties of the measurement results of temperature, pressure and relative humidity at the points of the determination of local values of the refractive index; u_g is the uncertainty of measurements of the air refractive index gradient.

In the case of equation (3):

$$u_T^2 \le 0.1\times10^{-14} L^2 \left(\frac{\partial n}{\partial T}\right)^{-2} \left(\sum_{i=1}^{N-1} A_i^2 + \sum_{i=0,N} A_{i0}^2\right)^{-1},$$

$$u_P^2 \le 0.1\times10^{-14} L^2 \left(\frac{\partial n}{\partial P}\right)^{-2} \left\langle \left(\sum_{i=1}^{N-1} A_i^2 \left(1-\frac{\sigma_i}{L}\right) + A_{00}\right)^2 + \left(\sum_{i=1}^{N-1} A_i \frac{\sigma_i}{L} + A_{N0}\right)^2 \right\rangle^{-1}, \tag{6}$$

$$u_h^2 \le 0.1\times10^{-14} L^2 \left(\frac{\partial n}{\partial h}\right)^{-2} \left\langle \left(\sum_{i=1}^{N-1} A_i \left(1-\frac{\sigma_i}{L}\right) + A_{00}\right)^2 + \left(\sum_{i=1}^{N-1} A_i \frac{\sigma_i}{L} + A_{N0}\right)^2 \right\rangle^{-1},$$

$$u_g^2 \le 0.7\times10^{-14} L^2 \left(A_{01}^2 + A_{N1}^2\right)^{-1}.$$

above range of meteorological conditions. Table 1 gives the results for a baseline of 1 km long at $N = 1$, and Table 2 gives the results for a baseline of 5 km long at $N = 4$.

Quantitative results for the developed method were also obtained for other values of N. For $N=2$, at $L=1$ km and the above meteorological conditions in the case of measurement model (2), for example, the data on u_P, u_h coincide with those given in Table 1. The requirements for u_T as it was to be expected, become less strict: $0.042\ °\text{C} < u_T < 0.055\ °\text{C}$.

Calculations using equations (5) and (6), which set the upper limits for the uncertainty of measurements of the corresponding quantities, were performed in [7] for normal atmospheric pressure in the range of relative humidity $0.3 < h < 0.8$ and temperatures $15\ °\text{C} < T < +25\ °\text{C}$ with a uniform distribution of intermediate measurement points along the trace. As an example, Table 1 and 2 show the calculated ranges of the variation of these upper limits in the above range of meteorological conditions.

The requirements for the accuracy of measurements of meteorological parameters (5) and (6), used in the developed method, were taken into account when equipping the corresponding meteorological sensors of a baseline of 1 km long of a reference geodetic polygon shown in Fig. 1.

The used sensors also allow satisfying the requirements given in equations (6) and (7) for the accuracy of measuring the refractive index gradient.

Fig. 1. General view of a baseline of 1 km long of a reference geodetic polygon of the NSC "Institute of Metrology"

To meet these requirements, which set the upper limit of the uncertainty value u_g of determining the refractive index gradient at the end points of the trace, the following conditions shall be met: firstly, the value of g must be determined by the finite difference method with a linear smoothing according to equation (4); secondly, the local values of the refractive index n_{01}, n_{-01} in equation (4) should be determined using meteorological sensors with accuracy characteristics u_T, u_P, u_h satisfying conditions (5) and (6). Under these conditions, the requirements for u_g can be met. According to the estimates made for case (5), for example, it is realized at $N \geq 2$ for $L = 1$ km and at $N \geq 4$ for $L = 5$ km.

Thus, the method of determining the gradient from data on additional refractive index measurements at two spaced points (equidistant from the end points) provides the required accuracy. For a measurement model described by equation (2), in particular, such accuracy is achieved provided that for a trace of $L = 1$ km, the local values of the refractive index are determined at two end points and at least one intermediate point, and for a trace of $L = 5$ km the values shall be determined at two end points and at least at three intermediate points. These conditions correspond to the requirements of the analysed procedure given earlier in this section of the paper, which allows not to take into account remainder terms in measurement models described by equations (2) and (3).

