UDC 001.4:006.91:621.317

Mathematical models of system-oriented measuring instruments

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Abstract

System-oriented measuring instruments (MIs) differ from traditional MIs in their ability to be integrated into complex control and automation systems. They are tools that are used in various complex technical systems to collect, analyse, transmit and use measurement data in real time. Such MIs are an important element in the implementation of the concept of Industry 4.0, where it is crucial not only to measure parameters, but also to use them for intelligent control of various technical processes.

Mathematical models allow to theoretically simulate the operation of a certain complex technical system to identify opportunities for its optimization and improvement of certain of its technical characteristics. The construction of a certain mathematical model of a technical system shall provide the maximum reflection of all the main properties of the modelled system. There are various methods for mathematical modelling of complex technical systems. However, there is no general procedure for creating such models.

The main components of system-oriented MIs are determined and their generalized structural diagrams of two types of such MIs are proposed. A generalized structural diagram of an information and measurement complex based on a combination of system-oriented MIs is presented, which allows performing measurements and processing of measurement data from numerous measurement objects.

For mathematical modelling of a system-oriented MI as a system, the graphical and analytical apparatus of the general systems theory was used. The combination of graphical and analytical interpretations of the obtained mathematical model provides the necessary information about the properties of the MI as a complex technical system. The obtained mathematical models can become the basis for evaluating the specific components of measurement uncertainties when using a system-oriented MI.

Keywords: mathematical model; hardware; software; measuring instrument; influential environment.

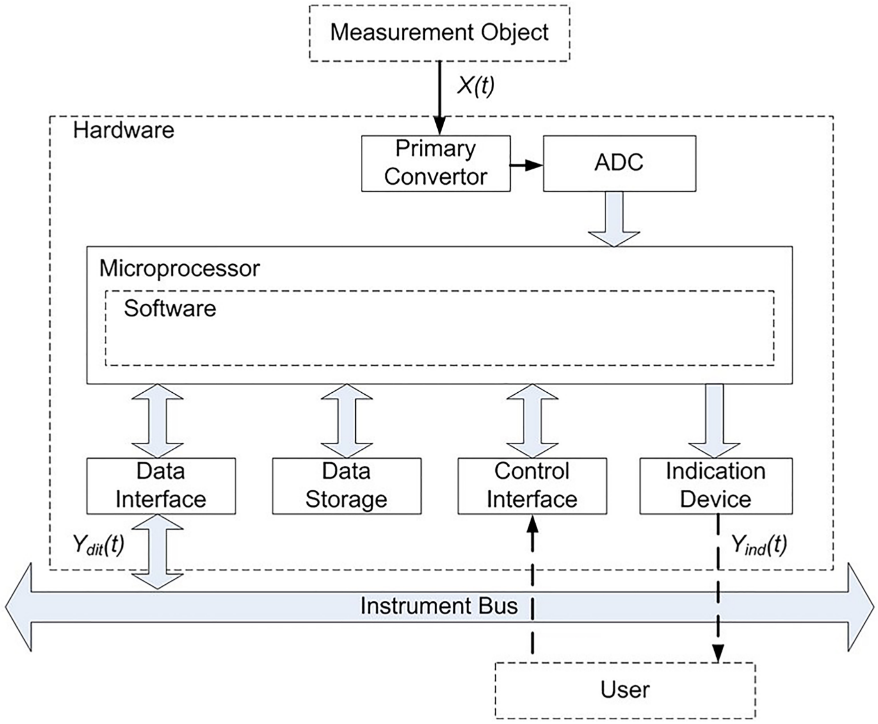
Received: 27.02.2025 Edited: 07.04.2025 Approved for publication: 10.04.2025

Introduction

System-oriented measuring instruments (MIs) differ from traditional MIs in their ability to be integrated into complex control and automation systems. They are tools that are used in various complex technical systems to collect, analyse, transmit and use measurement data in real time. Such MIs are an important element in the implementation of the concept of Industry 4.0 (The Fourth Industrial Revolution, Industry 4.0), where it is crucial not only to measure parameters, but also to use them for intelligent control of various technical processes.

