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Uncertainty of determining the Horst coefficient when processing and analysing thermographic images

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Abstract

Fractal forms are widespread in nature, for example, riverbeds, mountain ranges, coastlines, fractal structures of sets and random processes, etc. Fractal features can also be found in the structure of physical fields (thermal, acoustic, optical, etc.) and informative signals and functions that describe the distribution of physical quantities in space and time sequences of their measurement results. To analyse such objects, special methods of fractal geometry are used, which make it possible to effectively solve problems that, within classical methodological concepts, may cause certain difficulties. By using these methods when processing and analysing thermographic images, it will be possible to automate the detection of temperature anomalies in them and predict the development of defects. The main dynamics of fractal systems are often completely determined only by one characteristic of self-similarity – the Horst index. In this paper, the normalized range method to determine the index is used. It is known that the Horst index depends on the characteristics of the probability distribution. Therefore, to assess its accuracy and, ultimately, the reliability of the control of products made of non-metallic heterogeneous materials, the goal is to develop a method for calculating the measurement uncertainty of this index.

Keywords: non-destructive thermal imaging testing; reliability testing; testing automation; measurement accuracy; metrological characteristics.

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Introduction

Today, there is an ever-increasing interest in fractal geometry. This is due to the fact that it allows one to effectively solve problems that, within classical methodological concepts, may cause certain difficulties.

One of the central concepts of these methods is a measure of the structural nature of an object – fractal dimension, which is also known as the Hausdorff-Bezikovitch dimension. This concept was introduced by Mandelbort to measure fractal objects, which may include the surfaces of real physical objects, on which irregularities are randomly arranged [1]. In fact, this is a parameter that can be used to give an integral characteristic of the surface of the object under consideration or an image with a single number, regardless of the spatial scale and resolution of the measuring equipment used [2, 3].

In [4–6], the possibility of using fractal dimension in determining geometric parameters of defects in the structure of non-metallic heterogeneous materials and predicting their development by the complexity of the contour (defect display on the thermogram) is described. The use of a fractal approach in processing and analysing thermographic images will provide the possibility of automating the detection of temperature anomalies on them and predicting the development of defects.

A mandatory requirement for fractal objects is the fractional metric dimension. Its value shall be strictly greater than the topological one. It should be noted that the values of the fractional dimension are clearly determined only for classical fractals, for example, for the Koch snowflake. As for calculations of the fractal dimension of other objects, there are several methods of fractal geometry. In this paper, the method of determining the fractal dimension using the empirical Horst index is used.

The basic dynamics of the system are often completely determined by a single self-similarity characteristic – the Horst index (or exponent) [7]. Its value can be used to classify a series of sequential data as persistent, trend-stable, anti-persistent with expected changes in trends, or a random process [8].

The Horst exponent is a number $H \in [0.1]$, which characterizes the ratio of the component of the trend function to white noise. It can be used in the classification of time series when establishing non-random time series with a stable trend and random series (including non-Gaussian) [9]. With such an analysis, the following options are possible [7]:

• the Horst coefficient of time series less than 0.5 corresponds to anti-persistent series, if the system demonstrates growth in the previous period, then with a probability that is greater than the Horst coefficient less than 0.5, a decline will begin in the next period;

• with the Horst coefficient value of 0.5, a clear trend is not expressed, independence (absence of any memory of the past) of the time series values is demonstrated;

• with the Horst coefficient value of more than 0.5, non-randomness of an event is assumed, the series demonstrates trend-resistant behaviour, that is, if the series grows (falls) in the previous period, then with a probability that is greater than the Horst coefficient more than 0.5, it will maintain this trend in the next period further on.

The closer the value of the specified coefficient is to 1, the more persistent or trend-resistant the series is. If there is a clear trend of the time series to increase or decrease, then it is likely to persist in the future [10, 11–13]. It should be noted that to more or less accurately determine the value of the Horst index, the time series shall be long enough. At the same time, as noted in [14], for short information signals under consideration (corresponding to the processes of implementing thermal control of non-metallic heterogeneous materials), it is better to use the value of the constant $c = \frac{\pi}{2}$. By doing so, it will be possible to evaluate the properties of the information signal with greater probability and determine the conditions when it becomes persistent ($H > 0.5$).

When analysing thermograms obtained using infrared devices for flaw detection and defectometry of products made of non-metallic heterogeneous materials, it is possible to use changes in multifractal characteristics as a signal about the presence of sections with a characteristic change in this intensive parameter on the temperature dependence graphs. The latter indicates the presence of temperature anomalies on the thermogram, which is a sign of a hidden defect in the tested object. This opens up new possibilities in terms of automating the processing of thermograms obtained from infrared devices for thermal testing of products made of non-metallic heterogeneous materials.