Under experimental conditions (practical use of the discussed method on reference baselines), the combined standard uncertainty of the determination in accordance with the above described procedures is calculated using the following ratios:

$$u_{\bar{n}}^2 = \frac{1}{4N^2}\left(\frac{\partial n}{\partial T}\right)^2 \left(u_{cT_0}^2 + u_{cT_L}^2\right) + \frac{1}{N^2}\left(\frac{\partial n}{\partial T}\right)^2 \sum_{i=1}^{N-1} u_{cT_i}^2 + \frac{1}{4}\left(\frac{\partial n}{\partial P}\right)^2 \left(u_{cP_0}^2 + u_{cP_L}^2\right) + \frac{1}{4}\left(\frac{\partial n}{\partial h}\right)^2 \left(u_{ch_0}^2 + u_{ch_L}^2\right) + \frac{L^2}{144N^4}\left(u_{g_0}^2 + u_{g_L}^2\right) \quad (7)$$

for a measurement model described by equation (2), and

$$u_{\bar{n}}^2 = \frac{1}{L^2}\left(\frac{\partial n}{\partial T}\right)^2 \left[\sum_{i=1}^{N-1} A_i^2 u_{cT_i}^2 + A_{00}^2 u_{cT_0}^2 + A_{N0}^2 u_{cT_L}^2\right] + \frac{1}{L^2}\left(\frac{\partial n}{\partial P}\right)^2 \left\langle \left[\sum_{i=1}^{N-1} A_i \left(1 - \frac{\sigma_i}{L}\right) + A_{00}\right]^2 u_{cP_0}^2 + \left[\sum_{j=1}^{N-1} A_j \frac{\sigma_j}{L} + A_{N0}\right]^2 u_{cP_L}^2 \right\rangle +$$
$$+ \frac{1}{L^2}\left(\frac{\partial n}{\partial h}\right)^2 \left\langle \left[\sum_{i=1}^{N-1} A_i \left(1 - \frac{\sigma_i}{L}\right) + A_{00}\right]^2 u_{ch_0}^2 + \left[\sum_{i=1}^{N-1} A_i \frac{\sigma_i}{L} + A_{N0}\right]^2 u_{ch_L}^2 \right\rangle + \frac{1}{L^2}\left[A_{01}^2 u_{g_0}^2 + A_{N1}^2 u_{g_L}^2\right] \quad (8)$$

for a measurement model corresponding to equation (3).

Equations (7) and (8) use the previously introduced designations (for equations (5), (6)) supplemented with the indices corresponding to the specific points 0, i, and L, at which the measurements are performed:

$$u_{cT_{0,i,L}}^2 = u_{T_{0,i,L}}^2 + u_{instr.T}^2\left(\bar{T}_{0,i,L}\right),$$
$$u_{cP_{0,L}}^2 = u_{P_{0,L}}^2 + u_{instr.P}^2\left(\bar{T}_{0,L}\right),$$
$$u_{ch_{0,L}}^2 = u_{h_{0,L}}^2 + u_{instr.h}^2\left(\bar{T}_{0,L}\right).$$

Here $u_{cT_{0,i,L}}$, $u_{cP_{0,L}}$, and $u_{ch_{0,L}}$ are the combined standard uncertainties of the determination of meteorological parameters at points 0, i, and L, which are expressed through the Type B uncertainty components when calibrating the corresponding meteorological sensors $u_{instr.T}$, $u_{instr.P}$, and $u_{instr.h}$, and the uncertainty components $u_{T_{0,i,L}}$, $u_{P_{0,L}}$, and $u_{h_{0,L}}$ obtained with the sensors from experimental data at points 0, i, and L

when practically implementing the discussed method on a reference basis

$$u^2_{T_{0,i,L}} = \frac{\sum_{j=1}^{k}\left(T_j - \bar{T}_{0,i,L}\right)^2}{k(k-1)}, \quad \bar{T}_{0,i,L} = \frac{\sum_{j=1}^{k} T_j}{k},$$

$$u^2_{P_{0,L}} = \frac{\sum_{j=1}^{m}\left(P_j - \bar{P}_{0,L}\right)^2}{m(m-1)}, \quad \bar{P}_{0,L} = \frac{\sum_{j=1}^{m} P_j}{m},$$

$$u^2_{h_{0,L}} = \frac{\sum_{j=1}^{n}\left(h_j - \bar{h}_{0,L}\right)^2}{n(n-1)}, \quad \bar{h}_{0,L} = \frac{\sum_{j=1}^{n} h_j}{n}.$$