Mathematical models allow to theoretically simulate the operation of a certain complex technical system to identify opportunities for its optimization and improvement of certain of its technical characteristics. The construction of a certain mathematical model of a technical system shall provide the maximum reflection of all the main properties of the modelled system. This will allow obtaining a representative model of a real technical system and to establish the features of the processes studied by it, to predict these processes in time and to determine their quantitative characteristics [1].

There are various methods for mathematical modelling of complex technical systems. However, there is no general procedure for creating such mathematical models. A brief description of the use of different models in mathematical modelling of sensors and measuring transducers is provided in [2]. Due to the digital transformation towards the Internet of Things (IoT), data-driven modelling approaches have



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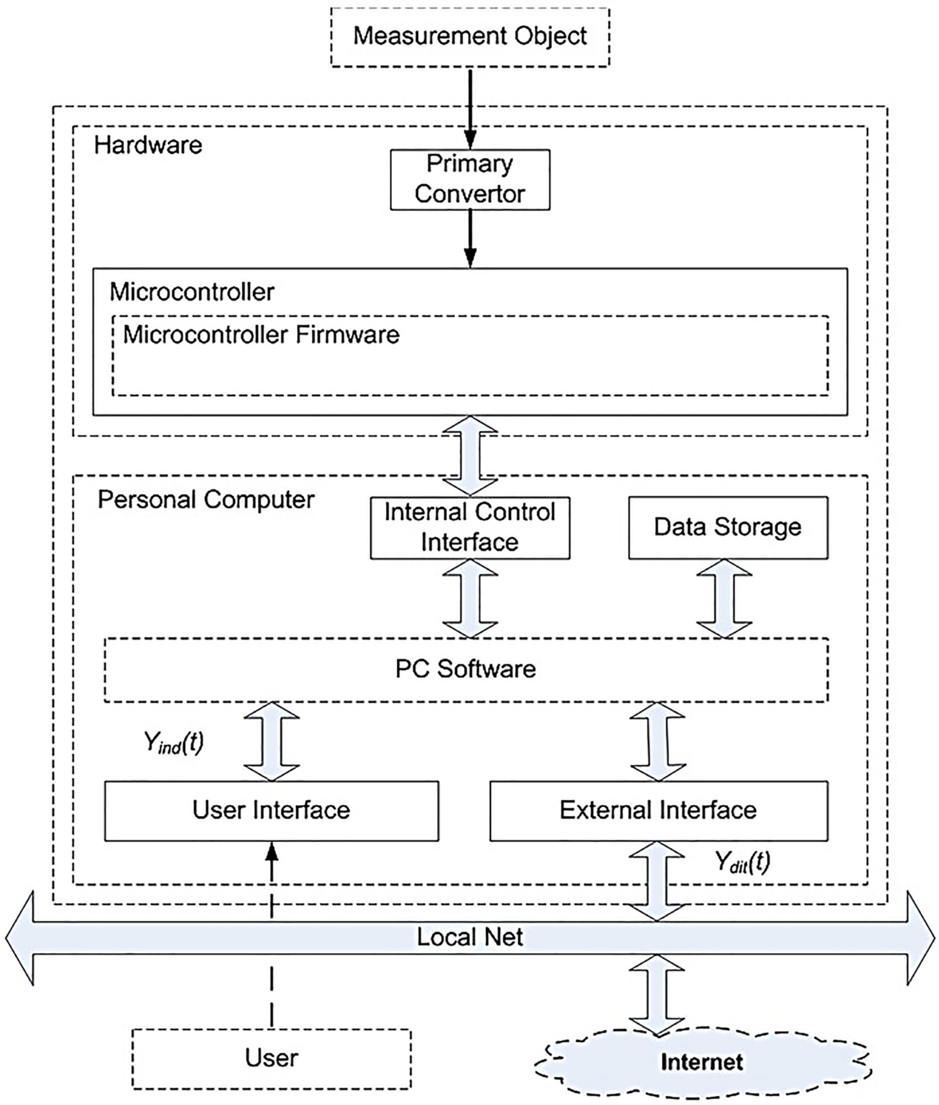

