

# Development of a method and a measuring instrument in the area of studying the parameters of the low-frequency magnetic field

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## Abstract

The study has been conducted in the field of improving the metrological support in measuring the magnetic quantities by developing a method and an instrument for measuring the magnitudes of dipole magnetic moments ( $\text{Am}^2$ ) and the strength of the low-frequency (50–1000 Hz) magnetic field of the source. A measurement method has been developed that refers to induction, namely the so-called point measurement methods, which involves the use of  $n$  primary measuring transducer  $s$  located at certain points in the space around the object under consideration.

The method is based on the analysis of multipole representation of the magnetic field and the use of twelve induction sensors placed on the equatorial plane and cylindrical surface. This allows excluding the influence of higher-order harmonics and reducing the methodological error of measurements.

A system of sensors has been proposed, which are located at points with given linear-angular coordinates and connected in a certain way. In addition, a structural diagram of the measuring system has been proposed. Analytical expressions for the measured parameters – the component of the dipole magnetic moment  $M_x$ ,  $M_y$ ,  $M_z$  – have been obtained. The methodological error has been analysed, and it has been proved that the proposed method provides measurement accuracy within 0.1–5.2% at a distance of 1.5...4 of overall dimensions of the source. The errors associated with the inaccuracy of positioning of the sensors have also been analysed.

The proposed method, as well as the instrument for measuring the low-frequency magnetic field parameters, are practically intended for measuring external magnetic field parameters, such as magnetic dipole moments and magnetic field strength, which is necessary for monitoring and addressing certain scientific and technological problems in various areas.

**Keywords:** external magnetic field; magnetic moment; magnetometric method; measuring system; methodological error.

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## Introduction

There are many areas of science and technology that require measurements of magnetic quantities, and thus require modern metrological support as well as its improvement. The main parameters of the external magnetic field of an object primarily include the magnetic field strength ( $H$ , A/m), which characterizes the field at a certain point in space, and the magnetic dipole moment ( $M$ ,  $\text{A}\cdot\text{m}^2$ ). According to DSTU ISO 80000-6:2016 "Quantities and units. Part 6: Electromagnetic quantities", the magnetic dipole moment is a physical quantity that characterizes magnetic properties of an object and determines the strength and direction of the magnetic field generated by this object at long distances.

Measurements of the magnetic dipole moment are crucial in certain fields of science and technology.

For example, the magnetic dipole moment of a space satellite is a critical parameter affecting its interaction with the Earth's magnetic field, influencing the orientation and stability control.

It is essential that the magnetic dipole moments of satellites may vary depending on their design, materials, and mission requirements ( $0.1\text{--}100 \text{ A}\cdot\text{m}^2$ ) [1]. Accurate characterization and control of these moments is critical to the mission success, as unintended magnetic interactions can cause perturbations in the satellite orientation and trajectory.

Ukrainian metrological support in this field of measurements needs some improvement, especially in terms of procedures of measuring the magnetic moment and the reference base of measurement standards in this field. For some time, DSTU GOST 8.030:2009 was in force, which determined the primary standard

Characteristics of point and integral methods

Characteristic	Contour system	Point sensor system
Principle of operation	Integral measurement through flow change	Local measurements at specific points
Scope	<ul style="list-style-type: none"> <li>• large objects,</li> <li>• global measurements</li> </ul>	<ul style="list-style-type: none"> <li>• local objects,</li> <li>• measurement of the magnetic moment of small objects,</li> <li>• detailed scanning</li> </ul>
Sensitivity	High with large field changes	High, but depends on the accuracy of the sensors
Components of magnetic moment	One winding measures one component either $M_x$ or $M_y$ or $M_z$	The system measures three components $M_x, M_y, M_z$
Order of magnetic moment	Dipole component. Measuring the quadrupole component requires differential loop windings and signal processing techniques	Dipole component, quadrupole component
Data processing complexity	Relatively simple	Requires complex data processing
Localization accuracy	Low (does not provide spatial information)	High (can obtain 3D field distribution)
Features	<ul style="list-style-type: none"> <li>• high sensitivity to changes in the magnetic moment,</li> <li>• monitoring of dynamic changes in the magnetic field,</li> <li>• possibility to control large objects</li> </ul>	<ul style="list-style-type: none"> <li>• flexibility in sensor placement and the ability to measure under difficult conditions,</li> <li>• the ability to obtain a detailed spatial structure of the magnetic field</li> </ul>
	High requirements for the accuracy of geometric contour parameters	Requires calibration of each sensor

of the magnetic moment, the verification scheme, and methods of the unit transfer. Methods with an error of up to 10% were used to measure the magnetic moment.

In the practice of measuring dipole magnetic moments, inductive methods have become widely used. This is due to the linearity of transfer function over a wide measurement range, high stability of characteristics, and low temperature error in the wide frequency range. Inductive methods are divided into point and integral methods. Point methods involve the use of a system of spatially distributed sensors around the object under consideration. The application of integral methods is associated with the use of contour measurement windings as primary converters of the magnetic field into an electrical signal, which enclose the source of the magnetic field. Integral (contour systems) and point methods have significant differences in their principles of operation, technical characteristics, and areas of application (Table 1).