Normalized range method for calculating the Hurst coefficient

There are many methods for determining the Hurst coefficient. One of the most frequently used in practice is the normalized range method. According to it, if a series of consecutive data is divided into ν sections of equal length, then the Hurst index can be determined as follows [9]:

$$\left(\frac{R}{S}\right)_n \approx cn^H, \quad (1)$$

where $\left(\frac{R}{S}\right)_n$ is a normalized range from the accumulated mean; S is the standard deviation of a series of observations; R is the range of accumulated deviation; n is the number of time points or number of observations; c = const, independent of n.

Let us consider the algorithm for implementing R/S analysis. Let there be a sample in the form of a series of consecutive values. It is desirable that the number of observed values be sufficiently high. Using logarithmic ratios, the specified series with the value of levels of length n is transformed:

$$\log\frac{U_i}{U_{i-1}}. \quad (2)$$

Let us find the value of the smallest proper divisor m (the number of elements in each group) for length n in such a way that it is not less than 10. Then let us denote the number of groups by $k = n/m$, and the elements of each of these groups by t_i. The average value of the series for each group will be [10]:

$$\overline{t_i} = \frac{1}{m}\sum_{i=1}^{m} t_i, \frac{1}{m}\sum_{i=m+1}^{2m} t_i \ldots \frac{1}{m}\sum_{i=(k-1)m+1}^{n} t_i. \quad (3)$$

In turn, the accumulated deviations from the X_i mean are calculated using the formula:

$$X_1 = t_1 - \frac{1}{m}\sum_{i=1}^{m} t_i,$$
$$X_2 = \left(t_2 - \frac{1}{m}\sum_{i=1}^{m} t_i\right) + X_1 \ldots \quad (4)$$
$$X_m = \left(t_m - \frac{1}{m}\sum_{i=1}^{m} t_i\right) + X_{m-1}.$$

For each group, the normalized range is calculated according to the expression:

$$R_k = X_{i\max} - X_{i\min}. \quad (5)$$

The biased standard deviation for each group is determined by the well-known formula:

$$S_k = \sqrt{\frac{1}{m}\sum_{i=1}^{m}\left(t_i - \overline{t}\right)^2}. \quad (6)$$

For each group, an R/S indicator is calculated. The average value of this indicator will be:

$$R/S_j = \frac{1}{k}\sum_{i=1}^{k} R/S_i. \quad (7)$$

This procedure is repeated, using all possible proper divisors as m. In the last step $m = n/2$ [10]. The next stage is to construct a linear regression equation, in which the variable is the logarithm of the R/S

index, and the factorial feature is the logarithm of the number of elements in the j-th group k [10]:

$$\log R/S = \log c + H \cdot \log k. \quad (8)$$

The parameters of equation (8) are found by the least squares method.

Uncertainty in determining the Hurst coefficient

The Horst coefficient is determined by the R/S-statistics method, which is based on real measurements of the characteristics of a series of consecutive data. However, it should be noted that the known formulas that allow calculating this parameter demonstrate its dependence on the characteristics of the probability distribution [15, 16]. Therefore, to assess the accuracy of the Horst coefficient and, ultimately, the reliability of the control of products made of non-metallic heterogeneous materials, the goal is to develop a method for calculating the measurement uncertainty of this indicator. To do so, first, one shall consider the algorithm for determining the Horst coefficient (Fig. 1).

The measurement uncertainty of the Horst coefficient is calculated by the expert method [17], accounting for incomplete information about the influencing quantities (components of the uncertainty budget).

The combined uncertainty u_C is calculated by the formula [17]:

$$u_C = \sqrt{u_A^2 + u_B^2}, \quad (9)$$

where u_A is type A uncertainty; u_B is type B uncertainty.

The first group of errors is associated with the dependence of the Hurst coefficient on the parameters of the probability distribution of a series of consecutive values and random influences when reading the readings while performing real measurements. The second group of errors is associated with the algorithm for determining the Hurst coefficient, including the accuracy of finding the normalized range, the standard deviation for each period, the number of periods into which a series of consecutive data is divided, the accuracy of constructing the linear regression equation, etc.

The standard uncertainty u_A of the determination of the Horst index is defined as the standard deviation of the measurement result and, in the case when the measurement result is evaluated as the arithmetic mean, is calculated by the expression:

$$u_A = \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}\left(H_i - \bar{H}\right)^2}, \quad (10)$$

where n is the number of observations; H_i is the i-th value of the Horst index; $\bar{H}$ is the average value of the Horst index.

The total value of the standard uncertainty for type B is calculated as follows: [17]:

$$u_B = \sqrt{\sum_{i=1}^{m}\left(\frac{\partial f}{\partial x_i}\right)^2 u_B^2(x_i)}, \quad (11)$$

where $\left(\frac{\partial f}{\partial x_i}\right)$ is partial derivatives of the function f against arguments x_i.