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Fig. 1. Generalized structural diagrams of two types of system-oriented MIs

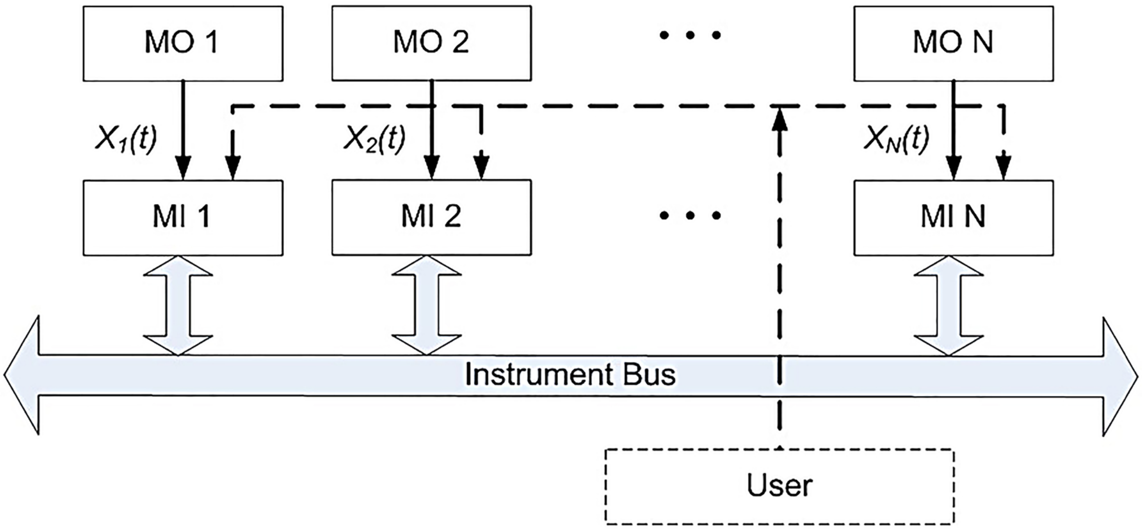

Fig. 2. Generalized structural diagram of the information and measuring complex based on system-oriented MIs

gained immense importance and this has led to the use of new approaches. A brief overview of modelling in measurement technologies from classic white box models to intelligent, data-driven cognitive solutions, with an identification of advantages and limitations, is given in [3].

Main types of system-oriented measuring instruments

The main components of system-oriented MIs are: sensors and primary converters of various physical quantities; modules for collecting and pre-processing measurement data (Analog-to-Digital Converter, ADC) for signal conversion, etc.); controllers and computing modules (microcontrollers, personal computers – PCs, etc.); communication modules and interfaces (wired and wireless communication devices, etc.); software (built into MI, installed on PCs, etc.); human-machine interaction interfaces (graphic panels, touch displays, etc.).

Generalized structural diagrams of two types of system-oriented MIs are presented in Fig. 1.

A system-oriented MI based on a microprocessor with embedded software is shown in Fig. 1, *a*. The hardware part of such a MI includes such devices as the primary convertor and ADC (for interaction with the measurement object), a control interface and an indication device (for interaction with the user), a storage device and a data exchange interface (for interaction via a common hardware bus).

A system-oriented PC-based MI is shown in Fig. 1, *b*. Such a MI includes a hardware component based on a microcontroller with firmware, the task of which is analog-to-digital conversion and primary processing of signals from the primary convertor, and the PC hardware: a data storage device, an interface for interaction with the microcontroller, an interface for interaction with the user, an interface for connecting to a local network. The measuring processes of such a MI are provided by the PC software, which includes both working programs and the operating system and corresponding hardware drivers. Using a local network, it is possible to connect to the general Internet network.