Relevant issues include the assessment and improvement of the accuracy of methods for measuring the dipole magnetic moment. In [1], which highlights the issue of increasing the accuracy of measurements of the magnetic moment of small-sized satellites, a

methodological error of 2% and a general standard error of 13% are specified. In [2], the errors of measurement results for the dipole magnetic functional elements of the satellite were estimated, but the methodological error and its definition are not specified.

In [3], a measurement method and methodological error of 4% are proposed, and the method is applied only to coil-type objects and requires three measurements. In [4], two methods of measurement are proposed, but for an object of a disk geometric shape. In [5], the light is shed on the procedure of evaluating the uncertainties of the magnetic moment, and the relevance of improving the metrological support in this field of measurements is confirmed.

The purpose of this paper is to develop a method of measuring the magnetic dipole moment, to increase the accuracy of measurements, to develop a procedure for the determination of methodological errors and to contribute to the development of national metrological support in this field.

### Theoretical relationships

Let us consider the analytical representation of the external magnetic field as the theoretical basis of

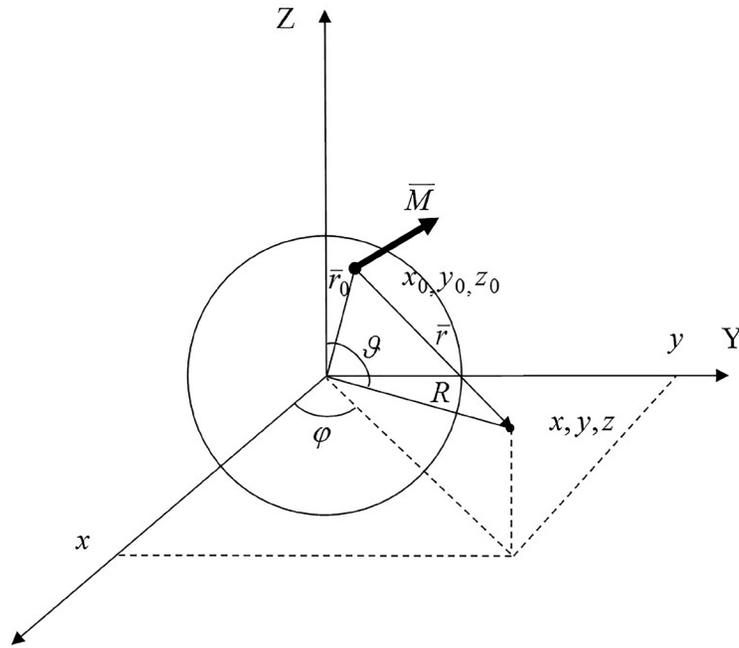


Fig. 1. Model of an eccentric inclined magnetic dipole

the method under development. The external magnetic fields of the currents at distances that are significant compared to the overall dimensions of the source have a dipole character. The model of an eccentric inclined magnetic dipole is used, which is theoretically substantiated in [6, 7] for the external magnetic field of an object that has electro-radio equipment and was practically used in [8]. The full magnetic potential of such a source is described by expression (1):

$$U = \frac{\bar{M}\bar{r}}{4\pi r^3}, \quad (1)$$

where  $\bar{M}$  is the magnetic moment of the external magnetic field source:  $\bar{M} = \bar{i} M_x + \bar{j} M_y + \bar{k} M_z$ ;

$\bar{r}$  is the radius vector from the dipole moment to the point of observation (Fig. 1).

Expression (1) is described in the Cartesian coordinate system by a mathematical expression (2):

$$U = \frac{M_x(x-x_0) + M_y(y-y_0) + M_z(z-z_0)}{4\pi \left[ (x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 \right]^{3/2}}, \quad (2)$$

where  $x_0 = R_0 \cos \varphi_0 \sin \vartheta_0$ ,  $y_0 = R_0 \sin \varphi_0 \sin \vartheta_0$ ,  $z_0 = R_0 \cos \vartheta_0$ ;  $x = R \cos \varphi \sin \vartheta$ ,  $y = R \sin \varphi \sin \vartheta$ ,  $z = R \cos \vartheta$ .

We expand expression (1) into a series by spherical functions and, after appropriate transformations, represent expression (3) in the form of a sum of spatial harmonics:

$$U = \frac{1}{4\pi} \sum_{n=1}^{\infty} \left( \frac{1}{R} \right)^{n+1} \sum_{m=0}^n (g_{nm} \cos m\varphi + h_{nm} \sin m\varphi) P_n^m(\cos \theta), \quad (3)$$

where  $R$ ,  $\varphi$ ,  $\theta$  are coordinates;  $g_{nm}$ ,  $h_{nm}$  are constant coefficients of the series, equal to the multipole magnetic

moments;  $P_n^m(\cos \theta)$  are Legendre polynomials;  $n$  is a serial number of the multipole;  $m$  is a serial number of an elementary multipole of the  $n$ -th order.