Taking formula (8) and assuming that the measured quantity is the R/S indicator, the sensitivity coefficient (derivative of the function H against the R/S variable) will have the form:

$$\frac{\partial H}{\partial(R/S)} = \left(\frac{\log(R/S) - \log(c)}{\log(k)}\right)'_{R/S} = \frac{1}{\log(k)\dfrac{R}{S}}. \quad (12)$$

Then formula (11) will be written as:

$$u_B = \sqrt{\left(\frac{1}{\log(k)\dfrac{R}{S}}\right)^2 \cdot u_B^2\left(\frac{R}{S}\right)}. \quad (13)$$

Then the combined measurement uncertainty of the Horst parameter will be determined by the formula:

$$u_C(H) = \sqrt{u_A^2(H) + \left(\frac{1}{\log(k)\dfrac{R}{S}}\right)^2 \cdot u_B^2\left(\frac{R}{S}\right)}. \quad (14)$$

As it is known, the expanded uncertainty U for the confidence level P is calculated as:

$$U = k \cdot u_C, \quad (15)$$

where k is the coverage coefficient, which depends on the confidence level P and the number of degrees of freedom v_{eff}, which is determined by the formula:

$$v_{eff} = \frac{u_c^4}{\sum_{i=1}^{m}\frac{u^4(x_i)}{v_i}\left(\frac{\partial f}{\partial x_i}\right)^4}, \quad (16)$$

moreover $v_i = n_i - 1$ – for type A uncertainties; $v_i = \infty$ – for type B uncertainties [17].

Conclusions

The possibility of using fractal geometry methods when testing products made of non-metallic heterogeneous materials using active thermography methods is shown. A fractal approach when processing and analysing thermographic images will provide the possibility of automating the detection of temperature anomalies on them and predicting the development of defects.

It is noted that the main dynamics of fractal systems is often completely determined by one characteristic of self-similarity – the Horst index. The normalized range method for determining the empirical Horst index is described in detail. Special attention is paid to the development of methods for calculating measurement uncertainties. General issues of calculating measurement uncertainties are considered.

A method for calculating the uncertainty of determining the Horst coefficient is developed. It is noted