By composing individual system-oriented MIs, it is possible to create complex information and measuring complexes for measuring and processing measurement data from numerous measurement objects (MOs). A generalized structural diagram of such an information-measuring complex based on a combination of system-oriented MIs is shown in Fig. 2, which interacts with N MOs.

Features of methods of mathematical modelling of system-oriented measuring instruments

Mathematical models are used for the design of measurement systems and are also the basis for the evaluation of their measurement uncertainties according to the requirements of special international and regional guidelines [4, 5]. Key aspects of traditional modelling approaches and a discussion of their transformation into data-driven modelling are presented in [6].

A systematic and universal modelling concept, which has evolved from the idea of classical measurement chain, is proposed in [7], which provides a theoretical justification of the concept and practical procedures for its application. This concept uses only a few typical model structures that are closely related to the measurement method used. However, these general model structures can be easily adapted to any specific measurement task.

The limitations of conventional functional-oriented models are considered in [8] in the light of a generalized concept of measurement. The proposed general object-oriented model of measurement systems identifies the following classes of objects: measured object – OB; measuring instrument – MI; standard; human observer; environment. Each of the classes of objects is characterized by its own attributes and operations or functions at three levels (internal, operational, external). The interaction between them is modelled, including the connection between all classes of objects.

Of particular interest for the mathematical modelling of system-oriented MIs is the study that defines the concepts of sensory, sensory-hardware, sensory-functional, and sensory-software infrastructure of a cyber-physical system [9]. It proposes a set-theoretic model of the formation of the information state of a cyber-physical system based on the model of sensory infrastructure, the decomposition of which allows determining the current state of its hardware, functional, and software components.

Mathematical modelling of system-oriented MIs as a system and its software as a subsystem using the apparatus of general systems theory is presented in [10]. The modelling performed allows to describe the functioning of a MI and to provide its graphical interpretation of the properties of the MI as a complex technical system. The basis for such modelling was the implementation of the apparatus of general systems theory for complex organizational and technical systems [11, 12].

Proposed mathematical models of system-oriented measuring instruments

To construct hierarchical levels of a mathematical model of a system-oriented MI, a block-hierarchical approach can be used. At each hierarchical level of the system model, the concept of a subsystem and a system element can be used as a set of mathematical objects (numbers, scalar variables, vectors, matrices, graphs, etc.) and their connecting relations, reflecting the properties of a real system-oriented MI.

The general approach to building a mathematical model of a system-oriented MI as a system includes the following basic procedures:

- determining the properties of the MI for building an adequate mathematical model and collecting initial information about the selected properties of the MI;
- obtaining mathematical expressions and equations of the model structure that generally describe the relationship between the applied properties and variables for the MI;
- determining numerical values of the parameters of the mathematical model for a specific MI and assessing the accuracy and adequacy of the resulting model, etc.

Graphical representations of mathematical models of system-oriented MIs using general systems theory are shown in Fig. 3, where:

- for MIs based on a microprocessor with embedded software (Fig. 3, *a*):

Sys_{mi} is a system-oriented MI as a system;

$SubSys_{hw}$ and $SubSys_{sw}$ are the hardware and software subsystems of the system-oriented MI Sys_{mi}, respectively;

$ReEx_{us}$, $ReEx_{ds}$, $ReEx_{ci}$ are the subsystems of external influence on the user's MI, data storage