### Development of a measurement method

The principle underlying the proposed measurement method is based on the use of a certain number  $n$  of primary measuring transducers of the induction type distributed in the space around the source under consideration of the magnetic field in certain coordinates. The primary measuring transducers are connected in a certain way, which is mentioned next. The technological performance of the sensors is of no fundamental importance.

Let us consider the implementation of the method of measuring the magnetic dipole moments. This involves the creation of a stationary magnetometric stand. Such a device is made of eight three-component and four one-component sensors (Fig. 2, Fig. 3).

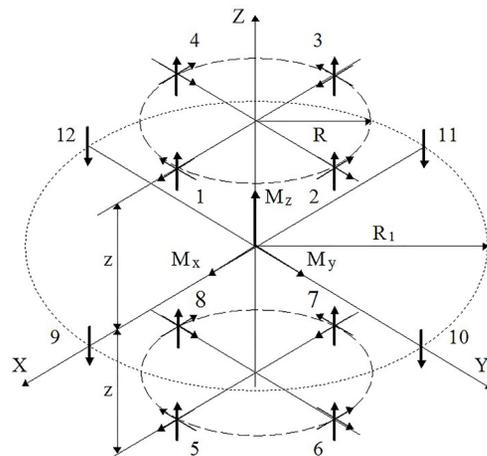


Fig. 2. Layout of primary measuring transducers

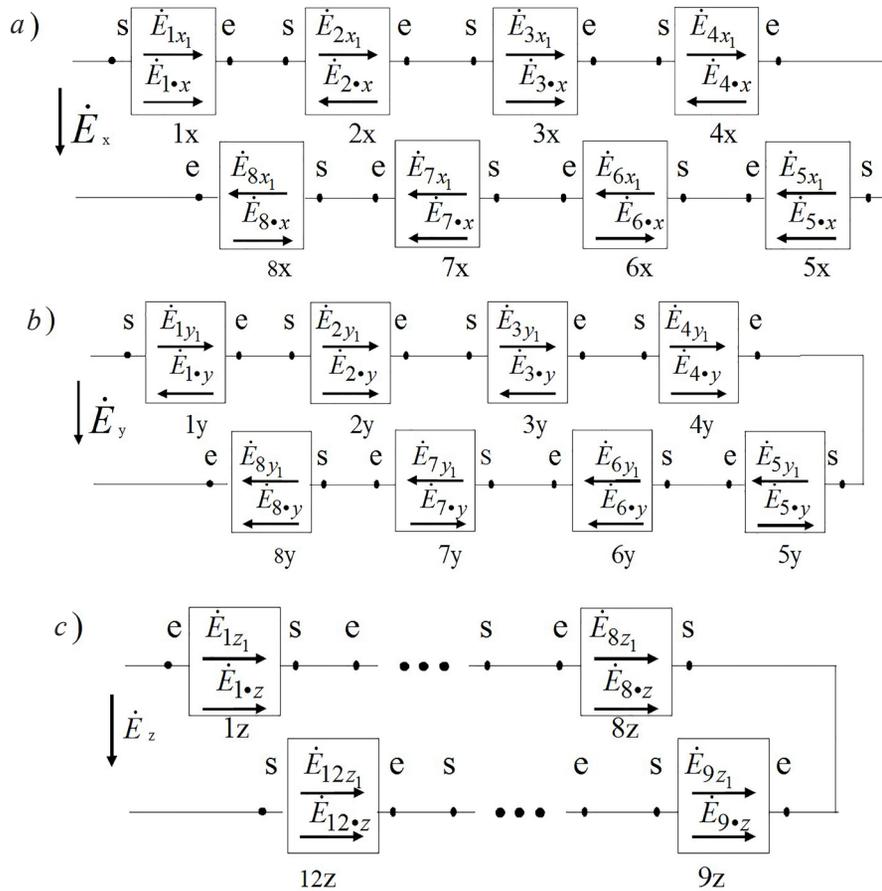


Fig. 3. Connection diagrams of sensors of measuring channels X(a), Y(b), Z(c)

Three-component sensors 1–8 are located on a cylindrical surface of radius  $R$ , four in two  $\pm Z$  planes. Single-component sensors 9–12 are located uniformly on a circle of radius  $R_1$  in the equatorial plane  $XOY(Z=0)$ .

The magnetic axes of the three-component sensors are oriented along the directions  $X$ ,  $Y$ ,  $Z$ , and the magnetic axes of the single-component sensors 9–12 are oriented along the coordinate direction  $Z$  and are parallel to the measured components  $M_x$ ,  $M_y$ ,  $M_z$  of the dipole magnetic moment.

