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New KCRV calculation method and validation methodology of the calculation algorithm

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Abstract

The purpose of this paper is to develop a new method for calculating the reference value of key comparisons (KCRV) to address the issue of unaccounted systematic uncertainty (Dark uncertainty) of comparison participants, as well as of a highly outdated approach when evaluating the systematic component of the uncertainty. Both of these issues negatively affect the accuracy of calculating the KCRV using the "weighted mean adjusted by threshold value" recommended in the rules for conducting key comparisons.

The paper presents a resulting developed algorithm for calculating the KCRV, which allows to significantly reduce the impact of "Dark uncertainty" on the calculation accuracy, as well as the effects of the outdated approach when evaluating the systematic component of the uncertainty. This was made possible by using a new method for calculating the weighted mean of the difference between measurement results and KCRV (calculated using the method currently used), a new approach to calculating the difference from KCRV, and by the development of a new formula for calculating the weighted mean.

In addition, special attention and relevance is paid to the modelling of comparisons.

The developed comparison models allowed one to assess the effectiveness of the calculation method in comparison with the available one.

Keywords: KCRV; key comparisons; modelling; Dark uncertainty; algorithm for calculating; DoE; standard uncertainty.

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Introduction

Increasing the accuracy of KCRV calculation is the most relevant task for modern metrology. This is because the degree of equivalence (DoE) of the leading national metrological laboratories is determined during international key comparisons of physical quantities, and is calculated as the difference between the results of measurements of a physical quantity in a laboratory and the KCRV itself, which is calculated based on the measurement results of all national metrological laboratories participating in a comparison.

The calculation of DoE has an impact on the publication of CMCs [1] that determine calibration capabilities of national metrological laboratories, which are at the top of the metrological hierarchy in those types of measurements where these laboratories are the main ones, as well as they determine the metrological level of these measurements worldwide. The uncertainties declared in the CMCs shall not be lower than DoE of a laboratory [2], being determined in the published comparison reports, based on which the CMCs are published.

Currently, the KCRV is calculated in accordance with the recommendations of the CIPM Consultative Committees [3]. The value of KCRV in key comparisons with the number of participants N is calculated as the weighted mean adjusted by threshold value:

$$x_{KCRV} = \frac{\sum_{i=1}^{N} \frac{x_i}{u_{adj}^2(x_i)}}{\sum_{i=1}^{N} \frac{1}{u_{adj}^2(x_i)}}, \quad (1)$$

where x_i is the result of measurements of a physical quantity of a key comparison participant (i); $u_{adj}(x_i)$ is uncertainties of x_i after adjustment by threshold value.

The adjustment is performed as follows.

The average value of the declared uncertainties of all comparison participants is calculated. The declared uncertainties that are not less than the average remain unchanged. The others are changed to the average value.

For comparisons, formula (1) is used, where there is only one artifact for measurements performed by all participants.

If each participant uses his artifact [4] and measures it with the result x_i, and the pilot laboratory measures all artifacts with the result x_{ip}, then the relative difference is determined as:

$$\Delta_i = \frac{x_i}{x_{ip}} - 1. \tag{2}$$

$u_{adj}(\Delta_i)$ is determined according to recommendations [4].

$$\Delta_{KCRV} = \frac{\sum_{i=1}^{N} \frac{\Delta_i}{u_{adj}^2(\Delta_i)}}{\sum_{i=1}^{N} \frac{1}{u_{adj}^2(\Delta_i)}}. \tag{3}$$

The unilateral DoE is defined as:

$$D_i = \Delta_i - \Delta_{KCRV}. \tag{4}$$

Development of a new method for calculating the KCRV

To significantly reduce the impact of "Dark uncertainty" in the standard uncertainties declared by the participants of comparisons, as well as the impact of a highly outdated approach to the evaluation of the systematic component of the uncertainty, special attention is paid to the use of such a parameter as the power difference of the measurement results x_i from x_{KCRV}.

Based on it, a new algorithm for calculating the KCRV was developed:

$$x^{new}_{KCRV} = \frac{\sum_{i=1}^{N} x_i^{((\log_{x_i} x_{KCRV})^2)}}{N}. \tag{5}$$

For key comparisons where each participant uses their artifact and measures only it with the result x_i, and the pilot laboratory measures all artifacts with the result x_{ip}:

$$\Delta^{new}_{KCRV} = \frac{\sum_{i=1}^{N} \Delta_i^{((\log_{\Delta_i} \Delta_{KCRV})^2)}}{N}. \tag{6}$$

Method for analysing the KCRV calculation algorithms based on key comparison modelling

The models were developed in stages.

Stage 1. The true value of a physical quantity was assigned to a virtual artifact, for example, the luminous flux of a precision lamp *Φ true* = 1000 Lm, or 2000 Lm.

Stage 2. The number of comparison participants was assigned, and so randomly was the uncertainty value of each participant in the range that is typical for this measurement type for previously conducted key comparisons.