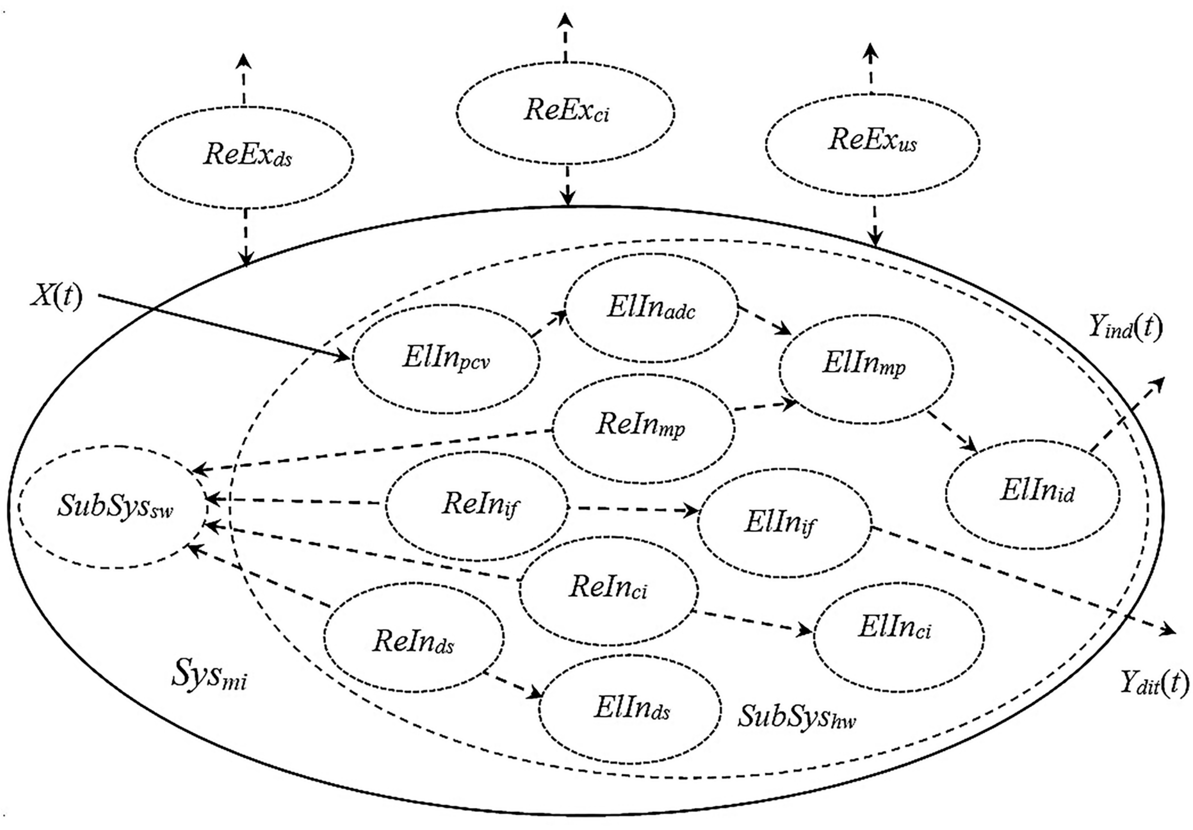

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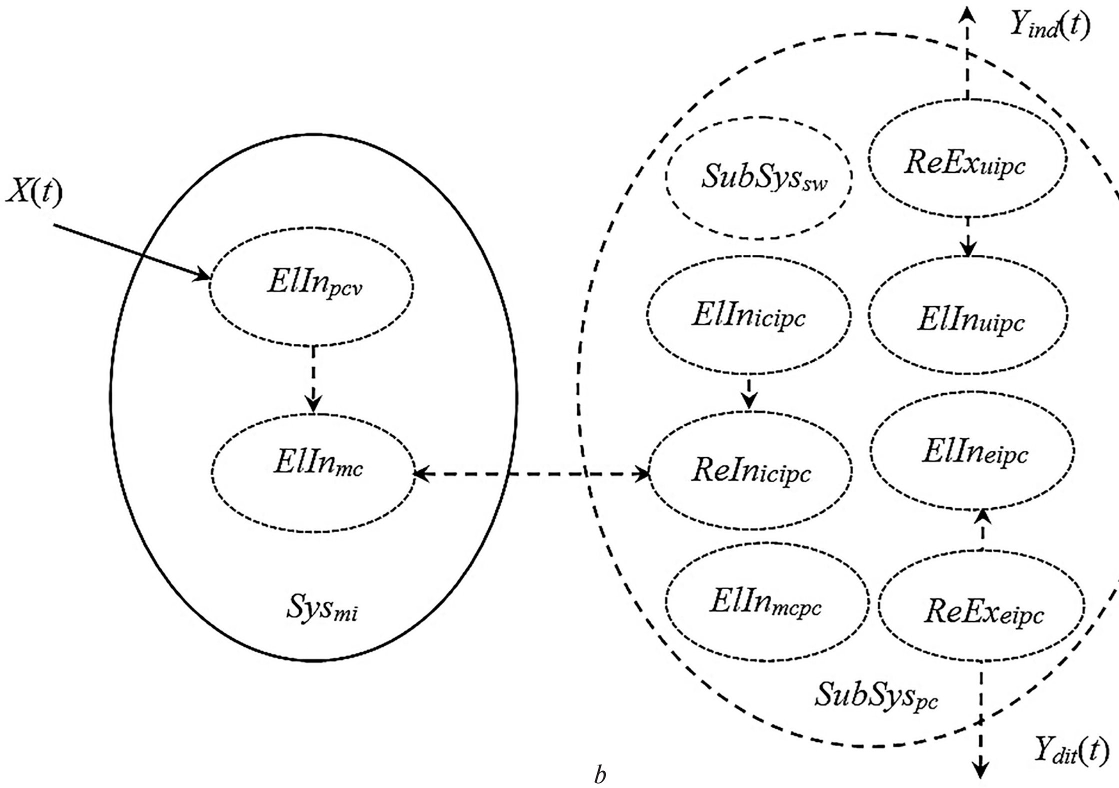
Fig. 3. Graphical representations of mathematical models of system-oriented MIs (*a*)