To find the values  $M_x$ ,  $M_y$ ,  $M_z$ , the magnetic potential (1) is written down, which characterizes the external magnetic field of the source, in the cylindrical coordinate system:

$$U = \frac{1}{4\pi} \sum_{n=1}^{\infty} \frac{1}{(R^2 + z^2)^{\frac{n+1}{2}}} \sum_{m=0}^n (g_{nm} \cos m\varphi + h_{nm} \sin m\varphi) P_n^m \left( \frac{z}{\sqrt{R^2 + z^2}} \right). \quad (4)$$

The components of the magnetic field strength in the cylindrical coordinate system are the differentiation of the magnetic potential by coordinates  $R$ ,  $\varphi$ ,  $z$ :

$$H_R = -\frac{\partial U}{\partial R}, H_\varphi = -\frac{\partial U}{R \partial \varphi}, H_z = -\frac{\partial U}{\partial z}.$$

For practical analysis, it is sufficient to limit the series to  $n = 3$ . The radial component will be equal to:

$$\begin{aligned} H_R = & \frac{1}{4\pi} \left[ 3g_{10} \frac{Rz}{(R^2 + z^2)^{\frac{5}{2}}} + (g_{11} \cos \varphi + h_{11} \sin \varphi) \frac{2R^2 - z^2}{(R^2 + z^2)^{\frac{5}{2}}} - \frac{3}{2} \frac{g_{20} R(R^2 - 4z^2)}{(R^2 + z^2)^{\frac{7}{2}}} + 3(g_{21} \cos \varphi + h_{21} \sin \varphi) \frac{z(4R^2 - z^2)}{(R^2 + z^2)^{\frac{7}{2}}} + \right. \\ & + 3(g_{22} \cos 2\varphi + h_{22} \sin 2\varphi) \frac{R(3R^2 - 2z^2)}{(R^2 + z^2)^{\frac{7}{2}}} - \frac{5}{2} g_{30} \frac{Rz(3R^2 - 4z^2)}{(R^2 + z^2)^{\frac{9}{2}}} + \frac{3}{2} (g_{31} \cos \varphi + h_{31} \sin \varphi) \frac{27R^2 z^2 - 4(R^4 + z^4)}{(R^2 + z^2)^{\frac{9}{2}}} + \\ & \left. + 15(g_{32} \cos 2\varphi + h_{32} \sin 2\varphi) \frac{Rz(5R^2 - 2z^2)}{(R^2 + z^2)^{\frac{9}{2}}} + 15(g_{33} \cos 3\varphi + h_{33} \sin 3\varphi) \frac{R^2(4R^2 - 3z^2)}{(R^2 + z^2)^{\frac{9}{2}}} + \dots \right]. \quad (5) \end{aligned}$$

The tangential component of the magnetic strength is determined by the expression:

$$H_\varphi = \frac{1}{4\pi} [(g_{11} \sin \varphi - h_{11} \cos \varphi) \frac{1}{(R^2 + z^2)^{3/2}} + 3(g_{21} \sin \varphi - h_{21} \cos \varphi) \frac{z}{(R^2 + z^2)^{5/2}} + 6(g_{22} \sin 2\varphi - h_{22} \cos 2\varphi) \frac{R}{(R^2 + z^2)^{5/2}} - \frac{3}{2}(g_{31} \sin \varphi - h_{31} \cos \varphi) \frac{R^2 - 4z^2}{(R^2 + z^2)^{7/2}} + 30(g_{32} \sin 2\varphi - h_{32} \cos 2\varphi) \frac{Rz}{(R^2 + z^2)^{7/2}} + 45(g_{33} \sin 3\varphi - h_{33} \cos 3\varphi) \frac{R^2}{(R^2 + z^2)^{7/2}} + \dots +]. \quad (6)$$

The axial component of the external magnetic field strength has the form:

$$H_z = \frac{1}{4\pi} [-g_{10} \frac{R^2 - 2z^2}{(R^2 + z^2)^{3/2}} + 3(g_{11} \cos \varphi + h_{11} \sin \varphi) \frac{Rz}{(R^2 + z^2)^{5/2}} - \frac{3}{2}g_{20} \frac{z(3R^2 - 2z^2)}{(R^2 + z^2)^{5/2}} - 3(g_{21} \cos \varphi + h_{21} \sin \varphi) \frac{R(R^2 - 4z^2)}{(R^2 + z^2)^{5/2}} + 15(g_{22} \cos 2\varphi + h_{22} \sin 2\varphi) \frac{R^2 z}{(R^2 + z^2)^{5/2}} + \frac{1}{2}g_{30} \frac{3R^2(R^2 - 8z^2) + 8z^4}{(R^2 + z^2)^{5/2}} - \frac{15}{2}(g_{31} \cos \varphi + h_{31} \sin \varphi) \frac{Rz(3R^2 - 4z^2)}{(R^2 + z^2)^{5/2}} - 15(g_{32} \cos 2\varphi + h_{32} \sin 2\varphi) \frac{R^2(R^2 - 6z^2)}{(R^2 + z^2)^{5/2}} + 105(g_{33} \cos 3\varphi + h_{33} \sin 3\varphi) \frac{R^3 z}{(R^2 + z^2)^{5/2}} + \dots +]. \quad (7)$$

Measuring channel  $X$ . Sensors 1x–8x of this channel are connected, as follows from Fig. 3(a), electrically in series, respectively.