Table 1

Results of comparison modelling and analysed KCRV calculation methods

***Φ true*, lm**	**1000**	***u*, %**		***Φ true*, lm**	**1000**	***u*, %**
1	1005	0.35		**1**	1001	0.35
2	998.3	0.5		**2**	998.3	0.5
3	997	0.15		**3**	997	0.15
4	997	0.2		**4**	997	0.2
5	1003.2	0.4		**5**	1002.2	0.3
6	1003	0.5		**6**	1002	0.3
7	997.5	0.25		**7**	997.5	0.25
8	993.4	0.5		**8**	993.4	0.5
Average	**999.3**			**Average**	**998.55**	
Weighted mean adjusted by threshold value	**999.604**			**Weighted mean adjusted by threshold value**	**998.970**	
New method	**999.925**			**New method**	**999.399**	
***Φ true*, lm**	**1000**	***u*, %**		***Φ true*, lm**	**2000**	***u*, %**
1	995	0.35		1	1995	0.7
2	1001.7	0.5		**2**	1998	0.2
3	1003	0.15		**3**	2003	0.4
4	1003	0.2		**4**	2001	0.2
5	996.8	0.4		**5**	2005	0.3
6	997	0.5		**6**	1993	0.4
7	1002.5	0.25		**7**	1996	0.3
8	1006.6	0.5				
Average	**1000.7**			**Average**	**1998.714**	
Weighted mean adjusted by threshold value	**1000.395**			**Weighted mean adjusted by threshold value**	**1999.348**	
New method	**1000.107**			**New method**	**1999.991**	

Table 2

Results of comparison modelling and analysed KCRV calculation methods

***Φ true*, lm**	**1000**	***u*, %**		***Φ true*, lm**	**1000**	***u*, %**
1	993	0.7		**1**	998.3	0.3
2	1002.5	0.4		**2**	1005.3	0.5
3	1008.4	0.6		**3**	1002.8	0.3
4	995.6	0.5		**4**	996.1	0.5
5	994.7	0.5		**5**	999.3	0.2
Average	**998.84**			**Average**	**1000.36**	
Weighted mean adjusted by threshold value	**998.964**			**Weighted mean adjusted by threshold value**	**1000.279**	
New method	**999.127**			**New method**	**1000.210**	
***Φ true*, lm**	**1000**	***u*, %**		***Φ true*, lm**	**1000**	***u*, %**
1	1005	0.35		**1**	1005	0.35
2	998.3	0.5		**2**	998.3	0.5
3	994	0.35		**3**	994	0.35
4	991	0.4		**4**	991	0.4
5	1007.2	0.35		**5**	1007.2	0.35
6	1003	0.5				
7	990	0.5				
8	993.4	0.5				
Average	**997.7375**			**Average**	**999.1**	
Weighted mean adjusted by threshold value	**997.966**			**Weighted mean adjusted by threshold value**	**999.256**	
New method	**998.241**			**New method**	**999.457**	
***Φ true*, lm**	**1000**	***u*, %**		***Φ true*, lm**	**1000**	***u*, %**
1	991	0.4		**1**	997	0.2
2	1007.2	0.35		**2**	1002.2	0.3
3	1003	0.5		**3**	1002	0.3
4	990	0.5		**4**	997.5	0.25
5	993.4	0.5		**5**	993.4	0.5
Average	**996.92**			**Average**	**998.42**	
Weighted mean adjusted by threshold value	**997.107**			**Weighted mean adjusted by threshold value**	**999.125**	
New method	**997.349**			**New method**	**999.843**	

Stage 3. In the ranges around *Φ true,* Lm ± *u,* Lm*,* the value of the measurement result of each comparison participant was randomly assigned.

The KCRV was calculated in three ways: 1. Average value. 2. Weighted mean adjusted by threshold value. 3. A new method (5) was developed.

Tables 1–2 show the results of the comparison modelling and the analysis of KCRV calculation methods.

Conclusions

Modern KCRV calculation algorithms were analysed.

A new KCRV calculation algorithm was developed, which significantly reduces the impact of "Dark uncertainty" and outdated approach to the uncertainty evaluation.

A methodology for analysing the calculation algorithms based on key comparison modelling was developed. 10 comparison models were developed, some of which accounted for the challenges related to "Dark uncertainty", as well as a highly outdated approach to evaluating the systematic component of the uncertainty.

Using the developed models, the methods for in key comparisons were studied, and a new method was developed. The results indicate the improved accuracy of KCRV calculation using the new method.

It is planned to further develop computer programs for generating the comparison models.

Новий метод розрахунку KCRV та методика валідації алгоритмів розрахунку

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Анотація

Метою цієї роботи є розробка нового методу розрахунку еталонного значення ключових звірень (KCRV) для вирішення проблеми неврахованої систематичної невизначеності (Dark uncertainty) учасників звірень, а також проблеми занадто консервативного підходу до оцінювання систематичної складової невизначеності. Обидві ці проблеми негативно впливають на точності обчислювання KCRV методом середньозваженого значення за невизначеністю кожного учасника звірень, корегованим за пороговим значенням, який є рекомендованим у правилах проведення ключових звірень.

Наведено результати розробки алгоритму розрахунку KCRV, який дозволяє значно знизити вплив "Dark uncertainty", а також занадто консервативного підходу до оцінювання систематичної складової невизначеності на точність розрахунку. Це стало можливим завдяки використанню нового підходу до розрахунку відмінності від KCRV, а також розробці нової формули розрахунку середньозваженого значення.

Також особливу увагу та актуальність викликає моделювання звірень. Створення численних моделей на основі віртуально заданого справжнього значення фізичних величин артефактів, які звичайно використовуються в консультативних комітетах та в регіональних організаціях за видами вимірювань, на основі віртуально заданих значень заявленої невизначеності та результатів вимірювань кожного віртуального учасника ключових звірень, таким чином, щоб вони відповідали реальним значенням невизначеності та результатам уже проведених ключових звірень, а також їх методології відбору моделей, дозволяє проводити аналіз алгоритмів розрахунку і порівнювати метод, який нині використовується, з новими розробленими методами.

Розроблені моделі звірень дозволили оцінити ефективність методу розрахунку в порівнянні з існуючим методом.

Ключові слова: KCRV; ключові звірення; моделювання; Dark uncertainty; алгоритм розрахунку; DoE; стандартна невизначеність.

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