b

Fig. 3. Graphical representations of mathematical models of system-oriented MIs (*b*)

device, control interface of the system-oriented MI Sys_{mi}, respectively;

$ElIn_{pcv}$, $ElIn_{adc}$, $ElIn_{mp}$, $ElIn_{id}$, $ElIn_{ci}$, $ElIn_{if}$, $ElIn_{ds}$ are the elements of the primary converter, ADC, microprocessor, display device, communication device, interfaces, data storage device of the hardware subsystem $SubSys_{hw}$, respectively;

$ReIn_{ci}$, $ReIn_{if}$, $ReIn_{ds}$ are the subsystems of mutual internal influences of the microprocessor, data exchange device, interfaces, data storage device between the hardware subsystem $SubSys_{hw}$ and the software subsystem $SubSys_{sw}$, respectively;

• for MIs based on a universal PC (Fig. 3, *b*) additionally:

$SubSys_{pc}$ is a PC software subsystem;

$ElInp_{cv}$, $ElIn_{mc}$, $ElIn_{eipc}$, $ElIn_{uipc}$, $ElIn_{icipc}$ are the elements of the primary converter, microcontroller, user interface, external bindings interface, internal control interface of the subsystem $SubSys_{pc}$, respectively;

$ReIn_{uipc}$, $ReIn_{eipc}$, $ReIn_{icipc}$ are the subsystems of external influence on the MI of the user interface, the external communications interface, the internal control interface of the subsystem $SubSys_{pc}$, respectively.

Considering all the components shown in Fig. 3, the analytical expression for two types of system-oriented MIs as complex technical systems [8–10] using general systems theory will take the form:

• for MIs based on a microprocessor with embedded software:

$$Sys_{mi1} \equiv \left\langle SubSys_{hw1}, SubSys_{sw}, Re_{hwsw}, Trg_{hw1}, Trg_{sw}, \Delta T_{mi} \right\rangle; \quad (1)$$

• for MIs based on a universal PC:

$$Sys_{mi2} \equiv \left\langle SubSys_{hw2}, SubSys_{pc}, Re_{hwpc}, Trg_{hw2}, Trg_{pc}, \Delta T_{mi} \right\rangle, \quad (2)$$

where Re_{hwsw} and Re_{hwpc} are the subsystem of interconnections between hardware and software subsystems of the system-oriented MI of the first type and subsystem of interconnections between hardware and software subsystems of the PC of the system-oriented MI of the second type, respectively;

Trg_{hw}, Trg_{sw}, Trg_{pc} are the subsystems of established sets of targets for the functioning of hardware and software subsystems of system-oriented MI, PC subsystem, respectively;

ΔT_{mi} is the period during which the modification of the MI as a system works as intended.