The magnetic axes of coils 1x, 3x and 5x, 7x are radial, and coils 2x, 4x and 6x, 8x are tangential to a circle of radius  $R$ . Therefore, the magnetic axes of these coils are exposed to the radial (5) and tangential (6) components of the source magnetic field, respectively, as well as the magnetic field of external sources, which will interfere. This field coincides

in direction with the components of the magnetic field in coils 1x, 4x, 5x, 8x and is opposite to them in coils 2x, 3x, 6x, 7x. Therefore, the total magnetic field, which the magnetic axes  $x$  of the coils are exposed to, is equal to the sum of the magnetic field component and the magnetic field of external sources for coils 1x, 4x, 5x, 8x and the difference of these fields for coils 2x, 3x, 6x, 7x.

The resulting electrical signal of the  $X$  channel is equal in total to:

$$\begin{aligned} \dot{E}_x &= \frac{1}{k_f} \cdot 2(\dot{H}_R \begin{pmatrix} R \\ \varphi=0^\circ \\ \pm z \end{pmatrix} + \dot{H}_\varphi \begin{pmatrix} R \\ \varphi=90^\circ \\ \pm z \end{pmatrix} - \dot{H}_R \begin{pmatrix} R \\ \varphi=180^\circ \\ \pm z \end{pmatrix} - \dot{H}_\varphi \begin{pmatrix} R \\ \varphi=270^\circ \\ \pm z \end{pmatrix}) = \\ &= \frac{1}{k_f} [12g_{11} \frac{R^2}{(R^2 + z^2)^{5/2}} + 30(g_{31} - 2g_{33}) \frac{R^2(6z^2 - R^2)}{(R^2 + z^2)^{9/2}} + \dots + \\ &\quad + \sum_{i=1}^8 (-1)^{i+1} H_{ix}] = \dot{E}_{x1} + \dot{E}_{x3} + \dots + \Delta \dot{E}_x, \end{aligned} \quad (8)$$

where  $\dot{E}_{x1}$  is a useful signal of the first harmonic, proportional to  $g_{11}$ ;  $\dot{E}_{x3}$  is the third harmonic interference signal proportional to the coefficients  $g_{31}$ ,  $g_{33}$ ;

$\Delta \dot{E}_x = \frac{1}{k_f} \sum_{i=1}^8 (-1)^{i+1} H_{ix}$  is the interference created

by the magnetic field  $\dot{H}_x$  of external sources.

It can be seen from expression (8) that the signal  $\dot{E}_x$  does not contain interference generated

by even harmonics. Third harmonic interference is excluded from the measurement result at  $6z^2 = R^2$ , i.e. when  $z = \sqrt{1/6} \cdot R$  the interference signal is close to zero. In this case, the expression for the resulting signal has the form:

$$\dot{E}_x = 8.16 \frac{g_{11}}{k_f R^3} + \dots + \Delta \dot{E}_x,$$

whence the magnitude of the magnetic dipole moment  $M_x = 0.12 E_x k_f R^3$ .

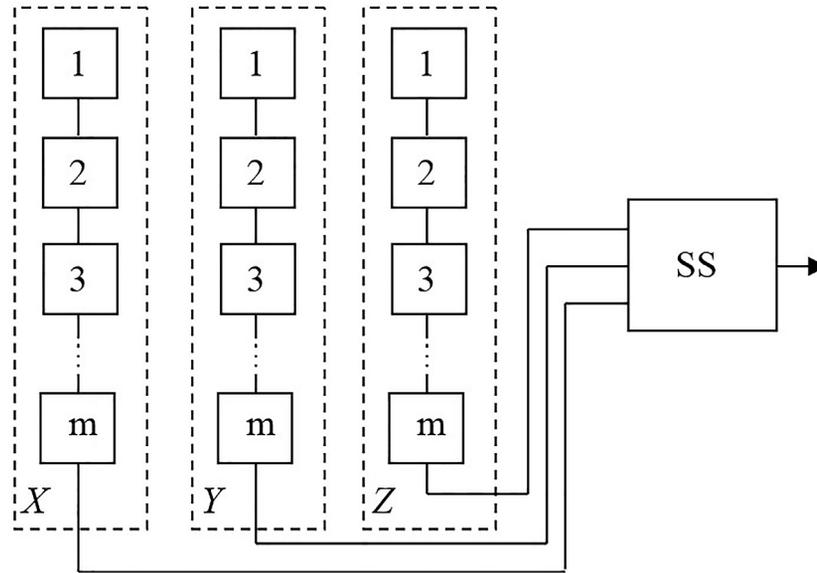


Fig. 4. Switching schemes of primary measuring transducers

The sensitivity of the measuring channel  $X$  is equal to  $S_{H_x} = \frac{8.16}{k_f} \cdot \frac{\text{mV}}{\text{mOe}}$ , where  $k_f$  is the conversion factor of a certain sensor.

Similar actions and transformations will be done for channels  $Y, Z$ . As a result, the following is obtained:

for channel  $Y$ :  $M_y = 0.12E_y k_f R^3$ ,  $S_{H_y} = \frac{8.16}{k_f} \cdot \frac{\text{mV}}{\text{mOe}}$ .

for channel  $Z$ :  $M_z = 0.23E_z k_f R^3$ ,  $S_{H_z} = \frac{2.2}{k_f} \cdot \frac{\text{mV}}{\text{mOe}}$ .