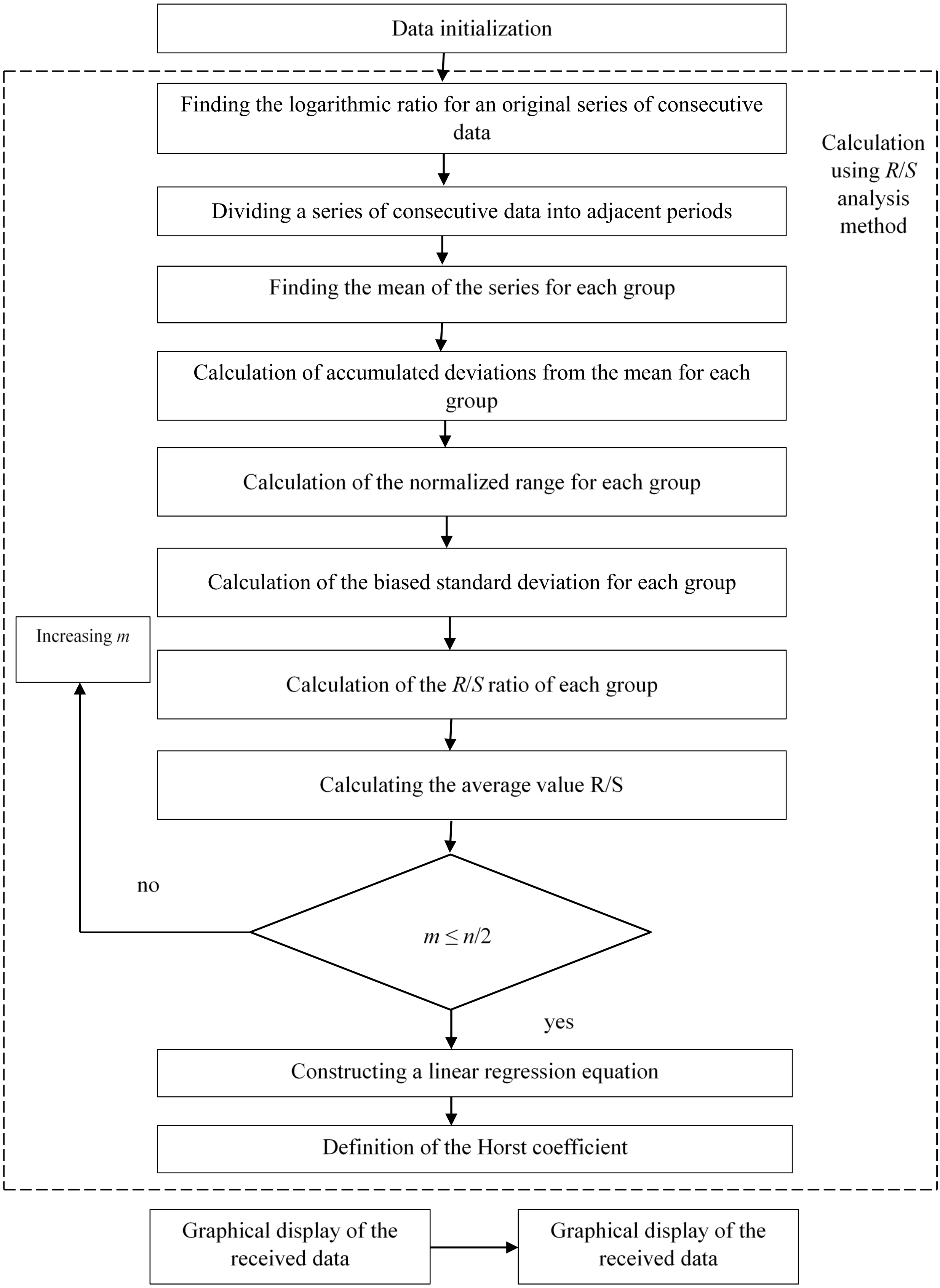

Fig. 1. Algorithm for determining the Horst coefficient

that the first group of errors is associated with the dependence of the Horst coefficient on the parameters of the probability distribution of a number of consecutive values and random influences when reading the readings while performing real measurements. The second group of errors is associated with the algorithm for determining the Horst coefficient, including the accuracy of finding the normalized range, the standard deviation for each period, the number of periods into which a series of consecutive data is divided, the accuracy in constructing the linear regression equation, etc.

This technique is versatile and can be used not only in determining the metrological characteristics of methods for controlling products made of non-metallic heterogeneous materials, but also for calculating the accuracy of determining the characteristics of fractal and multifractal objects of any nature.

An experiment was conducted to test the proposed approach to processing and analysing thermograms. As a result, the fractal dimensions of consecutive data series with a known Horst coefficient were determined. Each of the studied signals was analysed using the box counting method (cell fractal dimension method). It was noted that the value of this characteristic parameter proportionally (inversely) responds to the changes in the Horst coefficient value. At the same time, the expanded uncertainty values were obtained for a significance level of 0.05. The largest measurement uncertainty value was obtained for the Horst coefficient $H = 0.5$, and it does not exceed 10%.

Невизначеність встановлення коефіцієнта Гьорста при обробці та аналізі термографічних зображень

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Анотація

У природі дуже поширені фрактальні форми, наприклад, русло річки, рельєф гірських масивів, берегова лінія, фрактальні структури множин та випадкових процесів. Також фрактальні ознаки можуть бути знайдені в структурі фізичних полів (теплових, акустичних та оптичних тощо) та інформативних сигналів і функцій, що описують розподіл фізичних величин у просторі та часових послідовностей результатів їхніх вимірювань. Для аналізу таких об'єктів використовуються спеціальні методи фрактальної геометрії, що дають можливість ефективно розв'язувати задачі, які в рамках класичних методологічних концепцій викликають певні труднощі. Використання цього підходу при обробці та аналізі термографічних зображень забезпечить можливість автоматизації виявлення температурних аномалій на них та прогнозування розвитку дефектів. Під час аналізу термограм, отриманих за допомогою інфрачервоних приладів для дефектоскопії та дефектометрії виробів із неметалевих гетерогенних матеріалів, можна використовувати зміни мультифрактальних характеристик як сигнал про наявність ділянок із характерною зміною цього інтенсивного параметра на графіках температурної залежності. Останнє вказує на наявність температурних аномалій на термограмі, що є ознакою прихованого дефекту в об'ємі об'єкта, що контролюється. У багатьох випадках основна динаміка фрактальних систем повністю визначається однією характеристикою самоподібності – показником Гьорста. У роботі для визначення цього показника використовується метод нормованого розмаху. Відомо, що показник Гьорста залежить від характеристик ймовірнісного розподілу. Тому для оцінювання його точності та, зрештою, достовірності контролю виробів, виготовлених із неметалевих гетерогенних матеріалів, було поставлено мету розробити методику розрахунку невизначеності вимірювання цього показника.

Ключові слова: неруйнівний тепловізійний контроль; достовірність контролю; автоматизація контролю; точність вимірювання; метрологічні характеристики.

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