The relations for u_{g_0}, u_{g_L} in equations (7) and (8) in case of using model equation (4), by means of which the endpoint gradients are determined, can be represented as

$$u^2_{g_0} = \frac{1}{(\Delta\sigma)^2}\left\langle \left(\frac{\partial n}{\partial T}\right)^2 \left[u^2_{cT_{0+}} + u^2_{cT_{0-}}\right] + 2\left(\frac{\partial n}{\partial P}\right)^2 u^2_{cP_0} + 2\left(\frac{\partial n}{\partial h}\right)^2 u^2_{ch_0} \right\rangle,$$

$$u^2_{g_L} = \frac{1}{(\Delta\sigma)^2}\left\langle \left(\frac{\partial n}{\partial T}\right)^2 \left[u^2_{cT_{L+}} + u^2_{cT_{L-}}\right] + 2\left(\frac{\partial n}{\partial P}\right)^2 u^2_{cP_L} + 2\left(\frac{\partial n}{\partial h}\right)^2 u^2_{ch_L} \right\rangle,$$

where the + and – are the signs corresponding to the points shifted by distances $+\frac{\Delta\sigma}{2}$ and $-\frac{\Delta\sigma}{2}$ from points 0 and L, at which the gradient values are determined.

2.2. Discussion of prospects for further research

The formulas (7), (8) given in the previous section of the paper are valid for the conditions under which the influence of atmospheric turbulence can be neglected. Such conditions are usually considered when analysing the accuracy of methods for taking into account the influence of the Earth atmosphere on the results of distance measurements performed using electromagnetic waves [12]. For taking into account the atmospheric turbulence, it is necessary to conduct a corresponding additional study.

Conclusion

The accuracy capabilities of the gradient method for determining the mean integral group refractive index of air along the path of the laser radiation propagation in precision laser ranging are analysed. The method was developed at the NSC "Institute of Metrology" for use in measurements of near-Earth reference bases up to 5 km long with an expanded measurement uncertainty of about 1 mm. The results of theoretical analysis, modelling and numerical estimations (made for the conditions under which the effect of atmospheric turbulence is negligible) have confirmed high accuracy characteristics of the method and allow recommending it for practical use under the conditions for measurements on baselines equipped with instrumentation for the determination of meteorological parameters of the troposphere at the discrete points of the signal trajectory of the rangefinder (total station) on the baseline under consideration. The actual direction of further research is the improvement of this method for ensuring the possibility of its use under conditions when it is necessary to take into account the influence of the atmospheric turbulence on the uncertainty of determination of the mean integral refractive index of air.

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Про точність градієнтного методу визначення середнього інтегрального показника заломлення повітря для великомасштабних розмірних вимірювань

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Анотація

Наведено результати аналізу точності градієнтного методу визначення середньоінтегрального значення групового показника заломлення повітря на шляху поширення лазерного випромінювання в приземному шарі атмосфери. Середньоінтегральне значення показника заломлення повітря використовується в прецизійній лазерній далекометрії як поправка до результатів великомасштабних розмірних вимірювань, що враховує різницю між швидкістю поширення лазерного випромінювання в атмосфері та швидкістю світла у вакуумі. Проведений аналіз включає критичний огляд публікацій, що лежать в основі цього методу, й розгляд перспектив використання останнього у високоточних лазерних вимірюваннях горизонтальних базових ліній довжиною до 5 км із розширеною невизначеністю, що не перевищує 1 мм. Цей метод було досліджено в рамках проєкту 18 SIB01 GeoMetre "Великомасштабні вимірювання розмірів для геодезії", виконаного відповідно до Європейської метрологічної програми з інновацій та досліджень (EMPIR). Метод розроблено в ННЦ "Інститут метрології" для використання під час вимірювань навколоземних еталонних базисів довжиною до 5 км з розширеною невизначеністю вимірювань близько 1 мм. Результати теоретичного аналізу, моделювання та чисельних оцінок (проведених для умов, за яких вплив атмосферної турбулентності є несуттєвим) підтвердили високі точності характеристики методу й дозволяють рекомендувати його для практичного використання в зазначених умовах для вимірювань на базисах, обладнаних апаратурою для визначення метеорологічних параметрів тропосфери в дискретних точках траєкторії сигналу далекоміра (тахеометра). Актуальним напрямом подальших досліджень є вдосконалення цього методу з метою забезпечення можливості його використання в умовах, коли необхідне врахування впливу атмосферної турбулентності на невизначеність визначення середньоінтегрального показника заломлення повітря.

Ключові слова: показник заломлення; великомасштабні розмірні вимірювання; градієнтний метод.

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