The conducted analysis has shown that the proposed solution with placement of sensors in the equatorial plane and on the cylindrical surface excludes the influence of higher-order multipoles, including the fifth harmonic, on the accuracy of measuring the magnetic dipole moments.

The primary measuring transducers of the developed measuring instrument are located at specified points and data from them are sent sequentially in time to the measuring channel, where they are converted, measured, processed and displayed or memorized. Sensors are connected to the measuring channel using system switches (SS) (Fig. 4). The survey is

implemented using a system switch, which is managed by a microcontroller using the appropriate software.

When constructing the structural diagram of the developed measuring instrument, the functional composition of the microcontroller should be taken into account, as well as the features of the inclusion of measuring converters that ensure the performance of the functions of measurement conversion, measurement itself and information conversion. Taking into account the above and the scheme in Fig. 4, the structural diagram of the designed measuring instrument can be presented in the form of Fig. 5.

Measurements of dipole moments by point magnetometric devices are associated with a methodological error caused by spatial harmonics – higher-order multipoles of the external magnetic field under consideration. Therefore, the estimation of the error of the method of measuring dipole moments will be considered. In accordance with expressions (4–7):

$$\delta = \frac{H_n}{H_1} \cdot 100\%, \tag{9}$$

where  $H_n (n=3, 5, \dots)$  is  $n$ -th harmonic magnetic field strength;  $H_1 = M / (2\pi R^3)$  is the magnetic field strength of the first harmonic, proportional to the measured dipole moment  $M$ .

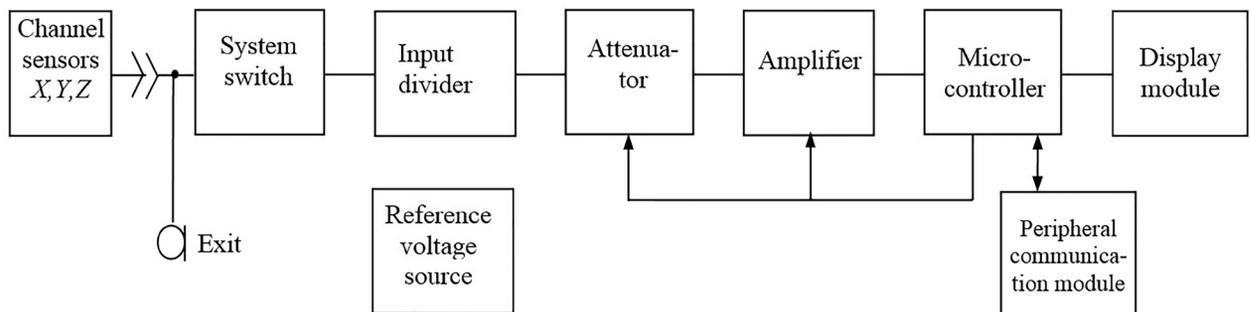


Fig. 5. Structural diagram of the developed measurement instrument

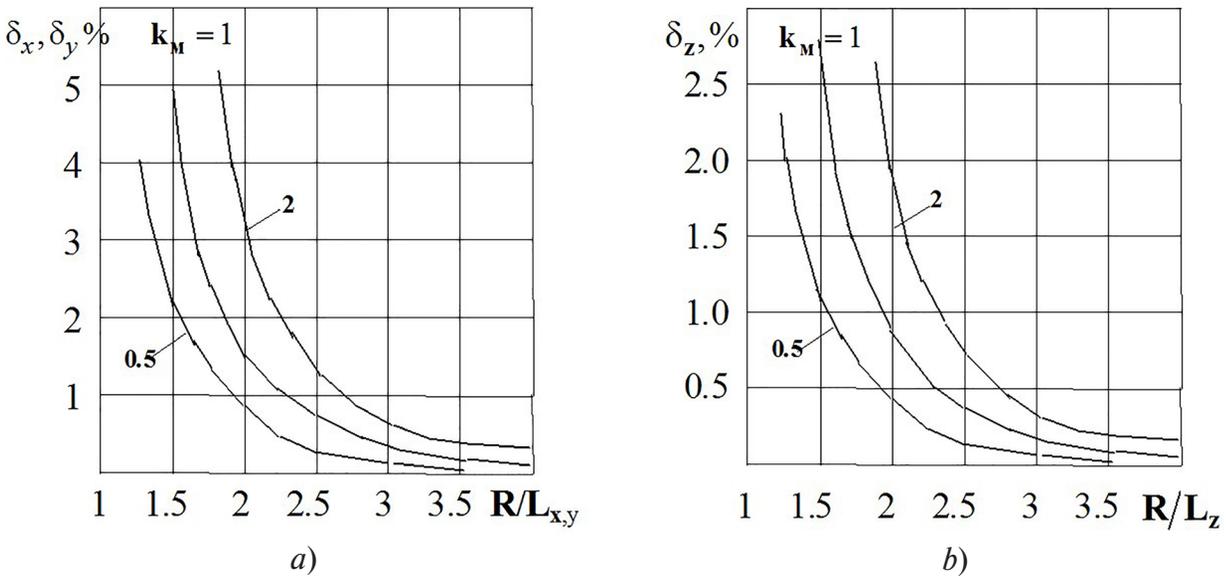


Fig. 6. Dependence  $\delta = f(R/L)$  for channels  $X, Y$  (a) and channel  $Z$  (b) of the device