As can be seen from the graphical representations of the mathematical models of the two types of system-oriented MIs in Fig. 3, these models have significant differences. Therefore, the analytical expressions for these models also differ significantly.

Analytical expressions for subsystems of two types of system-oriented MIs, considering their internal and external connections, have the form:

• for MIs based on a microprocessor with embedded software:

$$SubSys_{hw1} \equiv \left\langle ElIn_{pcv}, ElIn_{adc}, ElIn_{id}, ElIn_{mp}, ElIn_{ci}, ElIn_{if}, ElIn_{ds}, ReIn_{mp}, ReIn_{ci}, ReIn_{if}, ReIn_{ds} \right\rangle; \quad (3)$$

• for MIs based on a universal PC:

$$SubSys_{pc} \equiv \left\langle ElIn_{uipc}, ElIn_{eipc}, ElIn_{icipc}, ElIn_{mcpc}, ReEx_{uipc}, ReEx_{eipc}, ReEx_{icipc} \right\rangle. \quad (4)$$

It should be noted separately that the subsystem $SubSys_{sw}$ in two types of system-oriented MIs has significant differences. In a system-oriented MI based on a microprocessor with embedded software, it is a component of the system Sys_{mi}, and in a MI based on a universal PC, it is the PC software subsystem $SubSys_{pc}$. In this regard, the properties of these subsystems and the corresponding risks of the functioning of the system Sys_{mi} as a whole are assessed according to different procedures using special international and regional guidelines [13, 14].

Summary

For mathematical modelling of system-oriented MIs as a system, the graphical and analytical apparatus of the general systems theory can be used. The combination of graphical and analytical interpretations of the obtained mathematical model provides the necessary information about the properties of the MIs as a complex technical system. The resulting mathematical models have significant differences, and therefore the analytical expressions for these models also differ significantly. They can become the basis for evaluating the specific components of measurement uncertainties when applying system-oriented MIs.

Математичні моделі системно-орієнтованих засобів вимірювальної техніки

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Анотація

Системно-орієнтовані засоби вимірювальної техніки (ЗВТ) відрізняються від традиційних ЗВТ здатністю інтегруватися в складні системи керування та автоматизації. Це інструменти, які використовуються в різних складних технічних системах для збору, аналізу, передачі та використання даних вимірювань у реальному часі. Такі ЗВТ є важливим елементом у реалізації концепції Індустрії 4.0, де важливо не тільки вимірювати параметри, а й використовувати їх для інтелектуального керування різними технічними процесами.

Математичні моделі дозволяють теоретично змоделювати роботу певної складної технічної системи з метою виявлення можливостей її оптимізації та покращення окремих її технічних характеристик. Побудова певної математичної моделі технічної системи має передбачати максимальне відображення всіх основних властивостей модельованої системи. Існують різні методи математичного моделювання складних технічних систем. Однак загальної методології створення таких математичних моделей не існує.

Встановлено основні компоненти системно-орієнтованих ЗВТ та запропоновано їх узагальнені структурні схеми двох типів таких ЗВТ. Наведено узагальнену структурну схему інформаційно-вимірювального комплексу на основі комбінації системно-орієнтованих ЗВТ, який дозволяє проводити вимірювання та обробку даних вимірювань від великої кількості об’єктів вимірювання.

Для математичного моделювання системно-орієнтованого ЗВТ як системи використовувався графоаналітичний апарат загальної теорії систем. Поєднання графічної та аналітичної інтерпретацій отриманої математичної моделі дає необхідну інформацію про властивості ЗВТ як складної технічної системи. Отримані математичні моделі можуть стати основою для оцінки специфічних компонентів невизначеностей вимірювань при використанні системно-орієнтованого МІ.

Ключові слова: математична модель; обладнання; програмне забезпечення; вимірювальний прилад; впливове середовище.

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