For a measuring device whose sensors are located on a cylindrical surface (Fig. 2), the methodological error of measuring magnetic dipole moments  $M_x, M_y$  will be determined by the ratios:

$$\delta_x = \frac{3 \cdot 10^{-2}}{g_{11} R^4} (15g_{51} + 1122g_{53} + 61548g_{55}) \cdot 100\%,$$

$$\delta_y = \frac{3 \cdot 10^{-2}}{h_{11} R^4} (15h_{51} - 1122h_{53} + 61548h_{55}) \cdot 100\%.$$

Taking into account (9) and the value of the coefficients of the fifth harmonic, the methodological error will be equal to:

$$\delta_x = 0.266k_M \left(\frac{L_x}{R}\right)^4 \cdot 100\%, \quad \delta_y = 0.266k_M \left(\frac{L_y}{R}\right)^4 \cdot 100\%, \quad (10)$$

where  $k_M$  is a similarity coefficient for electrical devices of general industrial design containing first-order power circuits,  $0 < k_M \leq 2$ ;  $L_{(x,y,z)}$  are overall dimensions of the external magnetic field source in the orthogonal directions  $X, Y, Z$ .

When measuring the dipole moment  $M_z$ , the methodological error  $\delta_z$  for three options for the location of the  $z$ -th coils will be determined by the following expressions:

for coils  $1z-12z$  at  $z = \sqrt{1/6}R$  and  $R_1 = 1.31R$

$$\delta_{1z} = -\frac{8.6 \cdot 10^{-2} g_{50}}{R^4 g_{10}} \cdot 100\%;$$

for coils  $1z-8z$  at  $z_1 = 0.361R$   $\delta_{2z} = -\frac{0.262g_{50}}{R^4 g_{10}} \cdot 100\%$ ;

for coils  $1z-12z$  at  $z_1 = 0.3R$  and  $R_1 = 1.373R$

$$\delta_{3z} = -\frac{0.46g_{50}}{R^4 g_{10}} \cdot 100\%.$$

The methodological error of the  $M_z$  measurement will accordingly be equal to:

$$\delta_{1z} = 0.43k_M \left(\frac{z_z}{R}\right)^4 = 2.7 \cdot 10^{-2} k_M \left(\frac{L_z}{R}\right)^4 \cdot 100\%,$$

$$\delta_{2z} = 1.31k_M \left(\frac{z_z}{R}\right)^4 = 8.2 \cdot 10^{-2} k_M \left(\frac{L_z}{R}\right)^4 \cdot 100\%,$$

$$\delta_{3z} = 2.32k_M \left(\frac{z_z}{R}\right)^4 = 14.5 \cdot 10^{-2} k_M \left(\frac{L_z}{R}\right)^4 \cdot 100\%,$$

from where it is clear that  $\delta_{3z} > \delta_{2z} > \delta_{1z}$ .

Fig. 6 shows the dependences of

$$\delta_x = \delta_y = f(R/L_x) \quad (a) \quad \text{and} \quad \delta_z = f(R/L_z) \quad (b),$$

from which it follows that at a distance of 1.5...4 dimensions  $L$  of the magnetic field source, the maximum error value is 5.2...0.1% under  $k_M = 1$ . This is less than the methodological error of the known magnetometric method using four sensors (DSTU GOST 8.030) by 1.2...7 times.

It is also crucial to consider the estimation of positioning errors of sensors both in angular and linear coordinates, since they can significantly affect the accuracy of measurements. The inaccuracy of installing sensors in given coordinates on the values  $\Delta R = \Delta z$ ,  $\Delta \varphi = \Delta \theta = \Delta \alpha$  in a given coordinate system leads to the occurrence of a random linear-angular error in measuring the dipole moments of the magnetic field sources, the value of which is determined by the formula:

$$\sigma = \sqrt{\sum_{i=1}^k \left(\frac{\Delta E_i}{\sqrt{3}}\right)^2} / \sum_{i=1}^k E_i,$$

where  $\Delta E_i$  is an absolute error of the signal measurement by the  $i$ -th sensor;  $E_i$  is an effective value of the useful signal in the  $i$ -th sensor.

Taking into account expression (8), the expressions for the useful signal, the final expressions for

Calculated values of the linear-angular error  $\sigma$ , %

Sensor location	R/L					
	1.5	2.0	2.5	3.0	3.5	4.0
Equatorial plane	1.23	1.00	0.86	0.76	0.70	0.65
Spherical surface	0.51	0.41	0.35	0.31	0.28	0.26
Cylindrical surface	0.18	0.15	0.13	0.12	0.11	0.10

estimating the linear-angular error of measuring the magnetic moment  $M_i$  ( $i = x, y, z$ ) of the field source with the overall size  $L_i$  in the corresponding coordinate direction by the developed devices, have the form:

$$\sigma_{x(y)} = \frac{1}{5\sqrt{3}} \left( 2 \frac{\Delta R}{R} + 4 \sin^2 \frac{\Delta \alpha}{2} \right) \cdot 100\% \quad (z = \sqrt{1/6} \cdot R),$$

$$\sigma_z = \frac{1}{5\sqrt{3}} \left( 7 \frac{\Delta R}{R} + 3 \sin^2 \frac{\Delta \alpha}{2} \right) \cdot 100\% \quad (z = \sqrt{1/6} \cdot R; R_1 = 1.31R),$$

$$\sigma_z = \frac{1}{5\sqrt{3}} \left( 9 \frac{\Delta R}{R} + 4 \sin^2 \frac{\Delta \alpha}{2} \right) \cdot 100\% \quad (z_1 = 0.361R),$$

$$\sigma_z = \frac{1}{2\sqrt{3}} \left( 9 \frac{\Delta R}{R} + 4 \sin^2 \frac{\Delta \alpha}{2} \right) \cdot 100\% \quad (z_1 = 0.3R; R_1 = 1.373R).$$

Table 2 shows the calculated values of the linear-angular component of the measurement error of the dipole magnetic moment of a field source with a linear overall size  $L = 0.6$  m at different ratios  $R/L$  ( $\Delta R = 3$  mm,  $\Delta \alpha = 3^\circ$ ).

When assessing the overall accuracy of magnetic moment measurements, it should be taken into account that measurements are often performed under difficult conditions, as well as under the influence of temperature, mechanical and electromagnetic interference. It is also necessary to take into account the features of sensor calibration, their characteristics and other factors. In this context, within the specified research area, a procedure for evaluating the uncertainties of point methods of measuring the magnetic moment has been proposed, which is reflected in the [9–10] of the authors of this paper.

At the same time, some aspects of the uncertainty evaluation require further studies.

Thus, a method and instrument for measuring the magnetic dipole moments, which implements the method, are proposed. A feature is the possibility of excluding non-informative spatial harmonics and reducing methodological error.

### Conclusions

1. Based on the multipole representation of the external magnetic field, a method of measuring the magnetic dipole moments of magnetic field sources has been developed. The method involves the use of twelve induction sensors with their placement around the source under consideration in the equatorial plane and cylindrical surface, which ensures the exclusion of non-informative signals caused by higher-order multipoles. This method is implemented with the help of a measuring instrument that has correspondingly protected measuring channels. It is intended for use in industrial conditions on stationary magnetometric stands for monitoring the parameters of field sources.

2. It has been proven that by choosing the scheme of the optimal arrangement of sensors when combining them into a single system, the influence of multipoles of higher orders is eliminated, and the methodological measurement error is reduced.

3. It was identified that the measuring instrument, the sensors of which are placed on a cylindrical surface, has better metrological characteristics compared to instruments with other variants of sensor placement, and thus it is recommended to equip stationary magnetometric stands with these instruments.

## Розробка методу і засобу вимірювання в галузі дослідження параметрів низькочастотного магнітного поля

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### Анотація

Проведено дослідження в галузі вдосконалення метрологічного забезпечення вимірювань магнітних величин шляхом розробки методу та засобу вимірювання величин дипольних магнітних моментів ( $\text{Am}^2$ ) та напруженості низькочастотного (50–1000 Гц) магнітного поля джерела. Розроблено метод вимірювання, що відноситься до індукційних, а саме так званих точкових методів вимірювань, що передбачає використання розташованих у визначених точках простору навколо досліджуваного об'єкта  $n$  первинних вимірювальних перетворювачів. Показано переваги, недоліки та сфери застосування точкових магнітометричних методів.

Метод базується на аналізі мультипольного представлення магнітного поля та використанні дванадцяти індукційних давачів, розміщених на екваторіальній площині та циліндричній поверхні. Це дозволяє виключити вплив гармонік вищих порядків і зменшити методичну похибку вимірювань.

Запропоновано систему первинних вимірювальних перетворювачів, які розташовані в точках із заданими лінійно-кутовими координатами та з'єднані певним чином. Запропоновано структурну схему вимірювальної системи. Отримано аналітичні вирази для вимірюваних параметрів – компонент дипольного магнітного моменту  $M_x$ ,  $M_y$ ,  $M_z$ . Виконано аналіз методичної похибки та доведено, що запропонований метод забезпечує точність вимірювань у межах 0,1–5,2% на відстані 1,5...4 габаритних розмірів джерела. Виконано аналіз похибок, пов'язаних із неточністю позиціонування первинних вимірювальних перетворювачів.

Запропонований метод, а також пристрій для вимірювання параметрів магнітного поля низької частоти, практично призначено для вимірювання параметрів зовнішнього магнітного поля, таких як магнітні дипольні моменти та напруженість магнітного поля, що необхідно для моніторингу й розв'язання певних науково-технологічних завдань у різних галузях технологій.

**Ключові слова:** зовнішнє магнітне поле; магнітний момент; магнітометричний метод; вимірювальна система; методична похибка.

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